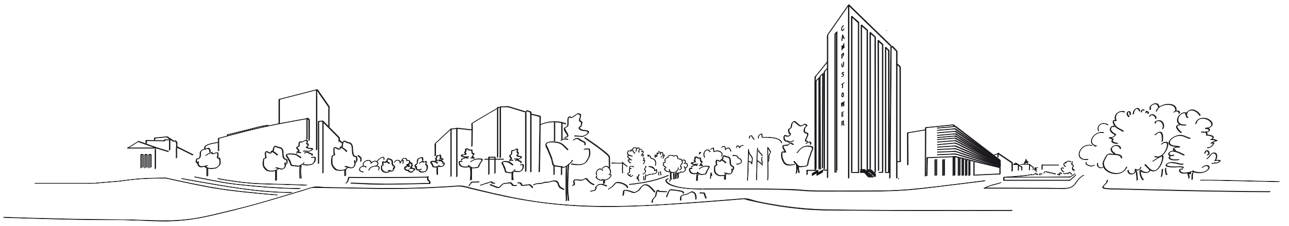
Generalized Continua as Models for Materials with Multi-Scale-Effects or under Multi-Field-Actions

September, 21st–25th 2015
Experimental Factory, Magdeburg, Germany

Organized by Holm Altenbach & Samuel Forest

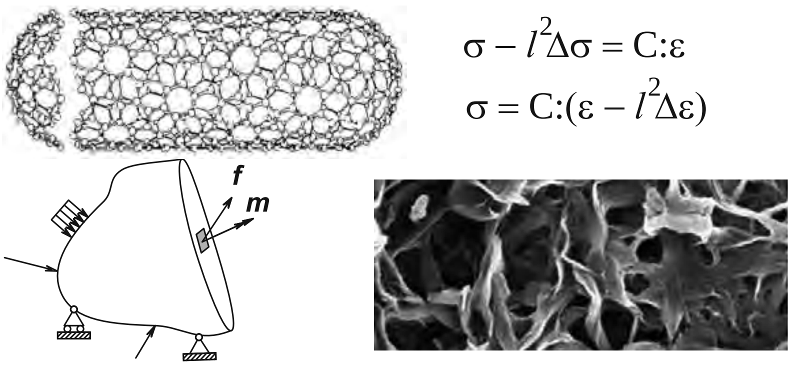


Book of Abstracts

Generalized Continua as Models for Materials with Multi-Scale-Effects or under Multi-Field-Actions

Advanced Seminar 2015

Edited by Holm Altenbach & Samuel Forest



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Preface

Generalized Continua have been in the focus of scientists from the end of the 19th century. A first summary was given in 1909 by the Cosserat brothers. After World War II, a true renaissance in this field occurred with a publication of Ericksen & Truesdell in 1958. Further developments were connected with the fundamental contributions of scientists from Germany, Russia, and France. During the last years, by two colloquia the centennial of the Cosserat book (both held in Paris in 2009) was celebrated. In addition, previous trilateral seminars *Mechanics of Generalized Continua - from Micromechanical Basics to Engineering Applications* (Wittenberg, 2010, 2012) and the CISM Course *Generalized Continua - from the Theory to the Engineering Applications* (Udine, 2011) discussed problems related to the theory and applications. During the new Advanced Seminar, attention will be paid on the most recent research items, i.e. new generalized models, materials with significant microstructure, multi-field loadings, or identification of constitutive equations. Last but not least, a comparison with discrete modeling approaches will be discussed.
This book of abstracts contains the abstracts submitted to the Advanced Seminar *Generalized Continua as Models for Materials with Multi-Scale Effects or Under Multi-Field Actions*, which will be organized at the *Experimental Factory* (Magdeburg, Germany) from September 21st–25th, 2015.

Magdeburg — *Holm Altenbach*
Paris — *Samuel Forest*
September, 2015

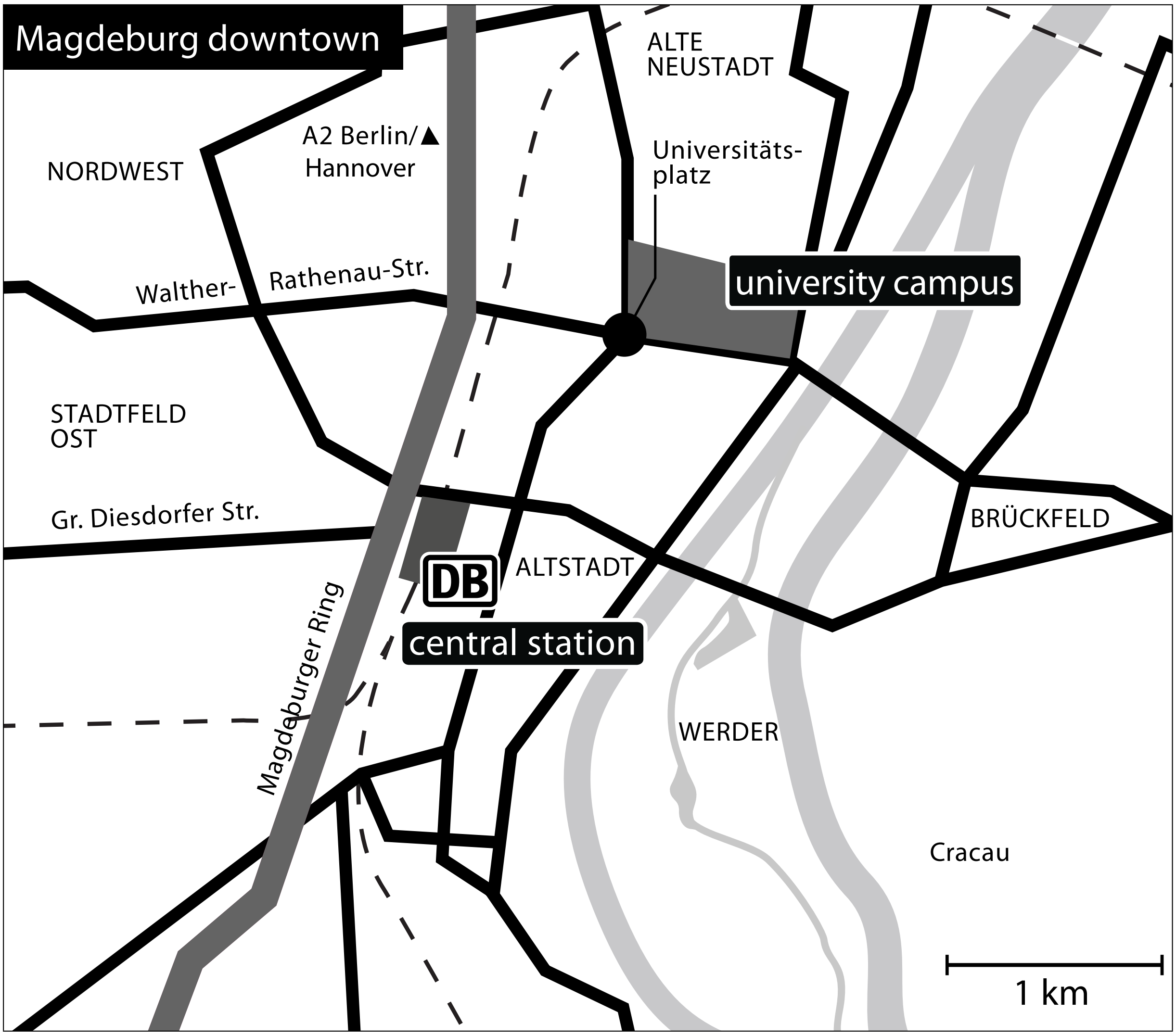
Contents

1 General Information

1.1 Location

1.1.1 Magdeburg

Magdeburg is the capital of the federal state Sachsen-Anhalt (Saxony-Anhalt). Magdeburg is situated at the Elbe River and was founded in 805. The Archbishopric of Magdeburg was founded in 968. In 1035 Magdeburg received the right to hold trade exhibitions and conventions. This city law later was established as the Magdeburg rights and were adopted and modified throughout Central and Eastern Europe. In the 17th century during the Thirty Years' War Magdeburg was destroid and the rebuilding was organized by Otto von Guericke - the famous German physicist establishing the physics of vacuums.
More information on www.wikipedia.org.
Magdeburg can be reached by train (www.bahn.de).



Some of the interesting sights in Magdeburg are The Green Citadel, *Unser Lieben Frauen* Monastery, Magdeburg equestrian, Millennium tower, Town hall, Otto von Guericke monument, Rotehorn-Park, Elbauenpark, St. John Church, The Gruson-Greenhouses and The Magdeburg Water Bridge.

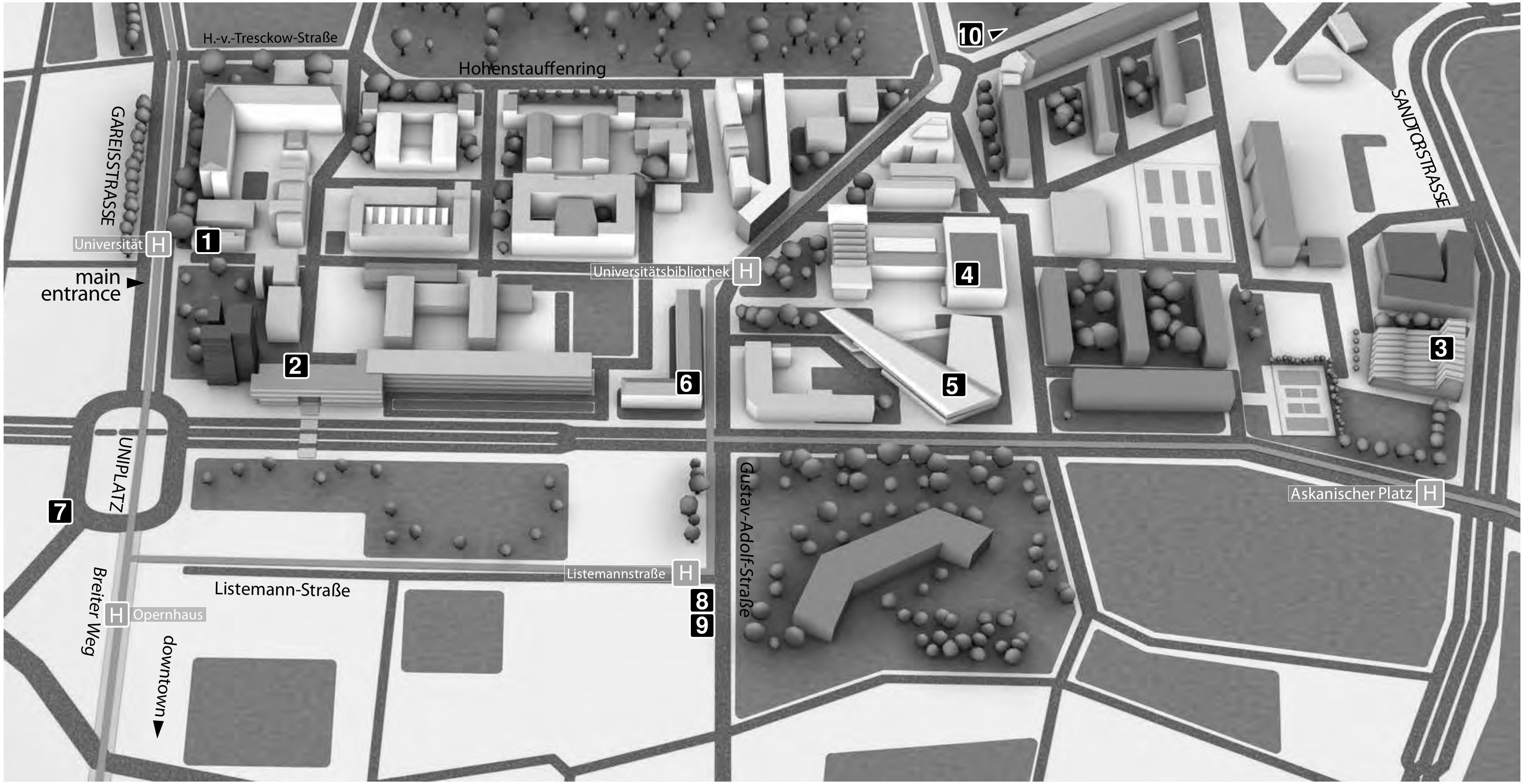
Some of these sights are presented in the following picture composition.

