Lösungen

zur

Technischen Mechanik

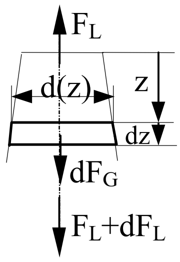
- Elastostatik -

Ausgabe 2016

Lösungen zur Aufgabensammlung Elastostatik (Ausgabe 2000, 2016)

2. Zug und Druck

Lösung 2.1



$$dF_G = \rho \cdot g \cdot \frac{\pi}{4} d(z)^2 \cdot dz \qquad d(z) = D\left(1+\frac{z}{h}\right)$$

$$\downarrow: dF_L + \rho \cdot g \cdot \frac{\pi}{4} d(z)^2 \cdot dz = 0$$

$$F_L(z) = -\int \rho \cdot g \cdot \frac{\pi}{4} \cdot D^2 \left(1+\frac{z}{h}\right)^2 dz + C^*$$

$$F_L(z) = -\rho \cdot g \cdot \frac{\pi}{4} \cdot D^2 \left(z + \frac{z^2}{h} + \frac{1}{3}\frac{z^3}{h^2}\right) + C$$

$$RB: F_L(z=0) = -F \quad \Rightarrow \quad C = -F$$

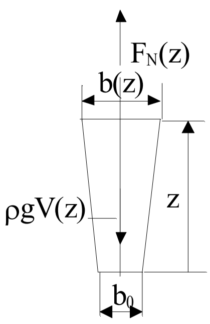
$$F_L(z) = -\rho \cdot g \cdot \frac{\pi}{4} \cdot D^2 \left(z + \frac{z^2}{h} + \frac{1}{3}\frac{z^3}{h^2}\right) - F$$

$$\sigma(z) = \frac{F_L(z)}{A(z)} \qquad A(z) = \frac{\pi}{4} d^2(z)$$

$$\sigma(z) = -\rho \cdot g \cdot z \frac{1+\frac{z}{h}+\frac{1}{3}\frac{z^2}{h^2}}{\left(1+\frac{z}{h}\right)^2} - \frac{4F}{\pi D^2 \left(1+\frac{z}{h}\right)^2}$$

$$\sigma(z=h) = -\frac{7}{12}\rho \cdot g \cdot h - \frac{F}{\pi D^2} = -2{,}68 \frac{N}{mm^2}$$

Lösung 2.2



$$\uparrow: \; F_N(z) = \rho g V(z) \qquad A(z) = a \cdot b(z) = a b_0 \left(1+\frac{z}{l}\right)$$

$$\sigma = \frac{F_N(z)}{A(z)} = \rho g \frac{V(z)}{A(z)} \qquad b(z) = b_0\left(1+\frac{z}{l}\right)$$

$$= \frac{1}{2}\rho g \frac{\left(2+\frac{z}{l}\right) z}{\left(1+\frac{z}{l}\right)} \qquad V(z) = \frac{1}{2}\left[b_0 + b(z)\right] a \cdot z$$

$$\boxed{\sigma = \frac{1}{2}\rho g \frac{(2l+z)\cdot z}{l+z}} \qquad = \frac{1}{2} a b_0 \left(2+\frac{z}{l}\right) z$$

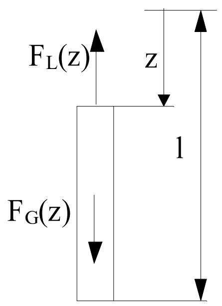
$$z = l \qquad \boxed{\sigma(l) = \frac{3}{4}\rho g l}$$

$$\Delta l = \int_0^l \frac{F_N(z)}{EA(z)}dz = \frac{1}{2}\frac{\rho g}{E}\int_0^l \frac{z^2+2lz}{z+l}dz = \frac{1}{2}\frac{\rho g}{E}\left\{\int_0^l z\,dz + l\int_0^l \frac{z}{z+l}dz\right\}$$

$$= \frac{1}{2}\frac{\rho g}{E}\left[\frac{z^2}{2} + l\big(z - l\ln(z+l)\big)\right]_{[0,l]} = \frac{1}{2}\frac{\rho g}{E}\left(\frac{l^2}{2} + l^2(1-\ln 2)\right)$$

$$\boxed{\Delta l = \frac{1}{4}\frac{\rho g l^2}{E}\left(3 - 2\ln 2\right)}$$

Lösung 2.3



$F_L - F_G(z) = 0 \qquad F_L = F_G(z) = \rho g A \cdot (l-z)$

$F_L(z=0) = F_{Lmax} = \rho g A l \qquad \sigma_{max} = \rho g l = 463\ \mathrm{Nmm^{-2}}$ für $l = 6000\mathrm{m}$

$$dw = \varepsilon dz = \frac{\sigma(z)}{E}dz$$

$$w(z) = \frac{\rho g}{E}\int_0^z (l - z^*)dz^* + w(z=0)$$

$$w(l) = \frac{\rho g}{E}\int_0^l (l-z)dz = \frac{1}{2}\frac{\rho g l^2}{E} = 6{,}61\mathrm{m}$$

$$l = 10000\mathrm{m}:\quad \sigma_{max} = 771\frac{\mathrm{N}}{\mathrm{mm}^2}$$

Mindeststreckgrenzen für

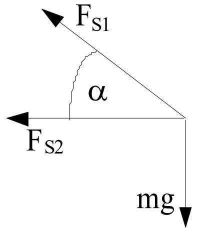
St 60 : $\sigma_F = 324\mathrm{Nmm^{-2}}$; 30MnCrSi6 : $\sigma_F = 590\mathrm{Nmm^{-2}}$; 50CrMo4 : $\sigma_F = 883\mathrm{Nmm^{-2}}$;

Lösung 2.5

$$\sigma = -\frac{F}{A};\quad A = \frac{\pi d^2}{4};\quad \varepsilon = \frac{\sigma}{E};\quad \varepsilon_q = -\nu\varepsilon;\quad d_1 = \left(1+\varepsilon_q\right)d$$

$$\sigma = -127{,}3\frac{\mathrm{N}}{\mathrm{mm}^2};\quad \varepsilon = -0{,}000618;\quad \varepsilon_q = 0{,}000185;\quad d_1 = 20{,}0037\mathrm{mm}$$

Lösung 2.6



$$\uparrow:\quad F_{S1}\sin\alpha - mg = 0 \quad F_{S1} = \frac{mg}{\sin\alpha} = 2mg \quad F_L = F_{S1}$$

$$\sigma = \frac{F_{S1}}{A} \le \sigma_{zul} \quad \Rightarrow \quad A_{erf} = \frac{2mg}{\sigma_{zul}} = 294{,}3\mathrm{mm}^2$$

$$A_{gew} = 297\mathrm{mm}^2 \text{ für T35} \quad \sigma_{vorh} = 99{,}1\frac{\mathrm{N}}{\mathrm{mm}^2} \le \sigma_{zul} = 100\frac{\mathrm{N}}{\mathrm{mm}^2}$$