1.1.2 Otto-von-Guericke-Universität (Otto von Guericke University)

Otto von Guericke University Magdeburg (www.ovgu.de) has a distinctive profile. It aims to create a lean and sharply-defined structure with its main focus of expertise in the traditional areas of engineering, the natural sciences, and medicine. It also views economics, management, the social sciences, and humanities as essential disciplines for a modern university in the information age.

Visitors coming by train can get off at Magdeburg-Hauptbahnhof (central station). After coming out of train station from the main entrance, cross the street with the tram tracks, and turn left in the direction of the main street. On your right behind the corner with the cinema, you can find the tram stop "City Carré/Hauptbahnhof". There, you can catch tram number 1 in direction "Lerchenwuhne" or number 8 in direction of "Neustädter See". Get off at the stop "Universität".

Further information in form of a campus and area map is given at page 12.



1 - Campus Service Center	6 - Guest Rooms (buildung 18)
2 - Gatekeeper (keys)	7 - Italian Restaurant ”*Lago di Garda*”
3 - Experimental Factory	8 - Asian Restaurant ”*Taipan*”
4 - University canteen	9 - Greek Restaurant ”*Korfu*”
5 - University library	10 - Italian Restaurant ”*Da Nino Dolce Vita*”

1.1.3 Experimentelle Fabrik (Experimental Factory)

The seminar on *Generalized Continua as Models for Materials with Multi-Scale-Effects or under Multi-Field-Actions* will take place in the lecture room of the Experimental Factory (www.exfa.de) at the Otto von Guericke University Magdeburg (www.ovgu.de).

The Experimental Factory is close to main campus of Magdeburg University. It is a research and transfer center for application orientated research in the areas of production and process innovation. In addition, there are several conference rooms. The building was awarded by the RIBA Prize for Architecture in 2002.

The seminar venue can be reached by the Tram line number 4 starting at Tram station "City Carré/Hauptbahnhof" in direction "Herrenkrug", exit at station "Askanischer Platz" (journey time 11 minutes). When arriving at this tram station, you will find the Experimental Factory on the left-hand side in driving direction. From tram stop it takes you 2 minutes to walk to the destination. For further information, please visit www.insa.de.

1.2 Committee

1.2.1 Scientific Committee

Co-Chairs

Holm Altenbach	(Magdeburg, Germany)
Samuel Forest	(Paris, France)

Scientific Board

Stéphane Berbenni	(Metz, France)
Albrecht Bertram	(Magdeburg, Germany)
Victor Eremeyev	(Magdeburg, Germany)
Vladimir Erofeev	(Nizhnii Novgorod, Russia)
Francesco dell'Isola	(Roma, Italy)
Elena Ivanova	(St. Petersburg, Russia)
Mikhail Karyakin	(Rostov, Russia)
Thomas Michelitsch	(Paris, France)
Wolfgang H. Müller	(Berlin, Germany)
Patrizio Neff	(Duisburg-Essen, Germany)
Paul Steinmann	(Erlangen, Germany)

1.2.2 Organising Committee

Victor A. Eremeyev	(Scientific Secretary)
Andreas Kutschke	(Technical Secretary)
Manuela Schildt	(Conference Office)
Johanna Eisenträger	(Conference Office)
Marcus Aßmus	(Conference Office)

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Homepage	www.ovgu.de/ifme/gc2015/gc2015.html

2 Programme

2.1 Timetable

Time	Monday 21.09.2015	Tuesday 22.09.2015	Wednesday 23.09.2015	Thursday 24.09.2015	Friday 25.09.2015
08:30	Registration				
09:30	Opening	Plenary 3	Plenary 5	Plenary 6	Plenary 8
10:00	Plenary 1	Forest	Ganghoffer	Freidin	Liebold
10:30	Bertram	Coffee-break	Coffee-break	Coffee-break	Coffee-break
11:00	Lecture 1	Lecture 5-7	Lecture 10	Lecture 12	Lecture 22
11:30	Lecture 2		Lecture 11	Lecture 13	Lecture 23
12:00	Lunch	Lunch	Lunch	Lunch	Lecture 24
12:30					Lecture 25
13:00					Lecture 26
13:30					Closing
14:00	Plenary 2	Plenary 4		Plenary 7	
14:30	Michelitsch	Berbenni		dell'Isola	
15:00	Lecture 3	Lecture 8		Lecture 14	
15:30	Coffee-break	Coffee-break	Excursion	Coffeebreak	
16:00	Lecture 4	Lecture 9		Lecture 15-21	
16:30	Come	City			
17:00	Together	tour			
17:30					
18:00	*German*		Banquet		
18:30	*barbecue*				
19:00					
19:30					
20:00					
20:30					
21:00					

2.2 Sessions Overview

2.2.1 Plenary Lectures

60 minutes including discussion

Plenary 1

Albrecht Bertram and Rainer Glüge, see Sect. 3.5 on p. 23

Plenary 2

Thomas Michelitsch and Bernard Collet, see Sect. 3.19 on p. 35

Plenary 3

Samuel Forest, Nicolas M. Cordero, and Esteban P. Busso, see Sect. 3.10 on p. 26

Plenary 4

Stéphane Berbenni, Vincent Taupin, Claude Fressengeas, and Laurent Capolungo, see Sect. 3.4 on p. 22

Plenary 5

Ibrahim Goda and Jean-François Ganghoffer, see Sect. 3.14 on p. 30

Plenary 6

Alexander B. Freidin, Elena Vilchevskaya, Igor K. Korolev, Sergey P. Aleshchenko , and S.E. Petrenko , see Sect. 3.11 on p. 27

Plenary 7

Francesco dell'Isola, Alessandro della Corte, Raffaele Esposito, and Lucio Russo, see Sect. 3.8 on p. 25

Plenary 8

Christian Liebold and Wolfgang H. Müller, see Sect. 3.17 on p. 32

2.2.2 Lectures

Lecture 1

Rainer Glüge, Albrecht Bertram, and Jan Kalisch see Sect. 3.13 on p. 29 (30 minutes including discussion)

Lecture 2

Victoria Yu. Presnetsova, Sergey N. Romashin, Larisa Yu. Frolenkova, and Vladimir S. Shorkin, see Sect. 3.26 on p. 39 (30 minutes including discussion)

Lecture 3

Claude Fressengeas, see Sect. 3.12 on p. 28 (30 minutes including discussion)

Lecture 4

Holm Altenbach and Victor A. Eremeyev, see Sect. 3.1 on p. 21 (30 minutes including discussion)

Lecture 5

Denis N. Sheydakov, see Sect. 3.28 on p. 40 (20 minutes including discussion)

Lecture 6

Sergei Khakalo, Jarkko Niiranen, Viacheslav Balobanov, and Bahram Hosseini, see Sect. 3.15 on p. 30 (20 minutes including discussion)

Lecture 7

Viacheslav Balobanov, Jarkko Niiranen, Sergei Khakalo, and Bahram Hosseini, see Sect. 3.3 on p. 22 (20 minutes including discussion)

Lecture 8

Christian G. Boehmer, Patrizio Neff, and Belgin Seymenoglu, see Sect. 3.6 on p. 24 (30 minutes including discussion)

Lecture 9

Sergey A. Lurie, Petr Belov, and Natalia Tuchkova, see Sect. 3.18 on p. 33 (30 minutes including discussion)

Lecture 10

Hossein Aminpour and Nicola Luigi Rizzi, see Sect. 3.2 on p. 21 (30 minutes including discussion)

Lecture 11

Markus Lazar, see Sect. 3.16 on p. 31 (30 minutes including discussion)

Lecture 12

Martin Dunn and Marcus Wheel, see Sect. 3.7 on p. 24

Lecture 13

Marcus Wheel, see Sect. 3.34 on p. 43

Lecture 14

Vladimir Erofeev, Sergey Kuznetsov, Alexey Malkhanov, and Igor Pavlov, see Sect. 3.9 on p. 25

Lecture 15

Ekatarina A. Podolskaya, A. Yu. Panchenko, and Anton M. Krivtsov, see Sect. 3.25 on p. 38 (20 minutes including discussion)

Lecture 16

Anatoli Stulov and Vladimir I. Erofeev, see Sect. 3.30 on p. 41 (20 minutes including discussion)

Lecture 17

Mikhail Nikabadze and Armine Ulukhanyan, see Sect. 3.24 on p. 37 (20 minutes including discussion)

Lecture 18

Mikhail Nikabadze , see Sect. 3.23 on p. 37 (20 minutes including discussion)

Lecture 19

Elena Vilchevskaya, see Sect. 3.33 on p. 43 (20 minutes including discussion)

Lecture 20

Andrey Vasiliev, Sergey Aizikovich, and Sergey Volkov, see Sect. 3.32 on p. 42 (20 minutes including discussion)

Lecture 21

Mahmoud Mousavi, see Sect. 3.20 on p. 35 (20 minutes including discussion)

Lecture 22

Roberto Serpieri, Francesco dell'Isola, Allesandro della Corte, Francesco Travascio, and Luciano Rosati, see Sect. 3.27 on p. 39 (30 minutes including discussion)

Lecture 23

Christian B. Silbermann , Khanh Chau Le, Matthias Baitsch, and Jörn Ihlemann, see Sect. 3.29 on p. 40 (30 minutes including discussion)

Lecture 24

Patrizia Trovalusci, Maria Laura de Bellis, and Martin Ostoja-Starzewski, see Sect. 3.31 on p. 41 (30 minutes including discussion)

Lecture 25

Andrey Nasedkin, see Sect. 3.21 on p. 36 (30 minutes including discussion)

Lecture 26

Patrizio Neff, see Sect. 3.22 on p. 36 (30 minutes including discussion)

2.3 Social Programme

2.3.1 Come Together

On Monday (4.30 pm) a *German barbecue* will be organized near the Experimentelle Fabrik. There is no additional fee.

2.3.2 City Tour

On Tuesday (4.30 pm) there is offered an English guided *City tour* (approximately 1,5 hours). The starting point is the Experimentelle Fabrik. There is an additional fee (10 EURO).

2.3.3 Excursion and Banquet

On Wednesday (2.00 pm) there is offered a bus tour to the Wasserstraßenkreuz (Magdeburg Water Bridge), where the Mitteland chanal is crossing the Elbe river, and to Pretziner Wehr (a flood prevention building near Magdeburg). The final stop will be in Magdeburg close to the restaurant, where the banquet will take place. There is an additional fee for the excursion (40 EURO) and the banquet (50 EURO).

3 Abstracts

3.1 On Strain Rate Tensors and Constitutive Equations of Inelastic Micropolar Materials

Holm Altenbach and Victor A. Eremeyev

Nonlinear micropolar continuum models allow to describe complex micro-structured media, for example, polycrystals, foams, cellular solids, lattices, masonries, particle assemblies, magnetic rheological fluids, liquid crystals, etc., for which the rotational degrees of freedom of material particles are important. In the case of inelastic behavior, the constitutive equations of the micropolar continuum have a more complicated structure, and the stress and couple stress tensors as well as other quantities depend on the history of strain measures. The aim of the lecture is to discuss the constitutive equations of the nonlinear micropolar continuum using strain rates. Following [1] we introduce a new family of strain rate tensors for micropolar materials. With the help of the introduced strain rates, we discuss the possible forms of constitutive equations of the nonlinear inelastic micropolar continuum, that is micropolar viscous and viscoelastic fluids and solids, hypo-elastic and viscoelastoplastic materials. Considering the fact that some of strain rates are not tensors but pseudotensors, we obtain some constitutive restrictions following from the material frame indifference principle. Using the theory of tensor invariants, we present the general form of constitutive equations of some types of inelastic isotropic micropolar materials including several new constitutive equations.

[1] H. Altenbach and V.A. Eremeyev. “Strain rate tensors and constitutive equations of inelastic micropolar materials”. In: *International Journal of Plasticity* 63 (2014), pp. 3–17.

3.2 On the Modeling of Carbon Nano Tubes as Generalized Continua

Hossein Aminpour and Nicola Luigi Rizzi

Carbon nano tubes have been given a large attention due to the fact that they show very peculiar mechanical properties [2]. Besides, as they can undergo very large deformations without losing the elastic behaviour, nonlinear models must be constructed in order to have a fair description of a number of very relevant phenomena.

Even though the molecular dynamic approach has been and is still largely used as a simulation tool, it has been recognized as cumbersome in many circumstances. For this reason, the attention of many researchers has been focused on the continuum modeling making recourse to both 3D and shell theory.

Following this line, the authors proposed a 1D model endowed with microstructure and non-linear hyperelastic constitutive relationships, showing that it can describe necking and kinking phenomena [3].

In this paper, the model is generalized by enriching its microstructure and a procedure for obtaining fair constitutive functions is proposed. Some bifurcation problems are solved in order to prove the effectiveness of the model proposed.

[2] H. Shima. “Buckling of Carbon Nanotubes: A State of the Art Review Hiroyuki Shima”. In: *Materials* 5 (2014), pp. 47–84.

[3] H. Aminpour and N. Rizzi. “A 1D continuum with microstructure for single-wall CNTs bifurcation analysis”. In: (2015 - submitted).

3.3 Isogeometric Analysis of Gradient-Elastic Rodsand 2D Gradient-Elastic Dynamic Problems

Viacheslav Balobanov, Jarkko Niiranen, Sergei Khakalo, and Bahram Hosseini

In the present contribution, isogeometric methods [4] are used to analyse statics and dynamics of rod and 2D problems based on gradient elasticity [5, 6]. Typically, the aim of the generalized theories of elasticity is to provide length scale parameters taking into account the effect of the microstructure of the material on its mechanical behaviour [5]. The current models, in particular, include one length scale parameter enriching the classical constitutive equations and resulting in fourth order partial differential equations instead of the corresponding second order ones based on the classical elasticity [5]. In our approach, the solvability of the problems is first formulated in a Sobolev space setting and then the problem is implemented by utilizing an isogeometric NURBS based discretization [6]. Computational results of the current approach cover different types of boundary conditions and are compared to analytical solutions or other type of reference solutions such as standard finite element approximations.

[4] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. “Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement”. In: *Computer Methods in Applied Mechanics and Engineering* 194 (2005), pp. 4135–4195.

[5] S. Papargyri-Beskou, A.E. Giannakopoulos, and D.E. Beskos. “Static analysis of gradient elastic bars, beams, plates and shells”. In: *The Open Mechanics Journal* 4 (2010), pp. 65–73.

[6] J. Niiranen et al. “Isogeometric static and dynamic analysis of gradient-elastic rod and plane strain problems”. In: (2015 in preparation).

3.4 A Fast Fourier Transform Method for Dislocation and Generalized Disclination Mechanics

Stéphane Berbenni, Vincent Taupin, Claude Fressengeas, and Laurent Capolungo

Recently, a small-distortion theory of coupled plasticity and phase transformation accounting for the mechanics and the thermodynamics of generalized defects called generalized disclinations (abbreviated g-disclinations) has been proposed [7]. Then, a numerical spectral approach has been developed to solve the elasto-static equations of field dislocation and g-disclination mechanics set out in this theory for periodic media and for linear elastic media [8]. Here, given a spatial distribution of g-disclination density tensors in heterogeneous or homogenous linear higher order elastic media, the incompatible and compatible elastic second and first distortions are obtained from the solution of Poisson and Navier-type equations in the Fourier space. The efficient Fast Fourier Transform (FFT) algorithm is used based on intrinsic Discrete Fourier Transforms that are well adapted to the discrete grid. Therefore, the stress and the skew-symmetric part of hyperstress (couple stress) [9] can be calculated using the inverse FFT. The numerical examples are given for periodic configurations of g-disclination dipoles like particular grain boundaries (symmetric tilt grain boundaries) and twin tips for different twinning configurations.

[7] A. Acharya and C. Fressengeas. “Coupled phase transformations and plasticity as a field theory of deformation incompatibility”. In: *International Journal of Fracture* 174 (2012), pp. 87–94.

[8] S. Berbenni et al. “A numerical spectral approach for solving elasto-static field dislocation and g-disclination mechanics”. In: *International Journal of Solids and Structures* 51 (2014), pp. 4157–4175.

[9] M. Upadhyay et al. “Elastic constitutive laws for incompatible crystalline media: the contributions of dislocations, disclinations and G-disclinations”. In: *Philosophical Magazine* 93 (2013), pp. 794–832.

3.5 Finite Gradient Elasticity with Internal Constraints

Albrecht Bertram and Rainer Glüge

A general framework for second gradient materials is given and specified for finite elasticity and plasticity [10]. This approach extends the linear theory given in [11]. The theory fulfils the Euclidean invariance requirement since it is formulated as a reduced form in a Langrangean setting. The isomorphy concept [12] is applied to (plastic) materials with elastic ranges and results in both multiplicative and additive decompositions of the plastic variables. The concept of material symmetry is enlarged to this class of materials, which gives rise to an interesting group structure.

By extending the classical notion of internal constraints [13], one can introduce kinematical restrictions for gradient materials, which essentially coincide with the concept of pseudo rigid bodies (see, e.g. [14] and further references therein). This opens the door for interesting applications in continuum mechanics.

Thermomechanical internal constraints can be defined which automatically fulfil the second law of thermodynamics.

[10] A. Bertram. “Finite gradient elasticity and plasticity: a constitutive mechanical framework”. In: *Continuum Mechanics and Thermodynamics* DOI 10.1007/s00161-014-0387-0 (2015).

[11] A. Bertram and S. Forest. “The thermodynamics of gradient elastoplasticity”. In: *Continuum Mechanics and Thermodynamics* 26 (2014), pp. 269–286.

[12] A. Bertram. *Elasticity and Plasticity of Large Deformations - an Introduction*. 3rd ed. Heidelberg: Springer, 2012.

[13] A. Bertram. “An introduction of internal constraints in a natural way”. In: *Zeitschrift für Angewandte Mathematik und Mechanik* 60 (1980), T100–T101.

[14] J. Casey. “Pseudo-Rigid Continua: Basic Theory and a Geometrical Derivation of Lagrange’s Equations”. In: *Proceedings of the Royal Society A* 460 (2004), pp. 2021–2049.

3.6 On Cosserat Model for Elastic Continua

Christian G. Boehmer, Patrizio Neff, and Belgin Seymenoglu

Elastic solids with microstructure can be studied using the Cosserat model for elastic continua. We formulate the complete dynamical Cosserat model including interaction terms between the deformation gradient and the Cosserat dislocation curvature tensor, and also take into account the Cosserat coupling term. This model is studied by assuming the deformations to be small, however, the rotations are kept fully nonlinear and multiple turns will be allowed in principle. The Euler-Lagrange equations of this model are derived by variations of the energy functional, and we are able to construct soliton-like solutions of this model. We can visualise our results with the help of suitable animations.

3.7 Computational Analysis of the Size Effects Displayed in Beams with Lattice Microstructures

Martin Dunn and Marcus Wheel

The mechanical behaviour of heterogeneous materials cannot be fully described by classical elasticity theories alone. However, higher order theories, such as couple stress, micropolar and micromorphic elasticity, are capable of forecasting phenomena such as the size dependent mechanical behaviour that is exhibited by such materials.

In this paper, the mechanical behaviour of finite element based computational representations of heterogeneous materials with regular or periodic cellular microstructure is compared to existing closed form analytical predictions of their constitutive behaviour available within the open literature. The principal motivation for undertaking this investigation is to inform the experimental mechanics community of the likely difficulties that may be encountered in testing finite sized samples of these low mass density materials.

During the computational investigation, slender, geometrically similar rectangular beams of different sizes which are comprised of regular, repeating arrangements of both square and equilateral triangular cellular microstructures were represented using the finite element analysis (FEA) software ANSYS. Flexural loading of the virtual samples reveals that the materials exhibit the theoretically forecast size effect from which the relevant material constitutive properties, notably the flexural modulus and characteristic length can be identified. These properties could then be compared to the published theoretical values for each microstructural configuration considered.

Initial findings suggest that while there is agreement between the numerically determined and theoretically predicted moduli the characteristic lengths in bending, ℓ_b, calculated from the numerical data appear to differ from the theoretical forecasts. Moreover, the computational representations indicate that finite sized material samples are capable of exhibiting size effects not predicted by the more general higher order constitutive theories. Results indicate that the nature of the size effect appears to depend on the prescription of the sample surfaces with respect to the specified microstructure of the material. While these unanticipated size effects show qualitative agreement with that forecast for a simple laminate material comprised of alternating stiff and compliant layers the consequences may be profound for experimental mechanical testing of such materials.