1. Verlängerung der einzelnen Abschnitte: Δl_1, Δl_2

   Für $\Delta l_1 + \Delta l_2 < \delta$ ist $F_L = 0$, für $\Delta l_1 + \Delta l_2 > \delta$ ist $F_L \neq 0$

$$\Delta l_1 = l_1\left(\frac{F_L}{E_1 A_1} + \alpha_{th}\cdot\Delta T\right); \qquad \Delta l_2 = l_2\left(\frac{F_L}{E_2 A_2} + \alpha_{th}\cdot\Delta T\right)$$

$$F_L\left(\frac{l_1}{E_1 A_1} + \frac{l_2}{E_2 A_2}\right) + \alpha_{th}\cdot\Delta T\left(l_1 + l_2\right) = \delta$$

Hookesches Gesetz:

$$F_L = -\frac{\alpha_{th}\cdot\Delta T\left(l_1 + l_2\right) - \delta}{\frac{l_1}{E_1 A_1} + \frac{l_2}{E_2 A_2}}$$

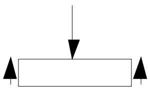
$$F_L = 0 \quad \Rightarrow \quad \Delta T = \Delta T^* = \frac{\delta}{\alpha_{th}\cdot\left(l_1 + l_2\right)}$$

$$\text{Für} \quad \Delta T < \Delta T^* \quad \text{ist} \quad F_L = 0$$

$$\text{Zahlenwerte}: \quad \Delta T^* = 28{,}6K \qquad F_L = -21{,}7\cdot 10^3\,N$$

$$\text{wegen} \quad A_2 < A_1 \quad \Rightarrow \quad \sigma_{\max} = \frac{F_L}{A_2} = -31\frac{N}{mm^2}$$

Lösung 2.7



$$\uparrow:\ \tau\cdot\pi\cdot d\cdot h - F = 0 \qquad F = \tau\cdot\pi\cdot d\cdot h = 15{,}7\ \text{kN}$$

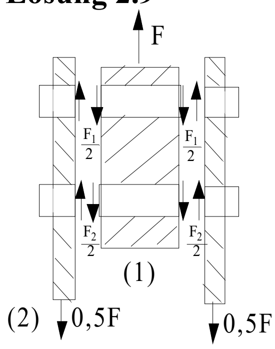
Lösung 2.8

1. Abscheren (Bolzen) $\quad \tau_{\text{vorh}} = \dfrac{2F}{\pi d^2} = 6{,}29\dfrac{\text{N}}{\text{mm}^2} < \tau_{\text{zul}} = 30\dfrac{\text{N}}{\text{mm}^2}$

2. Pressung (Bolzen) $\quad \sigma_{\text{Lvorh}} = \dfrac{F}{d\delta} = 9{,}88\dfrac{\text{N}}{\text{mm}^2} < \sigma_{\text{Lzul}} = 10\dfrac{\text{N}}{\text{mm}^2}$

3. Zug (Blech) $\quad \sigma_{\text{vorh}} = \dfrac{F}{(a-d)\delta} = 9{,}88\dfrac{\text{N}}{\text{mm}^2} < \sigma_{\text{zul}} = 50\dfrac{\text{N}}{\text{mm}^2}$

Lösung 2.9



Dehnung der Bleche zwischen den beiden Bolzen:

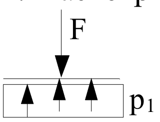
$$\varepsilon_1 = \frac{F_2}{Ea\delta_1} \qquad \varepsilon_2 = \frac{F_1}{2Ea\delta_2}$$

$$F_1 + F_2 = F \qquad \varepsilon_1 = \varepsilon_2 \quad \Rightarrow \quad \frac{F-F_1}{Ea\delta_1} = \frac{F_1}{2Ea\delta_2}$$

$$F_1 = \frac{2F\delta_2}{\delta_1 + 2\delta_2} = 5{,}714\text{kN} \qquad F_2 = F - F_1 = 4{,}286\text{kN}$$

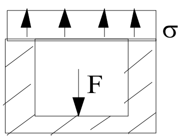
Lösung 2.10

1. Flächenpressung Bolzen/Buchse



$$\uparrow:\ p_1\cdot\frac{\pi}{4}d_1^2 - F = 0 \quad \Rightarrow \quad d_{1\text{erf}} = \sqrt{\frac{4F}{p_1\pi}} = 56{,}42\text{mm} \qquad d_{1\text{gew}} = 60\text{mm}$$

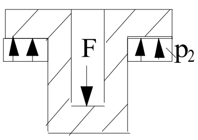
2. Zug in der Buchse



$$\uparrow:\ \sigma\cdot\frac{\pi}{4}\left(d_2^2 - d_1^2\right) - F = 0 \quad \Rightarrow \quad d_{2\text{erf}} = \sqrt{\frac{4F}{\sigma\pi} + d_1^2} = 69{,}81\text{mm}$$

$$d_{2\text{gew}} = 70\text{mm}$$

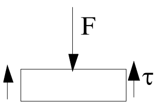
3. Flächenpressung Bund der Buchse



$$\uparrow:\ p_2\cdot\frac{\pi}{4}\left(d_3^2 - d_2^2\right) - F = 0 \quad \Rightarrow \quad d_{3\text{erf}} = \sqrt{\frac{4F}{p_2\pi} + d_2^2} = 78{,}57\text{mm}$$

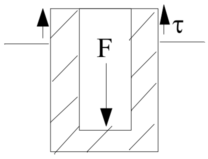
$$d_{3\text{gew}} = 80\text{mm}$$

4. Abscherung des Bodens der Buchse



$$\uparrow:\ \tau\cdot\pi d_1 h_1 - F = 0 \qquad h_{1\text{erf}} = \frac{F}{\tau\cdot\pi d_1} = 10{,}62\text{mm} \qquad h_{1\text{gew}} = 12\text{mm}$$

5. Abscherung des Bundes der Buchse



$$\uparrow: \quad \tau \cdot \pi d_2 h_2 - F = 0 \qquad h_{2erf} = \frac{F}{\tau \cdot \pi d_2} = 9{,}099\,mm \qquad h_{2gew} = 10\,mm$$

Spannungsnachweis für die Buchse:

1. $p_1 = \dfrac{4F}{\pi d_1^2} = 17{,}68 \dfrac{N}{mm^2} < p_{1zul} = 20 \dfrac{N}{mm^2}$

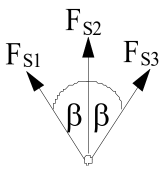
2. $\sigma = \dfrac{4F}{\pi\left(d_2^2 - d_1^2\right)} = 48{,}97 \dfrac{N}{mm^2} < \sigma_{zul} = 50 \dfrac{N}{mm^2}$

3. $p_2 = \dfrac{4F}{\pi\left(d_3^2 - d_2^2\right)} = 42{,}44 \dfrac{N}{mm^2} < p_{2zul} = 50 \dfrac{N}{mm^2}$

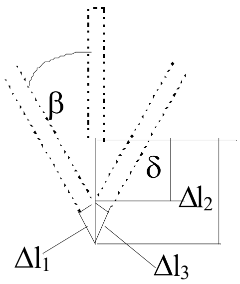
4. $\tau = \dfrac{F}{\pi d_1 h_1} = 22{,}11 \dfrac{N}{mm^2} < \tau_{zul} = 25 \dfrac{N}{mm^2}$

5. $\tau = \dfrac{F}{\pi d_2 h_2} = 22{,}74 \dfrac{N}{mm^2} < \tau_{zul} = 25 \dfrac{N}{mm^2}$

Lösung 2.12



Das System ist einfach statisch unbestimmt.
Gleichgewicht: $\rightarrow$: $F_{S1} = F_{S3}$
$\uparrow$: $F_{S2} = -2F_{S1}\cos\beta$



$$\Delta l_1 = \Delta l_3 = \left(\Delta l_2 - \delta\right)\cos\beta \qquad l_1 = l_3 = \frac{l}{\cos\beta}$$

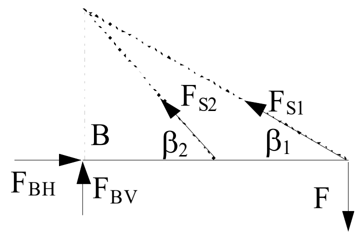
$$\Delta l_1 = \frac{F_{S1} l_1}{(EA)_1}; \qquad \Delta l_2 = \frac{F_{S2} l_2}{(EA)_2}$$

$$\frac{F_{S1}\dfrac{l}{\cos\beta}}{EA} = \left(\frac{F_{S2} l}{EA} - \delta\right)\cos\beta \qquad F_{S1} = \cos^2\beta\left(-2F_{S1}\cos\beta - \frac{AE}{l}\delta\right)$$

$$F_{S1} = F_{S3} = -\frac{EA\delta \cdot \cos^2\beta}{l\left(1 + 2\cos^3\beta\right)} = -45{,}6\,kN; \qquad F_{S2} = 2\frac{EA\delta \cdot \cos^3\beta}{l\left(1 + 2\cos^3\beta\right)} = 79\,kN$$

$$\sigma_{max} = \frac{F_{S2}}{A} = \frac{2E\delta \cdot \cos^3\beta}{l\left(1 + 2\cos^3\beta\right)} = 198 \frac{N}{mm^2}$$

Lösung 2.13



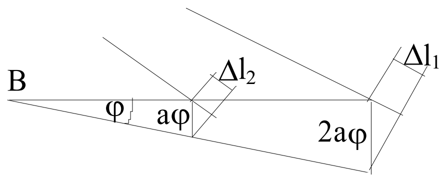
Das System ist einfach statisch unbestimmt.

$$\sigma_i = \frac{F_{Si}}{A_i} \quad i = 1,2$$

B: $F_{S1} \cdot \sin\beta_1 \cdot 2a + F_{S2} \cdot \sin\beta_2 \cdot a - F \cdot 2a = 0$

$$2F_{S1} \cdot \sin\beta_1 + F_{S2} \cdot \sin\beta_2 = 2F$$

$$\sin\beta_1 = \frac{\sqrt{5}}{5}; \quad \sin\beta_2 = \frac{\sqrt{2}}{2}; \quad l_1 = \frac{a}{\sin\beta_1}; \quad l_2 = \frac{a}{\sin\beta_2}$$



Kleine Verformungen:

$$\Delta l_1 = 2a \cdot \varphi \cdot \sin\beta_1$$
$$\Delta l_2 = a \cdot \varphi \cdot \sin\beta_2$$

$$\varepsilon_i = \frac{\sigma_i}{E_i} + \alpha_{th} \cdot \Delta T; \quad \varepsilon_i = \frac{\Delta l_i}{l_i}; \quad F_{Si} = E_i A_i \left(\frac{\Delta l_i}{l_i} - \alpha_{th} \cdot \Delta T \right)$$

$$\frac{\Delta l_1}{l_1} = 2\varphi \sin^2\beta_1 = \frac{2}{5}\varphi; \quad \frac{\Delta l_2}{l_2} = \varphi \sin^2\beta_2 = \frac{1}{2}\varphi$$

In die Gleichgewichtsbedingung eingesetzt, erhält man

$$\varphi = \frac{\frac{2F}{EA} + \alpha_{th}\Delta T\left(2\sin\beta_1 + \sin\beta_2\right)}{4\sin^3\beta_1 + \sin^3\beta_2} = \frac{\frac{2F}{EA} + \alpha_{th}\Delta T\left(\frac{2\sqrt{5}}{5} + \frac{\sqrt{2}}{2}\right)}{\frac{4\sqrt{5}}{25} + \frac{\sqrt{2}}{4}}$$

$$F_{S1} = \frac{4F\sin^2\beta_1 - EA\alpha_{th}\Delta T\left(\sin^2\beta_2 - 2\sin^2\beta_1\right)\sin\beta_2}{4\sin^3\beta_1 + \sin^3\beta_2} > 0$$

$$F_{S2} = \frac{2F\sin^2\beta_2 + 2EA\alpha_{th}\Delta T\left(\sin^2\beta_2 - 2\sin^2\beta_1\right)\sin\beta_1}{4\sin^3\beta_1 + \sin^3\beta_2} > 0$$

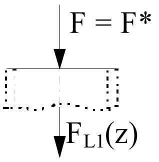
$$F_{S1} = 0 \quad \Rightarrow \quad \Delta T_1 = \frac{4F\sin^2\beta_1}{EA\alpha_{th}\left(\sin^2\beta_2 - 2\sin^2\beta_1\right)\sin\beta_2}$$

$$F_{S2} = 0 \quad \Rightarrow \quad \Delta T_2 = \frac{-F\sin^2\beta_2}{EA\alpha_{th}\left(\sin^2\beta_2 - 2\sin^2\beta_1\right)\sin\beta_1}$$

Verschiebungen des Kraftangriffspunktes für die beiden Grenzwerte:
$v_F(\Delta T_1) = 2a\,\varphi(\Delta T_1)$ und $v_F(\Delta T_2) = 2a\,\varphi(\Delta T_2)$

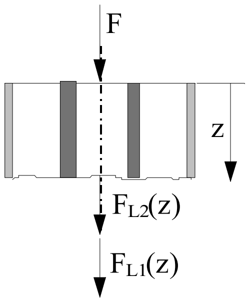
Zahlenwerte:
$\Delta T_1 = 44{,}9°K$; $\Delta T_2 = -44{,}4°K$;
$F_{S1}(\Delta T_1) = 0$; $F_{S2}(\Delta T_1) = 2\sqrt{2}\,F = 1414{,}2N$
$F_{S1}(\Delta T_2) = \sqrt{5}F = 1118{,}0N$; $F_{S2}(\Delta T_2) = 0$
$v_F(\Delta T_1) = 1{,}35mm$; $v_F(\Delta T_2) = 1{,}06mm$

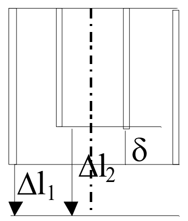
Lösung 2.14



$F_{L1}(z) = - F^*$ $\quad \Delta l_1 = -\frac{F^* l}{E_1 A_1}$; $\quad |\Delta l_1| = \delta = \frac{F^* l}{E_1 A_1}$ $\quad F^* = \frac{\delta}{l} E_1 A_1$

$F \geq F^*$





$$(1) \quad \downarrow: F_{L1}(z) + F_{L2}(z) + F = 0$$

$$(2) \quad \Delta l_2 = \delta + \Delta l_1 \qquad \Delta l_1 = \frac{F_{L1} l}{E_1 A_1}; \qquad \Delta l_2 = \frac{F_{L2} l}{E_2 A_2}$$

$$\frac{F_{L1} l}{E_1 A_1} - \frac{F_{L2} l}{E_2 A_2} + \delta = 0 \left| \cdot \frac{E_1 A_1}{l} \right.$$

$$(1) \quad F_{L1} + F_{L2} + F \qquad = 0 \; | +$$

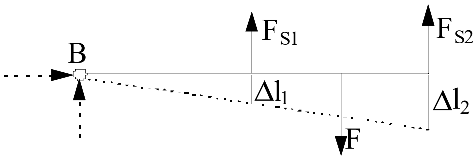
$$(2) \quad F_{L1} - \frac{E_1 A_1}{E_2 A_2} F_{L2} + F^* = 0 \; | -$$

$$F_{L2}\left(1 + \frac{E_1 A_1}{E_2 A_2}\right) + F - F^* = 0 \qquad F_{L2} = -\frac{F - F^*}{\left(1 + \frac{E_1 A_1}{E_2 A_2}\right)}$$

$$F_{L1} = -F_{L2} - F = -\frac{F \frac{E_1 A_1}{E_2 A_2} + F^*}{\left(1 + \frac{E_1 A_1}{E_2 A_2}\right)}$$

$$\Delta l_1 = \frac{F_{L1} l}{E_1 A_1} = -\frac{Fl + \delta E_2 A_2}{E_1 A_1 + E_2 A_2}$$

Lösung 2.15



$$\rightarrow: \quad F_{BH} = 0 \qquad (1)$$

$$\uparrow: \quad F_{BV} = F - F_{S1} - F_{S2} \qquad (2)$$

$$B: \quad 2F_{S1} a + 4F_{S2} a - 3Fa = 0 \qquad (3)$$

$$\Delta l_2 - 2\Delta l_1 = 0 \qquad (4)$$

Gleichungen (3) und (4):

$$4F_{S2} + 2F_{S1} - 3F = 0$$

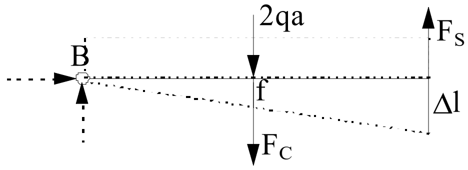
$$\frac{2F_{S2} l}{EA_2} - \frac{2F_{S1} l}{EA_1} = 0 \quad \Rightarrow \quad F_{S1} = \frac{A_1}{A_2} F_{S2}$$

$$4F_{S2} + 2\frac{A_1}{A_2}F_{S2} - 3F = 0$$

$$F_{S2} = \frac{3}{2}F \cdot \frac{1}{2 + \frac{A_1}{A_2}} = \frac{6}{11}F = 1090{,}9\,N \quad F_{S1} = \frac{3}{2}F \cdot \frac{1}{2\frac{A_2}{A_1} + 1} = \frac{9}{22}F = 818{,}2\,N$$

$$F_{BV} = 90{,}9\,N \qquad \Delta l_2 = \frac{2F_{S2}l}{EA_2} = 0{,}109\,mm$$

Lösung 2.16



$$\rightarrow: F_{BH} = 0 \qquad (1)$$

$$\uparrow: F_{BV} = F_C - F_S + 2qa \qquad (2)$$

$$B: 2F_S a - 2qa^2 - F_C a = 0 \qquad (3)$$

$$\Delta l = -2f \qquad (4)$$

Für die lineare Feder gilt: $f = \frac{F_C}{c}$ und damit wird aus Gleichungen (3) und (4)

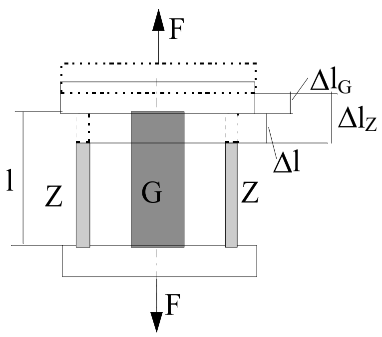
$$2F_S - F_C = 2qa$$

$$\frac{F_S l}{EA} + 2\frac{F_C}{c} = 0$$

$$F_S = \frac{qa}{1 + \frac{cl}{4EA}}; \quad F_C = -\frac{2qa\frac{cl}{4EA}}{1 + \frac{cl}{4EA}}; \quad F_{BV} = \frac{qa}{1 + \frac{cl}{4EA}}$$