3.8 Some Cases of Unrecognized Transmission of Scientific Knowledge: from Antiquity to Gabrio Piola's Peridynamics and Generalised Continuum Theories

Francesco dell'Isola, Alessandro della Corte, Raffaele Esposito, and Lucio Russo

In this paper we provide a few examples of cases of unrecognized transmission of knowledge in the history of science. We will start by considering some ancient examples concerning Democritus, Heron, Galileo and the history of the theory of tides. Then we will mainly focus on the works of the Italian scientist Gabrio Piola (1794-1850). In particular, we will clearly track in Noll's postulation of mechanics the echoes of the 'ancient' presentation by Piola of those fundamental ideas needed to found Analytical Continuum Mechanics. In this way we argue, basing our reasoning on modern and easily available sources, that, in the passage from one language and a cultural tradition to another, the main content of scientific texts may often be lost, and that more recent sources can be scientifically more primitive than some more ancient ones. The fully documented case of Piola, and the strong evidence that we provide in the parallel cases of Democritus and Heron, are mainly intended as a support for this general thesis. Moreover we discuss how the theory which has been called recently "peridynamics" was formulated in Piola's papers and how it has been recently rediscovered, having been the original papers forgotten, mainly because they were written in Italian.

3.9 Nonlinear Wave Propagation in Some Generalized Continua

Vladimir Erofeev, Sergey Kuznetsov, Alexey Malkhanov, and Igor Pavlov

A two-dimensional model representing a square lattice of round particles is proposed for description of auxetic properties of an anisotropic crystalline material with cubic symmetry. It is assumed that each particle has two translational and one rotational degree of freedom. Differential equations describing the propagation of elastic and rotational waves in such a medium have been derived. Relationships between the macroelasticity constants of the medium and the parameters of its inner structure have been found. It has been shown that the Poisson's ratios of the anisotropic material can be negative for certain values of the parameters of its inner structure.

In contrast to the continuum mechanics, a micropolar medium (Cosserat medium) is characterized by the vector of the center of mass and the rotation vector. The strain tensor is introduced as a bending-torsion tensor, instead of the stress tensor - the couple stress tensor. All tensors are nonsymmetrical. A solid can be characterized by six degrees of freedom and can be described by six equations of motion.

The work is devoted to the studying of the peculiarities of the propagation of different types of waves (longitudinal, shear, longitudinal-rotational, and shear-rotational) in nonlinear elastic and nonlinear thermo-elastic micropolar media.

We have carried out the research of the evolution of the profile of plane longitudinal Riemann waves which propagate in micropolar materials. We have found the dependencies of coordinates of a turnover point for Riemann waves on the parameters of a material. In addition, we have analyzed the influence of temperature on the evolution of such a wave.

It has been shown that the presence of nonlinear and dispersive effects in such media results in the formation of stationary waves, i.e. waves which propagate with a constant velocity and without changing their shape.

We have analyzed plane stationary waves and it has been found out that the formation of localized waves (solitons)is possible along with nonlinear periodical waves. We have calculated

the shapes of deformation and displacement waves. The shape of such waves is close to sinusoidal shape at low amplitudes. With the increase of the amplitude, the waveform becomes a sawtooth one, and deformation wave appears as a sequence of pulses. We have found the dependencies of waves' velocities on their amplitudes.
It has been shown that nonlinear stationary elastic rotation waves can be formed in a micropolar medium. Such a wave is periodic and propagates faster than waves in a linear medium. The wave has a sawtooth shape, and the wave length grows with the raise of its amplitude. We have analyzed a new type of stationary waves - nonlinear shear-rotation waves.
Materials with microstructure with high contrast in mass, stiffness, isolated and interconnected structural elements such as in 3d woven materials, demonstrate a number of complex phenomena under external excitation, including formation and interaction of localized waves, transfer of energy from macroscopic oscillations to microscopic, energy dissipation, internal resonances, etc. Such diversity is related to the dynamical processes occurring at the microscopic level.
It is very important to be able to model such phenomena for practical applications. At the same time, it is an extremely complex task due to coupling between spatial and temporal scales and the need to introduce very high spatial and temporal resolution.
An alternative approach is to consider an effective medium with internal degrees of freedom. The existence of a carrying medium is postulated, and it is assumed that this medium is described by a nonlinear elasticity theory. It is further assumed that an infinite number of non-interacting oscillators is coupled with each point of the carrying medium.
It is shown that the system of dynamic equations of generalized continua is equivalent to the Korteweg-de Vries-Burgers equation for one-dimensional wave processes. This equation has an analytical solution in the form of a spatially-localized wave. The influence of parameters such as stiffness, viscosity, and the ratio of the density of the carrying media to the oscillators' density on the magnitude, wave speed, and the thickness of an impulse-like wave is studied. A non-stationary process of deformation wave localization is studied numerically.

3.10 Second Strain Gradient Elasticity of Nano-Objects

Samuel Forest, Nicolas M. Cordero, and Esteban P. Busso

Mindlin's second strain gradient continuum theory for isotropic linear elastic materials is used to model two different kinds of size-dependent surface effects observed in the mechanical behaviour of nano-objects. First, the existence of an initial higher order stress represented by Mindlin's cohesion parameter, b_0, makes it possible to account for the relaxation behaviour of traction-free surfaces [15]. Second, the higher order elastic moduli, c_i, coupling the strain tensor and its second gradient are shown to significantly affect the apparent elastic properties of nano-beams and nano-films under uni-axial loading [16]. These two effects are independent from each other and allow for separated identification of the corresponding material parameters. Analytical results are provided for the size-dependent apparent shear modulus of a nano-thin strip under shear. Then Finite element simulations are performed to derive the dependence of the apparent Young's modulus and Poisson's ratio of nano-films with respect to their thickness, and to illustrate hole free surface relaxation in a periodic nano-porous material [17, 18].

[15] S. Forest, N.M. Cordero, and E.P. Busso. "First vs. second gradient of strain theory for capillarity effects in an elastic fluid at small length scales". In: *Computational Materials Science* 50 (2011), pp. 1299–1304.

[16] R.D. Mindlin. "Second gradient of strain and surface-tension in linear elasticity". In: *International Journal of Solids and Structures* 1 (1965), pp. 417–438.

[17] N.M. Cordero, S. Forest, and E.P. Busso. “Second Strain Gradient Elasticity of Nano-Objects”. In: *Journal of the Mechanics and Physics of Solids* (2015).

[18] A. Javili, F. dell’Isola, and P. Steinmann. “Geometrically nonlinear higher-gradient elasticity with energetic boundaries”. In: *Journal of the Mechanics and Physics of Solids* 61 (2013), pp. 2381–2401.

3.11 Chemical Reaction Fronts Propagation in Elastic and Inelastic Solids

Alexander B. Freidin, Elena N. Vilchevskaya, Igor K. Korolev, Sergey P. Aleshchenko, and Svetlana E. Petrenko

Interconnections between chemical reactions and deformation and fracture processes remain to be of significant interest for both fundamental science and engineering applications. We consider a reaction between gaseous and solid constituents similar to the reaction of silicon oxidation. To take into account stress effects, we develop a theory of stress-assist chemical reactions based on the notion of the chemical affinity tensor. We derive an expression of the chemical affinity tensor within the frames of mechanics of configurational forces as a consequence of the mass, momentum and energy balances and entropy inequality written down for an open system in which a chemical reaction takes place between a diffusive gaseous constituent and a solid of arbitrary rheology [19, 20, 21] . We formulate a kinetic equation in the form of the dependence of the reaction rate at the oriented surface element on the normal component of the chemical affinity tensor similar to the equation accepted in classical physical chemistry for the scalar chemical affinity in the case of reactions in gases and fluids (see, e.g. [22, 23]). Then, considering bulk reactions, we study the role of the affinity tensor invariants. We solve analytically a number of coupled boundary value problems of mechanochemistry [20, 24] and develop finite element procedures for numerical simulations of the reaction front propagation. We show how stresses can accelerate, retard and even block the front propagation. Having in mind experimental data (see, e.g., [25]) we study how stress relaxation due to viscosity affects the reaction front propagation and stress redistribution behind the front. On the whole, we demonstrate the variety of the reaction front behaviors in dependence on the reaction parameters, rheological models chosen for solid constituents, material parameters, external and internal stresses and temperature.

[19] A.B. Freidin. “Chemical affinity tensor and stress-assist chemical reactions front propagation in solids”. In: *ASME 2013 International Mechanical Engineering Congress and Exposition*. Vol. 9: Mechanics of Solids, Structures and Fluids. IMECE2013-64957. San Diego, California, USA: ASME, 2013, V009T10A102.

[20] A.B. Freidin, E.N. Vilchevskaya, and I.K. Korolev. “Stress-assist chemical reactions front propagation in deformable solids”. In: *International Journal of Engineering Science* 83 (2014), pp. 57–75.

[21] A.B. Freidin. “On the chemical affinity tensor for chemical reactions in deformable materials”. In: *Mechanics of Solids* 50 (2015), pp. 260–285.

[22] I. Prigogine and R. Defay. *Chemical Thermodynamics*. London: Longmans, Green, 1988.

[23] P. Glansdorff and I. Prigogine. *Thermodynamic Theory of Structure, Stability and Fluctuations*. New York: Wiley, 1971.

[24] A. Freidin, S. Petrenko N. Morozov, and E. Vilchevskaya. “Chemical reactions in spherically symmetric problems of mechanochemistry”. In: *Acta Mechanica* (2015).

[25] C. C. Buttner and M. Zacharias. "Retarded oxidation of Si nanowires". In: *Applied Physics Letters* 89 (2006), p. 263106.

3.12 Field Dislocation and G-Disclination Mechanics

Claude Fressengeas

In Volterra's construct of crystal defects [26], the presence of dislocations reflects a discontinuity of the elastic/plastic displacement across surfaces bounded by the dislocation lines, while the elastic/plastic rotation remains continuous. This discontinuity induces the incompatibility of the elastic/plastic strain tensor, which remains continuous however, while the elastic/plastic curvature tensor is left compatible [27]. If in addition disclinations are present in the body, the elastic/plastic rotation tensor becomes multi-valued across some bounded surface, and the elastic/plastic curvature tensor turns out to be incompatible [28]. However, the elastic/plastic strain tensor still remains continuous. Such a situation typically occurs along grain boundaries in polycrystals. Yet, occurrences of a discontinuity across some bounded surface of the entire elastic/plastic distortion tensor (including the elastic/plastic strain tensor) are commonplace in solids. They include recrystallization, twinning, phase transformation, shear bands, inclusions in a matrix of dissimilar material, etc. As shown by [7], the discontinuity of the elastic/plastic distortion is conveniently rejected by the incompatibility of the elastic/plastic 2-distortion and the existence of a non-vanishing smooth third order tensor of defect densities, referred to as the generalized disclination ("g-disclination") density tensor. For instance, introducing appropriate g-disclination density fields recently allowed computing the elastic stress fields of twin-tips for the first time, by using suitable FFT-based techniques [8].
Conservation of the g-disclinations topological content across arbitrary surfaces provides a natural framework for their dynamics, in terms of transport laws for the g-disclination densities. Whereas dislocations and conventional disclinations – *i.e.* g-disclinations associated with the incompatibility of the skew-symmetric (in the first two positions) 2-distortion – are the carriers of plasticity, the motion of the g-disclinations associated with the incompatibility of the symmetric (first two positions) 2-distortion mediates phase transformation and interphase dynamics. Thus, dissipation arises concurrently from the motion of dislocations and g-disclinations. Thermodynamic guidance provides elastic constitutive laws relating the elastic 1- and 2-distortions, and the stress and hyperstress tensors [9], as well as the driving forces for the motion of dislocations and g-disclinations. Remarkably, the Peach-Koehler force on dislocations now includes a cross-term containing the symmetric part of the hyperstress tensor, and we find a Peach-Koehler-type driving force on g-disclinations involving the hyperstress tensor and the g-disclination density. Positiveness of the dissipation provides guidelines for the constitutive relationships between the driving forces and the dislocation/g-disclination velocities.
The model contains the earlier dislocation and disclination theories [29, 30] as limiting cases when the incompatibility of the elastic/plastic 2-distortion and symmetric part of the 2-distortion are respectively overlooked. It is capable of addressing the evolution of the displacement and dislocation/g-disclination density fields along prescribed loading paths with the intent of understanding concurrent phase transformation and plasticity.

[7] A. Acharya and C. Fressengeas. "Coupled phase transformations and plasticity as a field theory of deformation incompatibility". In: *International Journal of Fracture* 174 (2012), pp. 87–94.

[8] S. Berbenni et al. "A numerical spectral approach for solving elasto-static field dislocation and g-disclination mechanics". In: *International Journal of Solids and Structures* 51 (2014), pp. 4157–4175.

[9] M. Upadhyay et al. “Elastic constitutive laws for incompatible crystalline media: the contributions of dislocations, disclinations and G-disclinations”. In: *Philosophical Magazine* 93 (2013), pp. 794–832.

[26] V. Volterra. “Sur l’équilibre des corps élastiques multiplement connexes”. In: *Ann. Sci. Ecol. Norm. Sup. III* 24 (1907), pp. 401–517.

[27] E. Kröner. “Continuum theory of defects”. In: *Les Houches, Session XXXV Physics of Defects*. Ed. by R. Balian. Amsterdam: North-Holland, 1981, pp. 217–315.

[28] R. deWit. “Linear theory of static disclinations”. In: *Fundamental aspects of dislocation theory*. Ed. by J.A. Simmons, R. deWit, and R. Bullough. Spec. Publ. 317. Amsterdam: Nat. Bur. Stand. (US), 1970, pp. 651–673.

[29] A. Acharya. “A model of crystal plasticity based on the theory of continuously distributed dislocations”. In: *Journal of the Mechanics and Physics of Solids* 49 (2001), pp. 761–784.

[30] C. Fressengeas, V. Taupin, and L. Capolungo. “An elasto-plastic theory of dislocation and disclination fields”. In: *International Journal of Solids and Structures* 48 (2011), pp. 3499–3509.

3.13 The Eigenmodes of Isotropic Strain Gradient Elastic Materials

Rainer Glüge, Albrecht Bertram, and Jan Kalisch

In a recent work, Bertram and Glüge proposed a framework for internal constraints in strain gradient materials [31]. Such an extension may be useful for the continuum modelling of micro-architectured materials, for example the pantographic truss structures in [32]. The proposal is basically to approach internal constraints on the strain gradient by taking parts of the higher order elastic stiffness hexadic to infinity, referred to as the natural approach to internal constraints [13]. For this purpose, we firstly examine the isotropic stiffness hexadic [33], and give a spectral decomposition of it. It turns out that the eigenmodes are hard to interpret geometrically, and that two eigenmodes depend on the specific material under consideration. We then use a finite element implementation of a strain gradient elasticity to demonstrate the feasibility of the natural approach to internal constraints.

[13] A. Bertram. “An introduction of internal constraints in a natural way”. In: *Zeitschrift für Angewandte Mathematik und Mechanik* 60 (1980), T100–T101.

[31] A. Bertram and R. Glüge. “Gradient Materials with Internal Constraints”. In: *Mechanics and Mathematics of Complex Systems* submitted (2015).

[32] P. Seppecher, J.J. Alibert, and F. dell’Isola. “Linear elastic trusses leading to continua with exotic mechanical interactions”. In: *Journal of Physics: Conference Series* 319 (2011), pp. 1–13.

[33] F. dell’Isola, G. Sciarra, and S. Vidoli. “Generalized Hooke’s law for isotropic second gradient materials”. In: *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* 456 (2009), pp. 2177–2196.

3.14 Limit Analysis of Lattices Based on the Asymptotic Homogenization Method: Application to the Prediction of Size Effects in Bone Plastic Collapse and Brittle Fracture

Ibrahim Goda and Jean-François Ganghoffer

Lattice structures possess a huge potential for energy absorbing applications, thus it is important to develop predictive tools for their mechanical response up to collapse. Yielding is generally premonitory of structural collapse for lattice structures, so a comprehensive and quantitative understanding of lattice yielding behavior is indispensable in engineering applications. In the present work, the overall plastic yield and brittle failure behaviors of three-dimensional lattices is investigated by a microstructural modeling approach based on the homogenization of the initially discrete microstructure. The multiaxial yield behavior of the lattice is analyzed to formulate a multiaxial plastic yield criterion. Furthermore, the brittle fracture of the lattice is modeled under triaxial stress states to construct the failure surfaces, defined in the tension-tension quadrant. In plastic yielding, the analysis is performed assuming an elastic perfectly plastic lattice, and a micromechanical model based on homogenization scheme is applied to a representative unit cell to determine the macroscopic plastic yield surfaces in stress space.
An adaptation and extension of the discrete homogenization method towards a micropolar effective medium has been introduced in order to construct the plastic yield surfaces for which the material point of the effective continuum supports couple stresses in addition to Cauchy-type stresses. The size effects in the ductile fracture mode have been addressed by considering a micropolar behavior, reflecting the influence of additional degrees of freedom and internal bending length effects on the initiation of plasticity.
This general framework is applied to evaluate the yield and failure properties of biological membranes (classified in terms of their nodal connectivity) and trabecular bone; these properties are of key interest in understanding and predicting the fracture of bones and bone implant systems. The effective strength of trabecular bone is evaluated in the two situations of fully brittle (fracture with no tissue ductility) and fully ductile failure (yield with no tissue fracture) of the trabecular tissue. A size-dependent non-classical plastic yield criterion is developed relying on the reduced Cosserat theory to capture the size-dependency of the trabecular bone structures. When the characteristic size of the bone sample is comparable to the bending length, a significant difference is shown between the results based on the non-classical theory and those obtained by the classical theory. Finite element analyses are performed to validate the plastic collapse stresses of trabecular bone in a 2D situation under uniaxial, shear, and biaxial tensile loadings. A validation of the construction yield and collapse surfaces is achieved by FE simulations.

3.15 Isogeometric Static and Dynamic Analysis of Gradient-Elastic Plane Strain/Stress Problems with Applications

Sergei Khakalo, Jarkko Niiranen, Viacheslav Balobanov, and Bahram Hosseini

In the present contribution, isogeometric methods [4] are used to analyse statics and dynamics of the plane strain/stress problems based on gradient-elasticity [5, 6, 34]. Typically, the aim of the generalized theories of elasticity is to provide length scale parameters taking into account the effect of the microstructure of the material on its mechanical behaviour [5]. The current models, in particular, include one length scale parameter enriching the classical constitutive equations and resulting in fourth order partial differential equations instead of the corresponding second order ones based on the classical elasticity [5, 34]. In our approach, the solvability of the problems is first formulated in a Sobolev space setting and then the problem is implemented by utilizing an isogeometric NURBS based discretization [6]. Computational results of the current approach cover different boundary condition types and are compared to analytical solutions or other type of reference solutions such as standard finite element approximations. Numerical results are obtained by implementing the isogeometric methods into a commercial software Abaqus.

[4] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. "Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement". In: *Computer Methods in Applied Mechanics and Engineering* 194 (2005), pp. 4135–4195.

[5] S. Papargyri-Beskou, A.E. Giannakopoulos, and D.E. Beskos. "Static analysis of gradient elastic bars, beams, plates and shells". In: *The Open Mechanics Journal* 4 (2010), pp. 65–73.

[6] J. Niiranen et al. "Isogeometric static and dynamic analysis of gradient-elastic rod and plane strain problems". In: (2015 in preparation).

[34] P. Fischer et al. "Isogeometric analysis of 2D gradient elasticity". In: *Computational Mechanics* 47 (2011), pp. 325–334.

3.16 Anisotropic Gradient Elasticity: Fundamentals and Singularity-Free Dislocations

Markus Lazar

A theory of anisotropic gradient elasticity including bulk anisotropy as well as weak nonlocal anisotropy is presented and used in order to model non-singular anisotropic dislocations.
The fundamental problem of non-singular anisotropic dislocations in the framework of anisotropic gradient elasticity theory will be presented in this talk. A general theory of non-singular dislocations is developed for linearly elastic and anisotropic media. Using the calculus of variations and the framework of incompatible elasticity, we derive the field equations of anisotropic gradient elasticity, which are inhomogeneous partial differential equations of fourth order. In order to solve "eigen-distortion problems" in such a framework, we derive the Green tensor of anisotropic gradient elasticity with up to six independent length scale parameters as a special version of Mindlin's form II anisotropic gradient elasticity theory [35, 36] and as the anisotropic generalization of gradient elasticity of Helmholtz type [37, 38, 39]. The framework models materials where anisotropy is twofold, namely the bulk material anisotropy and a weak non-local anisotropy relevant at the nano-scale [40, 41]. In this context, the Green tensor of the twofold anisotropic Helmholtz-Navier equation and the Green function of the anisotropic Helmholtz equation will be presented. The latter one plays the role of an anisotropic regularization function. The continuum theory of anisotropic gradient elasticity is an excellent candidate for eigenstrain-problems up to the nano-scale [42].
Using the Green tensor of the theory of anisotropic gradient elasticity, the non-singular fields which are produced by dislocations are given. The non-singular Mura, Peach-Koehler stress, and Burgers formulas are presented. In addition, the Eshelby stress tensor, the Peach-Koehler force and the J-integral of dislocations are given and discussed in the framework of anisotropic gradient elasticity.
All obtained dislocation fields are non-singular due to the regularization of the classical singular fields. The results have a direct application to numerical implementation and computer simulation of non-singular dislocations within the so-called (discrete) dislocation dynamics [43]. Therefore,

these non-singular formulas of the anisotropic dislocation fields are suitable for the numerical implementation in 3D anisotropic dislocation dynamics without singularities. Such a dislocation dynamics without singularities offers the promise of predicting the dislocation microstructure evolution from fundamental principles and on sound physical grounds. Thus, a dislocation-based plasticity theory can be based on the gradient theory of non-singular dislocations.

[35] R.D. Mindlin. "Micro-structure in linear elasticity". In: *Archive for Rational Mechanics and Analysis* 16 (1964), pp. 51–78.

[36] R.D. Mindlin. "Elasticity, piezoelectricity and crystal lattice dynamics". In: *Journal of Elasticity* 2 (1972), pp. 217–282.

[37] M. Lazar and G.A. Maugin. "Nonsingular stress and strain fields of dislocations and disclinations in first strain gradient elasticity". In: *International Journal of Engineering Science* 43 (2005), pp. 1157–1184.