Lösung 2.17

Allgemein gilt:



$$\Delta l + \Delta l_G = \Delta l_Z \qquad \frac{\sigma_Z}{E_Z}l - \frac{\sigma_G}{E_G}l = \Delta l$$

$$\uparrow: 2F_Z + F_G = F \quad 2\sigma_Z A_Z + \sigma_G A_G = F$$

$$1.\, \sigma_G = -\sigma_0 \quad und \quad F = 0$$

$$\sigma_Z = \frac{1}{2}\sigma_0 \cdot \frac{A_G}{A_Z} = +102\frac{N}{mm^2}; \qquad \Delta l = 0{,}3296\,mm$$

2. $\Delta l = 0{,}3296\,\mathrm{mm}$ und $F = 40\,\mathrm{kN}$

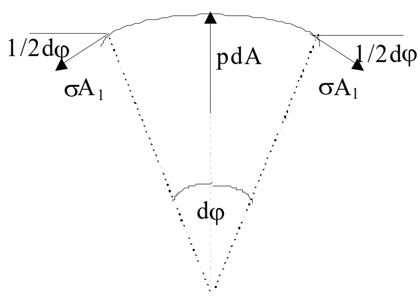
$$\sigma_Z = \frac{E_Z\left(\frac{\Delta l}{l} + \frac{F}{E_G A_G}\right)}{1 + 2\frac{E_Z A_Z}{E_G A_G}} = 104{,}6\,\frac{\mathrm{N}}{\mathrm{mm}^2}$$

$$\sigma_G = \frac{F - 2E_Z A_Z \frac{\Delta l}{l}}{A_G\left(1 + 2\frac{E_Z A_Z}{E_G A_G}\right)} = -18{,}5\,\frac{\mathrm{N}}{\mathrm{mm}^2}$$

3. $\Delta l = 0{,}3296\,\mathrm{mm}$ und $\sigma_G = 0$

$$F = 2E_Z A_Z \cdot \frac{\Delta l}{l} = 532{,}4\,\mathrm{kN}$$

Lösung 2.20



Forderung : $\varepsilon_T = \varepsilon$

$$\varepsilon(r) = \frac{2\pi r - 2\pi(r-\delta)}{2\pi(r-\delta)} = \frac{\delta}{r-\delta} \approx \frac{\delta}{r}$$

$$\varepsilon_T = \alpha_T \Delta T \qquad \boxed{\Delta T = \frac{\varepsilon_T}{\alpha_T} = \frac{\varepsilon}{\alpha_T} = \frac{\delta}{\alpha_T r}}$$

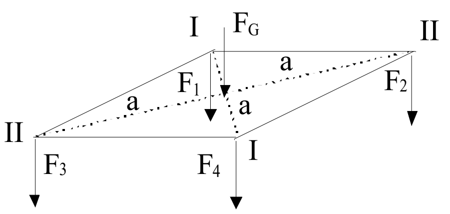
$$\varepsilon = \frac{\delta}{r} \qquad \boxed{\sigma = E \cdot \varepsilon = E\frac{\delta}{r}}$$

$$\uparrow: \quad p \cdot b \cdot r\,d\varphi - 2\sigma \cdot b \cdot t \sin\frac{d\varphi}{2} = 0$$

Linearisierung mit $\sin\frac{d\varphi}{2} \approx \frac{d\varphi}{2}$

$$\boxed{p = \sigma \cdot \frac{t}{r} = E\frac{\delta t}{r^2}}$$

Lösung 2.21



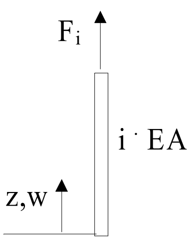
Gleichgewichtsbedingungen:

$\downarrow: \; F_1 + F_2 + F_3 + F_4 + F_G = 0$ (1)

$d_I: \; F_2\,a - F_3\,a = 0 \qquad F_3 = F_2$ (2)

$d_{II}: \; F_4\,a - F_1\,a = 0 \qquad F_4 = F_1$ (3)

Verformungsbedingung : $w_{F_G} = \frac{1}{2}(w_1 + w_4) = \frac{1}{2}(w_2 + w_3)$ (4)



Allgemein : $w_i = \frac{F_i l}{(EA)_i} \qquad (EA)_i = i \cdot EA$

(2) und (3) in (4):

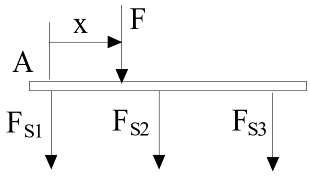
$$\frac{F_1 l}{EA} + \frac{F_1 l}{4EA} = \frac{F_2 l}{2EA} + \frac{F_2 l}{3EA} \qquad \frac{5}{4}F_1 = \frac{5}{6}F_2 \qquad (4a)$$

(2), (3) und (4a) in (1) : $2F_1 + 3F_1 + F_G = 0$

$$\boxed{\begin{aligned} F_1 &= F_4 = -\frac{1}{5}F_G \\ F_2 &= F_3 = -\frac{3}{10}F_G \\ w_G &= -\frac{1}{8}\frac{F_G l}{EA} \end{aligned}}$$

$$w_G = \frac{1}{2}\left(\frac{F_1 l}{EA} + \frac{F_1 l}{4EA}\right) = \frac{5}{8}\frac{F_1 l}{EA}$$

Lösung 2.22



$\downarrow$: $F_{S1} + F_{S2} + F_{S3} + F = 0$ (1)

$\overset{\curvearrowright}{A}$: $F_{S2}\, a + 2\, F_{S3}\, a + F\, x = 0$ (2)

$$w_i = \frac{F_{Si} l_i}{EA} \qquad (3), (4), (5)$$

1.) $w_1 = w_2 = w_3$

$$\frac{F_{S1} a}{EA} = \frac{3F_{S2} a}{2EA} = \frac{2F_{S3} a}{EA} \qquad F_{S2} = \frac{2}{3}F_{S1} \quad F_{S3} = \frac{1}{2}F_{S1} \qquad (6), (7)$$

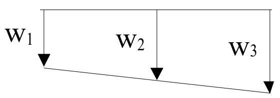
(6) und (7) in (1) : $F_{S1} + \frac{2}{3}F_{S1} + \frac{1}{2}F_{S1} = -F \qquad F_{S1} = -\frac{6}{13}F$

(6) und (7) in (2) : $\frac{2}{3}F_{S1} a + F_{S1} a = -Fx \qquad \boxed{x = -\frac{5}{3}\frac{F_{S1}}{F}a = \frac{10}{13}a}$

2.)

$$\boxed{\sigma_1 = \frac{F_{S1}}{A} = -\frac{6}{13}\frac{F}{A} \qquad \sigma_2 = \frac{F_{S2}}{A} = -\frac{4}{13}\frac{F}{A} \qquad \sigma_3 = \frac{F_{S3}}{A} = -\frac{3}{13}\frac{F}{A}}$$

3.)