[38] M. Lazar. "The fundamentals of non-singular dislocations in the theory of gradient elasticity: Dislocation loops and straight dislocations". In: *International Journal of Solids and Structures* 50 (2013), pp. 352–362.

[39] M. Lazar. "On gradient field theories: gradient magnetostatics and gradient elasticity". In: *Philosophical Magazine* 94 (2014), pp. 2840–2874.

[40] M. Lazar and G. Po. "The non-singular Green tensor of gradient anisotropic elasticity of Helmholtz type". In: *European Journal of Mechanics A/Solids* 50 (2015), pp. 152–162.

[41] M. Lazar and G. Po. "The non-singular Green tensor of Mindlin's anisotropic gradient elasticity with separable weak non-locality". In: *Physics Letters A* 379 (2015), pp. 1538–1543.

[42] D. Seif et al. "Atomistically enabled nonsingular anisotropic elastic representation of near-core dislocation stress fields in α-iron". In: *Physical Review B* 91 (2015), p. 184102.

[43] G. Po et al. "Singularity-free dislocation dynamics with strain gradient elasticity". In: *Journal of the Mechanics and Physics of Solids* 68 (2014), pp. 161–178.

3.17 Applications of Higher-Order Continua to the Size Effect in Bending - Theory and Recent Experimental Results

Christian Liebold and Wolfgang H. Müller

It is well known that elastic deformations may depend on the size of the material volume. Mechanical characteristics vary in the intrinsic design of the material's substructures. A size effect is reflected in a stiffer or softer elastic response at different scales. A continuum mechanical understanding is necessary, for example, for the design phase of micro- and nanosize systems, often accompanied by finite element (FE) simulations. The knowledge of additional material parameters is of great importance. In the present work, flexural characteristics of slender beams of different sizes are investigated corresponding to a method from LAKES [44] of analyzing the size effect. Higher-order continuum theories as the *strain gradient theory* of MINDLIN's types (cf., [45]), *modified strain gradient theory* (cf., LAM et al. [46]), *micropolar theory* (cf., [47]), *couple stress theory* (abbr. CS) as well as *surface elasticity theory* (abbr. SE, cf., GURTIN & MURDOCH [48]) are presented. Considering EULER-BERNOULLI beam assumptions, Young's moduli of bending structures in extended continua are derived analytically, e.g.:

$$E^{\mathsf{CS}} = E\left(1 + 6\frac{\ell^2}{t^2}\right), \qquad E^{\mathsf{SE}} = E + E^{\mathsf{surf}}\left(\frac{6}{t} + \frac{2}{w}\right),$$

where E denotes the conventional YOUNG's modulus, t the thickness of beams with rectangular cross-sections only, w the width and ℓ, as well as E^{surf} the corresponding additional material parameter, which allow to describe the overall bending behavior of the tested specimens. Atomic force microscopy- (abbr. AFM), micro-RAMAN spectroscopy (abbr. MRS) and a bending-vibration method will be presented as experimental assessment to the bending behavior of beam structures. YOUNG's modulus was measured and evaluated. The results are fitted to the predictions of each higher-order theory in good accordance and the material parameters are extracted, respectively. In particular, deflection measurements were performed, and force, as well as strain data was recorded for micro beams with decreasing thicknesses, using the AFM method in combination with a RAMAN spectroscope. Measurements of the first flexural eigenfrequency were carried out, using a fast FOURIER transformation (abbr. FFT) of the acoustic signal of the bending vibration. The results imply a positive and negative correlation to the size of the chosen specimens. The tested materials include single crystalline silicon, amorphous silicon nitride, epoxy polymer, SU-8 polymer, aluminum foams and aluminum with artificial heterogeneities.
In addition, the couple stress model (a.k.a. pseudo-COSSERAT continuum, or indeterminate COSSERAT model) is used in order to provide a FE strategy. The variational formulation of the couple stress theory:

$$\int_V (\sigma_{ij}\delta\varepsilon_{ij} - \mu_{ij}\delta\chi_{ij})\,\mathrm{d}V = \oint_{\partial V} t_k \delta u_k \mathrm{d}A - \int_{A_\mathrm{s}} \frac{1}{2}\left[\left[\mu_{li}\delta u_{m,n}\right]\right]\epsilon_{imn} n_l \mathrm{d}A_\mathrm{s} = 0$$

was implemented into the open-source finite element project FEniCS$^{©}$, using equidistantly distributed tetrahedral and continuous LAGRANGE elements with a polynomial degree of two in observance to the rank of the resulting elliptical partial differential equation. The enhanced numerical simulation of simple beam bending, using different sizes of the model is in good agreement with recent experimental results and results from literature.

[44] R. Lakes. "Experimental methods for study of Cosserat elastic solids and other generalized elastic continua". In: *Continuum Models for Materials with Micro-Structure*. Ed. by H. Mühlhaus. New York: J. Wiley, 1995, pp. 1–22.

[45] R.D. Mindlin and N.N. Eshel. "On first strain-gradient theories in linear elasticity". In: *International Journal of Solids and Structures* 4 (1968), pp. 109–124.

[46] D.C.C. Lam et al. "Experiments and theory in strain gradient elasticity". In: *Journal of the Mechanics and Physics of Solids* 51 (2003), pp. 1477–1508.

[47] V.A. Eremeyev, L.P. Lebedev, and H. Altenbach. *Foundations of Micropolar Mechanics*. Heidelberg: Springer, 2013.

[48] M.E. Gurtin and A.I. Murdoch. "A continuum theory of elastic material surfaces". In: *Archive for Rational Mechanics and Analysis* 57 (1975), pp. 291–323.

3.18 On Generalized Continual Model of Adhesion Interaction in Mechanics of Solid and Thin Structures

Sergey A. Lurie, Petr Belov, and Natalia Tuchkova

We develop a continuum model of media, in which known superficial effects such as a superficial tension, friction of rest of bodies with ideally smooth surface of contact, the meniscus, wettability and capillarity are modeled within the framework of the unified continuous description as particular scale effects in the media. Early in the works [49, 50, 51, 52, 53], the applied variants of the adhesion interactions were proposed based on the variants of gradient theories of elasticity. Using them, it was possible to explain many known nonclassical effects of the mechanics of the continuous media, connected with scale effects. As an example, the variant of the theory of thin films with face adhesive properties was examined.

In the present work, we elaborate the generalized model of adhesion. Using the variational approach within the framework of the generalized theory of media with fields of defects, the continuum theory of adhesion interactions is studied. We define the surface potential energies (the energy of adhesion interactions), edges energy, and the energy of specific points of the surface edges. We also formulate the constitutive equations on the surface and establish the consistent treatment of all physical constants. We consider the test problems for definition of new physical constants. A generalized Pascal equation for a surface tension pressure and a generalized Young law for the description of wettability will be received theoretically whereas earlier Pascal law for capillarity and Young law for wettability were known as empirical.

The authors use the theory of media with microstructures with common description of the spectrum of the superficial phenomena in contrast to other variants of theories (of Mindlin types) of the media with microstructures. Moreover, it will be shown that the part of the superficial energy directly connected to defectiveness (occurrence of superficial defects) is allocated within the framework of the given model. This superficial energy explicitly defines through parameters of the model (adhesive characteristics of environments), and on the other hand this superficial energy is actually a specific energy of fracture.

As application, the dispersed composites with micro- and nano-inclusions were considered. An approach and the model have been validated to predict some basic mechanical properties of composite materials reinforced with micro- and nano-scale particles/fibres/tubes.

At last, the gradient models of a two-dimensional defectless medium are formulated. Two problems for a graphene sheet and for the single walled nanotubes are examined as two examples of such two-dimensional media. We show that the characteristic feature of both problems is the fact that the mechanical properties of such structures are not defined by "volumetric" moduli but by adhesive ones which have different physical dimensions which coincide with the dimension of the corresponding stiffness of classical plates and shells.

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[49] S. Lurie et al. “Interphase layer theory and application in the mechanics of composite materials”. In: *International Journal of Material Science* 41 (2006), pp. 6693–6707.

[50] P.A. Belov and S.A. Lurie. “Theory of ideal adhesion interactions”. In: *Mechanics of Composite Materials and Designs* 14 (2007), pp. 519–536.

[51] S.A. Lurie and N.P. Tuchkova. “Continuous models of the adhesion for deformable solids and the media with nanostructures”. In: *Kompozity i nanostruktury [Composites and nanostructures]* 2 (2009), pp. 25–43.

[52] S. Lurie et al. “Advanced theoretical and numerical multiscale modeling of cohesion/adhesion interactions in continuum mechanics and its applications for filled nanocomposites”. In: *Computational Material Science* 45 (2009), pp. 709–714.

[53] S. Lurie and P. Belov. “Gradient effects in fracture mechanics for nano-structured materials”. In: *Engineering Fracture Mechanics* 130 (2014), pp. 3–11.

3.19 A Matrix Function Approach to Generate Nonlocal Constitutive Behavior

Thomas Michelitsch and Bernard Collet

We analyze one-dimensional discrete and quasi-continuous linear chains of $N \gg 1$ equidistant and identical mass points with periodic boundary conditions and generalized non-local interparticle interactions in the harmonic approximation. We introduce elastic potentials which define by Hamilton's principle discrete "Laplacian operators" ("Laplacian matrices") which are operator functions ($N \times N$-matrix functions) of the Laplacian of the Born-von-Karman linear chain with next neighbor interactions. The nonlocality of the constitutive law of the present model is a natural consequence of the *non-diagonality* of these Laplacian matrix functions in the N dimensional vector space of particle displacement fields where the (Bloch) eigenvectors and as a consequence the periodic boundary conditions of the linear chain are maintained. In the quasi-continuum limit (long-wave limit) the Laplacian matrices yield "Laplacian convolution kernels" (and the related elastic modulus kernels) of the non-local constitutive law. The elastic stability is guaranteed by the positiveness of the elastic potentials. We establish criteria for "weak" and "strong" nonlocality of the constitutive behavior which can be controlled by scaling behavior of material constants in the continuum limit when the interparticle spacing $h \to 0$. The approach provides a general method to generate physically admissible (elastically stable) *non-local constitutive laws* by means of "simple" Laplacian matrix functions. The model can be generalized to model non-locality in $n = 2, 3, \ldots$ dimensions of the physical space.

3.20 Dislocation-Based Fracture Mechanics Within Generalized Continua

Mahmoud Mousavi

Dislocations can be used as macro elements to achieve the elastic model of material weakened by defects. In this talk, the application of the dislocation-based fracture mechanics is presented for the analysis of materials within **generalized continuum mechanics** including **gradient elasticity** and **nonlocal elasticity**. The motivation for this study is the fact that the singularity of the dislocation is regularized within these frameworks. Consequently, it is expected that the crack (which is modeled by distributing the dislocations along the crack path) have also nonsingular fields.

The dislocation is a line defect which gives rise to elastic and plastic distortion. The dislocation density of a single dislocation can be convoluted with a so-called distribution function, so that the boundary conditions of the crack-faces are satisfied. The unknown distribution function is to be determined using the appropriate boundary conditions. Using such distribution of the dislocations and considering the superposition principle, all field quantities of the material including stress, strain, displacement, dislocation density and plastic distortion are derived. The crack tip plasticity is also captured without any ad-hoc assumption.

[54] S.M. Mousavi and E.C. Aifantis. "A Note on Dislocation-based Mode III Gradient Elastic Fracture Mechanics". In: *Journal of the Mechanical Behavior of Materials* in press (2015).

[55] S.M. Mousavi and S.J. Fariborz. "Anti-plane elastodynamic analysis of cracked graded orthotropic layers with viscous damping". In: *Applied Mathematical Modelling* 36 (2012), pp. 1626–1638.

[56] S.M. Mousavi and A.M. Korsunsky. "Non-singular fracture theory within nonlocal anisotropic elasticity". In: *Material and Design* accepted (2015).

[57] S.M. Mousavi and M. Lazar. “Distributed dislocation technique for cracks based on non-singular dislocations in nonlocal elasticity of Helmholtz type”. In: *Engineering Fracture Mechanics* 136 (2014), pp. 79–95.

[58] S.M. Mousavi, J. Paavola, and D. Baroudi. “Distributed nonsingular dislocation technique for cracks in strain gradient elasticity”. In: *Journal of the Mechanical Behavior of Materials* 23 (2014), pp. 47–58.

3.21 Size-Dependent Modeling of Piezoelectric Materials and Devices under Acoustic Loads

Andrey Nasedkin

This research presents the new size-dependent models of piezoelectric materials oriented to finite element applications. The proposed models include the facilities of taking into account different mechanisms of damping for mechanical fields as well as for electric and magnetic fields. The coupled models also incorporate the equations of the theory of acoustics for viscous fluids. In particular cases, these models permit to use the mode superposition method with full separation of the finite element systems into independent equations for the independent modes for transient and harmonic problems.
The main boundary conditions were supplemented with the facilities of taking into account the surface effects, allowing to explore the nanoscale piezoelectric materials in the framework of theories of continuous media with surface stresses and their generalizations.
For piezoelectric nanocomposite material we develop the general models in the framework of classic continuum approaches of solid mechanics and methods of composite mechanics account for their internal microstructure and surface effects.
For all the problems we have implemented the finite element method technologies based on the generalized statements. Here we have used various numerical algorithms to maintain a symmetrical structure of the finite element quasi-definite matrices (matrix structure for the problems with a saddle point), as well as the algorithms of the mode superposition methods.
To increase the accuracy of calculations, especially for nanoscale and nonlinear problems, we have provided the ability of automatic transfer to the dimensionless problem statements.
Analysis of the well-known finite element software shows that the proposed models and technologies significantly increase the facilities for analyzing complex active materials and provide the methods of solving new problems with coupled physico-mechanical fields, including the problems for nanoscale and composite bodies.
This work was supported by the Russian Science Foundation (grant number 15-19-10008).

3.22 A Variant of the Indeterminate Couple Stress Model with Symmetric Force Stress Tensors and Symmetric Couple Stress Tensors

Patrizio Neff

We present a new model variant of the indeterminate couple stress model and discuss its connection to existing formulations. Of central importance is the prescription of boundary conditions, which may differ for the same bulk equations. This is a reminder that a variational formulation in itself is not yet complete without prescribing boundary conditions. Our new model has the decisive advantage that all stress and couple stress tensors remain symmetric, a feature impossible in the classical approach. We also discuss and refute recent claims in the literature that the couple stress tensor must be skew-symmetric. Our results motivate, to the contrary, that the couple stress tensor should be symmetric.

3.24 Some Analytical Solutions in the Theory of Thin Bodies

Mikhail Nikabadze and Armine Ulukhanyan

The inverse tensor-operators to a tensor-operator of the equations of motion in terms of displacements for an isotropic homogeneous material and to a stress-operator are found. They allow decomposing equations and boundary conditions. The inverse matrix differential tensor-operator to the matrix differential tensor-operator of the micropolar theory of elasticity equations of motion in displacements and rotations is built as for isotropic homogeneous materials with a center of symmetry as for materials without one. We obtain the equations as in vector of displacement as in vector of rotation. As a special case, a reduced continuum is considered. Cases in which it is easy to invert the stress-operator and the moment stress-operator are found out.
From the decomposed equations of classical (micropolar) theory of elasticity, the corresponding decomposed equations of quasistatic problems of theory of prismatic bodies with constant thickness in displacements (in displacements and rotation) are obtained. From these systems of equations, we derive the equations in moments of unknown vector functions with respect to any system of orthogonal polynomials. We obtain the various approximations systems of equations (from zero to eighth order) in moments with respect to the systems of Legendre and second kind Chebyshev polynomials. The system splits and for each moment of unknown vector-function we obtain an equation of elliptic type of high order (order of the system depends on the order of approximation), the characteristic roots of which are easily found. Using the method of Vekua, we can get their analytical solution.
For micropolar theory of thin prismatic bodies with two small sizes having the rectangular cross-section, the split equations in moments of displacement and rotation vectors via an arbitrary the system of polynomials (Legendre, Chebyshev) are obtained. We also deduce similar equations for the reduced continuum involving the classical equation of the continuum.
The split systems of equations of eight approximation for micropolar theory of multilayer prismatic bodies of constant thickness in the moments of the displacement and rotation vectors are obtained. Using Vekua method, we can found the analytical solution for this system and for equations for reduce continuum.

3.23 Eigenvalue Problems of any even Rank Tensor and Tensor-Block Matrix with some Applications in Mechanics

Mikhail Nikabadze

Eigenvalue problems of the tensor and the tensor-block matrix of any even rank are considered. Formulas expressing classical invariants of any even rank tensor as with the help of the tensors and the extended tensors of minors as with the help of the tensors and the extended tensors of algebraic cofactors are given. We also obtain formulas for the classical invariants of the any even rank tensor through the first invariants of degrees of this tensor. The inverse formulas to these formulas are also given. Some definitions, statements and theorems are formulated about tensors and tensor-block matrix of any even rank. A complete orthonormal system of eigentensor of symmetric tensor, as well as a complete orthonormal system of eigentensor columns of the symmetric tensor-block matrix are construct. As a special case, we consider the fourth rank tensor and tensor-block matrix, and sixth rank tensor. It is proved that a tensor-block matrix of elastic moduli tensor is positive-definite. In micropolar theory, the characteristic equation of the tensor-block matrix has 18 positive roots counted each root according to its multiplicity. Therefore, the complete orthonormal system of eigentensor columns of the tensor-block matrix consists of 18 tensor-columns. Canonical representation of the tensor-block matrix is given. Using this presentation, we get the canonical form of the elastic strain energy and constitutive relation.

We introduce the conception of symbol of structure of the tensor-block matrix and give a classification of the tensor-block matrix of elastic modulus tensors of the micropolar linear elasticity of anisotropic bodies without a center of symmetry. All linear micropolar anisotropic elastic materials which do have not a center of symmetry, are divided into 18 classes according to the number of different eigenvalues, and the classes depending on the multiplicity of the eigenvalues are subdivided into subclasses. All told above is equally true to linear micropolar theory of elasticity of anisotropic bodies with a center of symmetry. In this case, it is enough to study the internal structure of each of the two positive-definite tensors of elastic moduli separately. In the latter case, in contrast to the classical case, the characteristic equation for each elastic modulus tensor has the 9th degree (in the classical theory of elasticity, characteristic equation has the 6th degree).

It is shown that, if we make a classification of the set of positive-definite symmetric four rank tensors, then we get the 9 major classes according to the number of different eigenvalues and the classes depending on the multiplicities of the eigenvalues are subdivided into subclasses. In total, we have 256 subclasses (in classical case we have six classes consisting of 32 subclasses). Thus, if each of the anisotropic materials corresponds to the elastic modulus tensors of the same structure, the number of anisotropic materials is 256. If the elastic modulus tensors have the same symbol of structure and belong to the different subclasses, the number of linearly elastic anisotropic materials with a center of symmetry in the sense of the elastic properties is equal to 12870. If tensors have different structures, then the number of materials is 65536. The number of anisotropic materials without a center of symmetry equals 131072.

In an explicit form, we have constructed a complete orthonormal system of eigentensor-columns of tensor-block matrix of elastic modulus tensors using 153 independent parameters as well as a complete orthonormal system of eigentensor-columns of tensor-block-diagonal matrix of elastic modulus tensors with the use of 72 independent parameters and a complete orthonormal system of eigentensors for positive-definite symmetric elastic modulus tensor of the micropolar elasticity theory with the help of 36 independent parameters. The orthonormal system of eigentensors for elastic modulus tensor with 15 independent parameters in the classical theory of elasticity was explicitly completed by N.I. Ostrosablin. We also consider the classification of classical anisotropic materials.

The eigenvalues and eigentensors for classical materials of crystallographic syngonies different from the forms produced by N.I. Ostrosablin as well as for some micrpolar materials are found.

3.25 On Stability of Planar Square Lattice

Ekatarina A. Podolskaya, Artem Yu. Panchenko, and Anton M. Krivtsov

This work focuses on the stability investigation of planar square lattice. Analysis based on atomistic methods for determination of the generalized continuum parameters is proposed. It is known that the pair force interaction does not allow the stability of stress-free square lattice because its shear modulus is zero within nearest neighbor interaction. However, strong ellipticity analysis shows that the continuum, which is constructed from square lattice using Cauchy-Born rule, can exist at certain all-round extension. This effect is similar to that arising in strings. The size and the shape of strong ellipticity domain in strain space depends on the parameters of the interaction potential. Numerical experiment (molecular dynamics simulation) is done to refine the obtained continuum result from discrete point of view as strong ellipticity is necessary stability condition only. The dependence of sample lifetime on its size and on the perturbation magnitude is additionally looked into. The influence of pair torque interaction, which stabilizes the stress-free square lattice in continuum analysis, is discussed.

3.26 Method for Calculating the Characteristics of Elastic State Media with Internal Degrees of Freedom

Victoria Yu. Presnetsova, Sergey N. Romashin, Larisa Yu. Frolenkova, and Vladimir S. Shorkin

In practical use of non-traditional models of local Cosserat, Leroux, Tupin, Mindlin, Aero, the problem of determining the elastic constants arise. This work is devoted to solving this problem. The solution is based on a comparison of the defining relation of non-traditional model with its counterpart. It is obtained from the conversion of a specially constructed for this purpose variant nonlocal elastic medium in the local model.

3.27 Variational Derivation of General Finite-Deformation Two-Phase Poroelasticity

Roberto Serpieri, Francesco dell'Isola, Allesandro della Corte, Francesco Travascio, and Luciano Rosati

Biphasic poro-elasticity is considered a very useful tool in a multiplicity of applications in biomechanics, geomechanics, and impact engineering. Nevertheless, the achievement of a general consistent theory of two-phase poroelasticity capable of addressing media with any degree and range of compressibility of the constituent phases still represents an open problem in theoretical and computational solid mechanics [59]. Since the first seminal works by Mindlin [35] and Bedford and Drumheller [60], a considerable amount of effort has been spent to frame the relevant governing equations into a purely variational derivation [61, 62]. The present contribution expands on the formulations proposed in these two previous works, and provides a finite-deformation variational derivation of a biphasic poro-elastic generalized continuum theory accounting for full compressibility of both phases. Specifically, the considered strain energy density functional includes the dependency on the first deformation gradient of the solid phase and on its effective volumetric strain. Upon determining the relevant Euler-Lagrange equations, it is shown that such a variational formulation admits Terzaghi's formula [63] as a general partitioning law between stress tensors. The reported results suggest that this variational theory represents an adequate methodological approach for pursuing a general formulation of multiphase continuum mechanics.