$$w_2 = \frac{1}{2}(w_1 + w_3) \qquad (6')$$

(1) bis (5) und (6') liefern $F_{S1} + F_{S2} + F_{S3} = - F$ (1')

$$F_{S2} a + 2F_{S3} a = -\frac{2}{3}Fa \qquad (2')$$

$$\frac{3}{2}F_{S2} = \frac{1}{2}F_{S1} + F_{S3} \qquad (6')$$

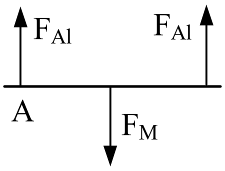
$F_{S1} + F_{S2} + F_{S3} = - F$ (I) $\qquad F_{S1} - 3\, F_{S2} + 2F_{S3} = 0$ (II) $\qquad F_{S2} + 2F_{S3} = -\frac{2}{3}F$ (III)

I - II : $4\, F_{S2} - F_{S3} = -F \qquad$ III + 2·(I - II) : $9F_{S2} = -\frac{8}{3}F$

$$\boxed{\begin{array}{lll} F_{S2} = -\frac{8}{27}F & F_{S3} = -\frac{5}{27}F & F_{S1} = -\frac{14}{27}F \\ w_2 = -\frac{4}{9}\frac{Fa}{EA} & w_3 = -\frac{10}{27}\frac{Fa}{EA} & w_1 = -\frac{14}{27}\frac{Fa}{EA} \end{array}}$$

Lösung 2.23

1.)

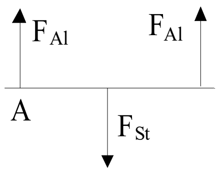


$\curvearrowleft A: \; F_{Al} \cdot 2a - F_M\, a = 0$

$$F_{Al} = \frac{1}{2} F_M \qquad \Delta l_{Al} = \frac{F_{Al} l_{Al}}{E_{Al} A_{Al}} = \frac{F_M l_{Al}}{2 E_{Al} A_{Al}} = \delta$$

$$\boxed{F_M = 2 \frac{\delta}{l_{Al}} \cdot E_{Al} A_{Al} = 2200\,\mathrm{N}}$$

2.)



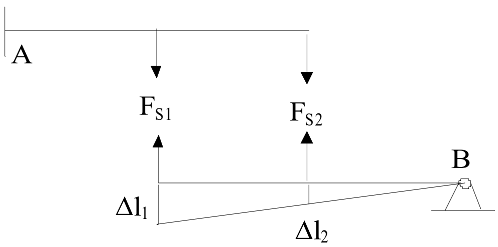
$\curvearrowleft A: \quad F_{Al} \cdot 2a - F_{St}\, a = 0 \qquad F_{st} = 2\, F_{Al} \quad (1)$

$$\Delta l_{Al} + \Delta l_{St} = \delta = \frac{F_{Al} l_{Al}}{E_{Al} A_{Al}} + \frac{F_{St} l_{St}}{E_{St} A_{St}} \qquad (2)$$

(1) in (2) liefert

$$\boxed{F_{Al} = \frac{\delta}{l_{Al}} \cdot \frac{E_{Al} A}{1 + 2 \frac{l_{St}}{l_{Al}} \frac{E_{Al}}{E_{St}}} = 550N \qquad F_{St} = 1100N \qquad w_K = \Delta l_{Al} = \frac{F_{Al} l_{Al}}{E_{Al} A} = 2{,}5mm}$$

Lösung 2.24



$\curvearrowright B: \; 2\, F_{S1}\, a + F_{S2}\, a = 0 \qquad (1)$

$$\Delta l_1 = \frac{F_{S1} a}{EA} \qquad \Delta l_2 = \frac{F_{S2} a}{EA} + \alpha_T \Delta T \cdot a$$

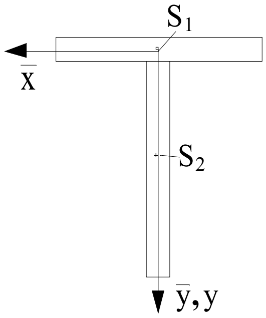
$$\Delta l_1 = 2 \Delta l_2 \qquad \frac{F_{S1} a}{EA} = \frac{2 F_{S2} a}{EA} + 2 \alpha_T \Delta T \cdot a$$

$$F_{S2} = -2 F_{S1} \qquad F_{S1} + 4 F_{S1} = 2 \alpha_T \Delta T EA$$

$$\boxed{F_{S1} = \frac{2}{5} EA \alpha_T \Delta T \qquad F_{S2} = -\frac{4}{5} EA \alpha_T \Delta T}$$

3.Flächenträgheitsmomente

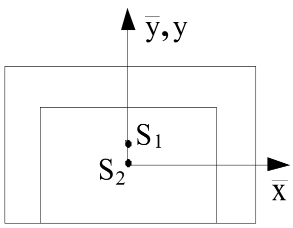
Lösung 3.1



i	$A_i\left[c^2\right]$	$\bar{y}_{Si}\left[c\right]$	$\bar{y}_{Si}A_i\left[c^3\right]$	$I_{xxi}\left[c^4\right]$	$I_{yyi}\left[c^4\right]$	$\bar{y}_{Si}^2A_i\left[c^4\right]$
1	6	0	0	$\frac{1}{2}$	18	0
2	9	5	45	$\frac{243}{4}$	$\frac{3}{4}$	225
Σ	15		45	$\frac{245}{4}$	$\frac{75}{4}$	225

Damit $\bar{y}_S = \frac{45}{15}c = 3c$; $I_{\bar{x}\bar{x}} = \left(\frac{245}{4}+225\right)c^4$; $I_{xx} = \left(\frac{1145}{4}-135\right)c^4 = \frac{605}{4}c^4$; $I_{yy} = \frac{75}{4}c^4$

Lösung 3.2

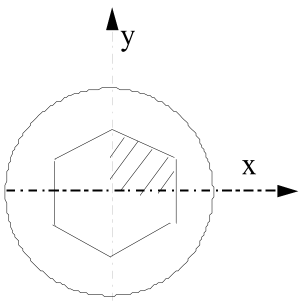


i	$A_i\left[h^2\right]$	$\bar{y}_{Si}\left[h\right]$	$\bar{y}_{Si}A_i\left[h^3\right]$	$I_{xxi}\left[h^4\right]$	$I_{yyi}\left[h^4\right]$	$\bar{y}_{Si}^2A_i\left[h^4\right]$
1	40	0,5	20	83,33	213,33	10
2	-24	0	0	-32	-72	0
Σ	16		20	51,33	141,33	10

$\bar{y}_S = \frac{20}{16}h = 1{,}25h$; $I_{\bar{x}\bar{x}} = 61{,}33h^4$; $I_{xx} = \left(61{,}33 - \frac{25}{16}\cdot 16\right)h^4 = 36{,}33h^4$; $I_{yy} = 141{,}33h^4$

$I_{xy} = 0$; $I_1 = I_{yy}$; $I_2 = I_{xx}$

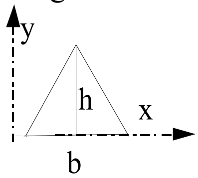
Lösung 3.3



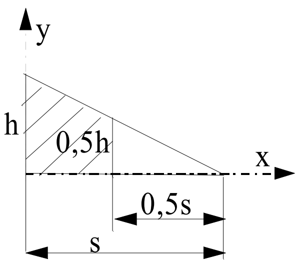
Zerlegung der Sechskantfläche in vier gleiche Teilflächen. Diese werden weiter in zwei Dreiecke zerlegt.

Das Flächenträgheitsmoment des Kreises ist $I_{xxK} = I_{yyK} = \frac{\pi d^4}{64}$

Wegen mehrfacher Symmetrie gilt: $I_{xx} = I_{yy}$ und $I_{xx} = I_{xxK} - I_{xxS}$



$$I_{xxD} = \frac{bh^3}{12}$$

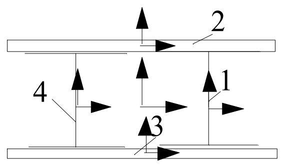


$$h = \frac{s}{\sqrt{3}}$$

$$I_{xxS} = 4\left\{\frac{s\left(\frac{s}{\sqrt{3}}\right)^3}{12} - \frac{\frac{s}{2}\left(\frac{s}{2\sqrt{3}}\right)^3}{12}\right\} = \frac{5\sqrt{3}}{144}s^4$$

$$I_{xx} = I_{yy} = \frac{\pi d^4}{64} - \frac{5\sqrt{3}}{144}s^4$$

Lösung 3.4



Für I20 entnimmt man DIN1025:
$A = 3{,}35 \cdot 10^3 mm^3$; $I_{xx} = 21{,}4 \cdot 10^6 mm^4$; $I_{yy} = 1{,}17 \cdot 10^6 mm^4$;
h = 200mm; b = 300mm; c = 400mm

Gurte:

$$A_2 = t \cdot c = 4 \cdot 10^3 mm^2; \quad I_{xx} = \frac{ct^3}{12} = 33{,}33 \cdot 10^3 mm^4; \quad I_{yy} = \frac{tc^3}{12} = 53{,}33 \cdot 10^6 mm^4$$

i	$A_i\left[mm^2\right]$	$x_{Si}[mm]$	$y_{Si}[mm]$	$I_{xxi}\left[mm^4\right]$	$I_{yyi}\left[mm^4\right]$	$y_{Si}^2 A_i\left[mm^4\right]$	$x_{Si}^2 A_i\left[mm^4\right]$
1	$3{,}35 \cdot 10^3$	150	0	$21{,}4 \cdot 10^6$	$1{,}17 \cdot 10^6$	0	$75{,}38 \cdot 10^6$
2	$4 \cdot 10^3$	0	105	$0{,}03 \cdot 10^6$	$53{,}3 \cdot 10^6$	$44{,}1 \cdot 10^6$	0
3	$4 \cdot 10^3$	0	-105	$0{,}03 \cdot 10^6$	$53{,}3 \cdot 10^6$	$44{,}1 \cdot 10^6$	0
4	$3{,}35 \cdot 10^3$	-150	0	$21{,}4 \cdot 10^6$	$1{,}17 \cdot 10^6$	0	$75{,}38 \cdot 10^6$
Σ	$14{,}7 \cdot 10^3$			$42{,}87 \cdot 10^6$	$109{,}0 \cdot 10^6$	$88{,}2 \cdot 10^6$	$150{,}8 \cdot 10^6$

$I_{xx} = (42{,}87 + 88{,}2) \cdot 10^6 mm^4 = 131{,}1 \cdot 10^6 mm^4$;
$I_{yy} = (109{,}0 + 150{,}8) \cdot 10^6 mm^4 = 259{,}8 \cdot 10^6 mm^4$;

Lösung 3.5

1. Differenz zweier Rechtecke

$$I_{xx} = \frac{(b+\delta)(h+\delta)^3}{12} - \frac{(b-\delta)(h-\delta)^3}{12} = \frac{\delta h^2}{6}(h+3b) + \frac{\delta^3}{6}(3h+b) = 5{,}352 \cdot 10^4 mm^4$$

$$I_{yy} = \frac{(h+\delta)(b+\delta)^3}{12} - \frac{(h-\delta)(b-\delta)^3}{12} = \frac{\delta b^2}{6}(3h+b) + \frac{\delta^3}{6}(h+3b) = 1{,}88 \cdot 10^4 mm^4$$

2. Dünnwandiger Träger, d.h. δ << h,b (Vernachlässigung von Größen, die in δ klein von höherer Ordnung sind)

$$I_{xx} = 2\left[\frac{\delta h^3}{12} + b\cdot\delta\left(\frac{h}{2}\right)^2\right] = \frac{\delta h^2}{6}(h+3b) = 5{,}333\cdot 10^4\,\mathrm{mm}^4$$

$$I_{yy} = 2\left[\frac{\delta b^3}{12} + h\cdot\delta\left(\frac{b}{2}\right)^2\right] = \frac{\delta b^2}{6}(b+3h) = 1{,}867\cdot 10^4\,\mathrm{mm}^4$$

Lösung 3.6

i	$A_i\left[a^2\right]$	$\bar{x}_{Si}[a]$	$\bar{y}_{Si}[a]$	$\bar{x}_{Si}A_i\left[a^3\right]$	$\bar{y}_{Si}A_i\left[a^3\right]$
1	15	-1,5	2,5	-22,5	37,5
2	7,5	1	1,667	7,5	12,5
Σ	22,5			-15	50

$\bar{x}_S = -\frac{2}{3}a = -0{,}667a;$ $\bar{y}_S = \frac{20}{9}a = 2{,}222a;$

Fortsetzung der Tabelle:

$I_{xxi}\left[a^4\right]$	$I_{yyi}\left[a^4\right]$	$I_{xyi}\left[a^4\right]$	$\bar{y}_{Si}^2A_i\left[a^4\right]$	$\bar{x}_{Si}^2A_i\left[a^4\right]$	$\bar{x}_{Si}\bar{y}_{Si}A_i\left[a^4\right]$
31,25	11,25	0	93,75	33,75	-56,25
10,42	3,75	3,125	20,84	7,5	12,5
41,67	15	3,125	114,59	41,25	-43,75