[35] R.D. Mindlin. "Micro-structure in linear elasticity". In: *Archive for Rational Mechanics and Analysis* 16 (1964), pp. 51–78.

[59] R. de Boer. "Theoretical poroelasticity - a new approach". In: *Chaos, Solitons & Fractals* 25 (2005), pp. 861–878.

[60] A. Bedford and D.S. Drumheller. “Theories of immiscible and structured mixtures”. In: *Int. J. Eng. Sci.* 21 (1983), pp. 863–960.

[61] F. dell’Isola, A. Madeo, and P. Seppecher. “Boundary conditions at fluid-permeable interfaces in porous media: A variational approach”. In: *International Journal of Solids and Structures* 46 (2009), pp. 3150–3164.

[62] R. Serpieri and L. Rosati. “Formulation of a finite deformation model for the dynamic response of open cell biphasic media”. In: *Journal of the Mechanics and Physics of Solids* 59 (2011), pp. 841–862.

[63] K. Terzaghi. “The shearing resistance of saturated soils and the angle between the planes of shear”. In: *International Conference on Soil Mechanics and Foundation Engineering*. Cambridge, MA, 1936.

3.28 Buckling of Sandwich Tube with Foam Core under Combined Loading

Denis N. Sheydakov

In the framework of a general stability theory for three-dimensional bodies the buckling analysis is carried out for the nonlinearly elastic three-layer cylindrical tube subjected to axial compression, torsion, and inflation. It is assumed that the middle layer (core) of the tube is made of metal or polymer foam, and the model of micropolar continuum is used to describe its behavior. Such approach allows to study in detail the influence of foam microstructure on the deformation stability, which is especially important when the macroscopic dimensions of the tube are comparable with the average size of the foam cells. The inner and outer layers (coatings) of the tube are assumed to be made of the classic non-polar materials. Applying linearization, the neutral equilibrium equations have been derived, which describe the perturbed state of the cylindrical sandwich tube. By solving these equations numerically for some specific materials, the critical surfaces and corresponding buckling modes have been found and the stability regions have been constructed in the space of loading parameters (relative axial compression, twist angle, internal pressure). Using the obtained results, the influence of coatings properties, as well as the overall size of the tube, on the loss of stability has been analyzed.
This work was supported by the Russian Science Foundation (grant number 14-19-01676).

3.29 Thermodynamic and Kinematic Aspects of the Continuum Dislocation Theory

Christian B. Silbermann, Khanh Chau Le, Matthias Baitsch, and Jörn Ihlemann

In order to simulate the mechanical behavior of metals with dislocation cells under load path changes, a model was developed in [64] which operates with scalar dislocation densities. A desired validation of model assumptions as well as the estimation of introduced microstructural parameters requires a lower scale theory. An appropriate means is the Continuum Dislocation Theory (CDT) [65] which relates macroscopic plastic deformation with Geometrically Necessary Dislocations (GNDs). Within CDT, the dislocation density tensor is a thermodynamic state variable, which reflects tensorial dislocation properties and allows the consideration of large dislocation ensembles. The thermodynamically consistent CDT of Le and coworkers [66, 67] captures both the strain energy of the deformed crystal and the elastic energy of dislocations plus the dissipation of energy due to dislocation motion. This enables the analysis of dislocation structures during plastic deformation. To this end, the FEM code of Baitsch [68] is used, and simulation results of plane strain deformation processes with single slip condition are presented. Special attention is put on the effect of time-dependent boundary conditions in order to simulate load path changes as well as the effect of dissipation on the resulting dislocation structures. Therefore, different dissipation approaches are investigated. Furthermore, the thermodynamical meaning of the dissipation potential and the microstructural interpretation of underlying dissipative mechanisms are discussed. Finally, some challenges of the theory's enhancement towards multi-slip conditions are discussed.

[64] C.B. Silbermann, A.V. Shutov, and Ihlemann J. "Modeling the evolution of dislocation populations under non-proportional loading". In: *International Journal of Plasticity* 55 (2014), pp. 58–79.

[65] B.A. Bilby, L.R.T. Gardner, and A.N Stroh. "Continuous distributions of dislocations and the theory of plasticity". In: *Proc. XIth ICTAM*. Vol. 8. 1957, pp. 35–44.

[66] K.C. Le and P. Sembiring. "Analytical solution of plane constrained shear problem for single crystals within continuum dislocation theory". In: *Archive of Applied Mechanics* 78 (2008), pp. 587–597.

[67] K.C. Le and Q.S. Nguyen. "On bending of single crystal beam with continuously distributed dislocations". In: *International Journal of Plasticity* 48 (2013), pp. 152–167.

[68] K.C. Le, M. Baitsch, and T.M. Tranc. "Evolution of dislocation structure during indentation". In: *International Journal of Engineering Science* 94 (2015), pp. 195–211.

3.30 Frequency-Dependent Attenuation and Phase Velocities Dispersion of an Acoustical Wave Propagation in the Media with Damages

Anatoli Stulov and Vladimir I. Erofeev

In frame of the self-consistent mathematical model, which includes the dynamics of a material and the state of its defects, the particular qualities of acoustic wave propagation in the material with damages are considered. In this study, a constitutive equation of the damaged medium is derived, and the similarity between the models for damaged materials and the medium with memory is confirmed. The dispersion analysis of the model is carried out, and it is shown that the damage of the material gives rise to frequency-dependent attenuation and anomalous dispersion of phase velocity of acoustic wave propagating through that material. This makes it possible to estimate the damage of the material by means of a nondestructive acoustic method.

3.31 Particle Random Composites as Micropolar Continua: A Statistically Based Multiscale Procedure

Patrizia Trovalusci, Maria Laura de Bellis, and Martin Ostoja-Starzewski

A wide range of composite materials display random morphologies. Among others, we focus on particle composites, where randomly distributed inclusions are embedded in a different matrix. Examples range from polycrystals up to concrete and masonry-like materials.

The aim of this work is the definition of a statistically-based scale-dependent multiscale procedure for the evaluation of the 2D effective mechanical behavior of such random composites, [69]. The homogenized response of the heterogeneous medium is investigated resorting to a two-scale micropolar computational approach, able to naturally account for length scales parameters and skew-symmetric shear effects.

Micropolar continua are adopted at both scales in order to account for a physically-based length scale of the microstructure and for the skew-symmetric part of shear stress and strain. More specifically, at the macro-scale the micropolar continuum is particularly suitable in the case the characteristic length at the meso-scale is not negligible with respect to the macroscopic specimen size. At the meso-scale, instead, the micropolar continua is the result of a further homogenization process, involving a micro-scale where the microstructure of the actual material can be modeled as a network of interconnected beams.

The key idea of the procedure is to approach the so-called Representative Volume Element (RVE) using finite-size scaling of Statistical Volume Elements (SVEs). To this end properly defined Dirichlet, Neumann and periodic-type, [70], non-classical boundary value problems are numerically solved on the SVEs defining hierarchies of constitutive bounds.

Two material cases of inclusions, either stiffer or softer than the matrix, are studied and it is found that, independent of the contrast in moduli, the RVE size for the bending micropolar moduli is smaller than that obtained for the classical moduli. The results of the performed numerical simulations point out the worthiness of accounting spatial randomness as well as the additional degrees of freedom of the Cosserat continuum.

[69] P. Trovalusci et al. “Particulate random composites homogenized as micropolar materials”. In: *Meccanica* 49 (2014), pp. 2719–2727.

[70] P. Trovalusci et al. “Scale-dependent homogenization of random composites as micropolar continua”. In: *European Journal of Mechanics A/Solids* 49 (2014), pp. 396–407.

3.32 Indentation of a Rigid Stamp into an Elastic Transversely-Isotropic Functionally-Graded Layer Lying on a Transversely-Isotropic Half-Plane

Andrey Vasiliev, Sergey Aizikovich, and Sergey Volkov

Indentation of a rigid spherical punch or punch with flat base into an elastic transversely-isotropic functionally-graded layer lying on an elastic transversely-isotropic half-plane is considered. Elastic moduli of the layer vary with depth according to arbitrary functions. Boundaries of the contact can be free or fixed (contact stresses are not equal zero at the boundary of the contact area) for a case of a spherical punch. Two types of boundary conditions on the interface between the functionally-graded layer and homogeneous half-plane are also considered.

A semi-analytical method is used for construction of an approximated solution of the problem in an analytical form. The method is efficient for the layer of arbitrary thickness.

The main results include computations of the profiles of the contact stresses under the punch for different types of variation of elastic properties in the layer. Cases of a much softer or harder substrate (value of a Young’s modulus of the substrate is up to 100 times smaller or bigger than that of the layer) are analysed.

3.33 Micropolar Continuum in Eulerian Description

Elena Vilchevskaya and Elena Ivanova

Within the spatial description it is customary to present thermodynamic state quantities with respect to an elementary volume fixed in space containing an ensemble of particle. During its evolution the elementary volume is occupied by different particles, where each of these particles has its own mass, inertia tensor, angular and translational velocities. The aim of the present paper is to answer the question of how inertia and kinematic characteristics of the elementary volume can be determined. In order to model structural transformations related to the consolidation or defragmentation of particles or to an anisotropy change one should take into account that the inertia tensor of the elementary volume may change. It means that an additional constitutive equation has to be formulated. Some kinetic equations for the inertia tensor of the elementary volume are suggested. A specificity of inelastic polar continuum description in the frame of the spatial description is discussed.

3.34 Are Composite Laminates Cosserat Materials?

Marcus Wheel

More generalized continuum theories such as couple stress, Cosserat and micromorphic elasticity apparently describe the mechanical behaviour of loaded heterogeneous materials when the size scale approaches that of the intrinsic microstructure of the material. Such materials are forecast to exhibit a size effect in which stiffness increases as overall size is reduced to that of the microstructure, a phenomenon that is not predicted by classical or Cauchy elasticity theory. Materials comprised of lattices of straight, slender connectors are known to exhibit behaviour consistent with that forecast by Cosserat elasticity and analytically obtained estimates of the relevant constitutive parameters have been widely reported in the literature. Constitutive parameter data, obtained experimentally by determining the size effects exhibited by real materials when loaded in a non-uniform state of stress, usually by bending or twisting, have also been reported. Simple composite laminate materials are also believed to display generalized continuum behaviour. However, it can readily be shown that finite sized samples of such materials when loaded in bending can actually exhibit a much more fascinating variety of size effects, including an increase in compliance with reducing size, which evidently contradicts forecast effects. The conditions under which these materials exhibit this wealth of size effects will be presented and the practical implications for the experimental testing of representative samples of such materials to identify their relevant constitutive properties will be fully discussed.

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