$I_{\bar{x}\bar{x}} = (41{,}67 + 114{,}59)a^4 = 156{,}26a^4;$ $I_{\bar{y}\bar{y}} = (15 + 41{,}25)a^4 = 56{,}25a^4$

$I_{\bar{x}\bar{y}} = (3{,}125 + 43{,}75) = 46{,}875a^4$

$$I_{xx} = I_{\bar{x}\bar{x}} - \bar{y}_S^2A = (156{,}26 - 111{,}09)a^4 = 45{,}17a^4$$

$$I_{yy} = I_{\bar{y}\bar{y}} - \bar{x}_S^2A = (56{,}25 - 10)a^4 = 46{,}25a^4$$

$$I_{xy} = I_{\bar{x}\bar{y}} + \bar{x}_S\bar{y}_SA = (46{,}875 - 33{,}317)a^4 = 13{,}56a^4$$

Hauptträgheitsmomente und Hauptachsen:

$$I_{1,2} = \left(45{,}71 \pm \sqrt{(-0{,}54)^2 + (13{,}56)^2}\right)a^4 = (45{,}71 \pm 13{,}57)a^4$$

$I_1 = 59{,}28a^4;$ $I_2 = 32{,}14a^4$

$$\tan\varphi_{01} = \frac{I_1 - I_{xx}}{I_{xy}} = \frac{59{,}28 - 45{,}17}{13{,}56} = 1{,}04056 \qquad \varphi_{01} = 46{,}1°$$

Lösung 3.7

$A = 13{,}0\,\mathrm{cm}^2;$ $x_S = 1{,}6538\,\mathrm{cm};$ $y_S = -2{,}6538\,\mathrm{cm};$ $I_{xx} = 80{,}78\,\mathrm{cm}^4;$ $I_{yy} = 38{,}78\,\mathrm{cm}^4;$
$I_{xy} = -32{,}31\,\mathrm{cm}^4;$ $I_1 = 98{,}31\,\mathrm{cm}^4;$ $I_2 = 21{,}24\,\mathrm{cm}^4;$ $\varphi_{01} = -28{,}49°$

Lösung 3.8

$A = 18{,}0\text{cm}^2$; $x_S = 0$; $y_S = 0$; $I_{xx} = 246\text{cm}^4$; $I_{yy} = 61{,}5\text{cm}^4$;
$I_{xy} = -90\text{cm}^4$; $I_1 = 282{,}63\text{cm}^4$; $I_2 = 24{,}87\text{cm}^4$; $\varphi_{01} = -22{,}15°$

Lösung 3.9

$A = 530\text{cm}^2$; $y_S = 36{,}54\text{cm}$; $I_{xx} = I_1 = 39{,}756 \cdot 10^4\text{cm}^4$; $I_{yy} = I_2 = 30{,}71 \cdot 10^4\text{cm}^4$

Lösung 3.10

$A = 105\text{cm}^2$; $y_S = 8{,}63\text{cm}$; $I_{xx} = I_1 = 1579\text{cm}^4$; $I_{yy} = I_2 = 817{,}5\text{cm}^4$

Lösung 3.11

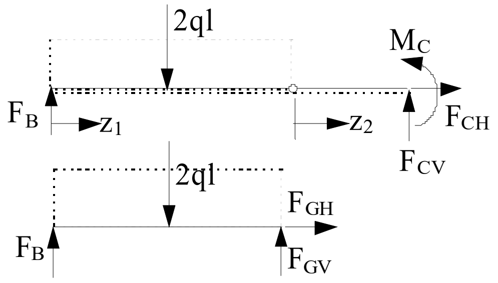
$$A = \frac{\pi D^2}{4} - 6\frac{\pi d^2}{4} = \frac{\pi}{4}\left(10^4 - 6 \cdot 10^2\right)\text{mm}^2 = 73{,}83\text{cm}^2$$

$$I_p = 2I_{xx} = 2I_{yy} = 2I_1 = 2I_2 = \frac{\pi D^4}{32} - 6\left(\frac{\pi d^4}{32} + 35^2 \cdot \frac{\pi d^2}{4}\right) = 923{,}43\text{cm}^4$$

$$I_1 = I_2 = 461{,}72\,\text{cm}^4$$

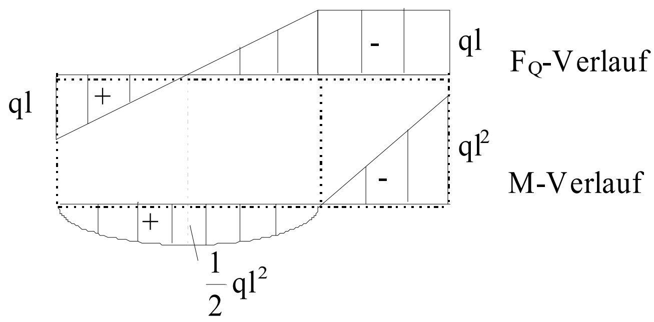
Biegung

Lösung 4.1



$$\rightarrow: F_{CH} = 0$$
$$\uparrow: F_B + F_{CV} - 2ql = 0$$
$$C: 3F_B l - 4ql^2 - M_C = 0$$
$$G: 2F_B l - 2ql^2 = 0$$
$$F_B = ql; \quad F_{CH} = 0; \quad F_{CV} = ql; \quad M_C = -ql^2$$
$$F_{GH} = 0; \quad F_{GV} = ql$$

$0 \le z_1 \le 2l$	$0 \le z_2 \le l$
$F_L(z_1) = 0$	$F_L(z_2) = 0$
$F_Q(z_1) = q(l - z_1)$	$F_Q(z_2) = -ql$
$M(z_1) = qlz_1 - \frac{1}{2} qz_1^2$	$M(z_2) = -qlz_2$



Aufgabe 3.2 (Flächenmomente): $I_{xx} = 36{,}33h^4$

Maximalwerte für y: Unterseite $y = e_1 = 3{,}25h$

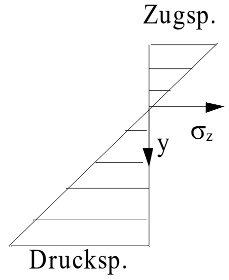
Oberseite $y = -e_2 = -1{,}75h$

Spannungsverteilung an der Einspannstelle (Maximales Moment)

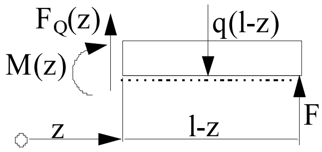
$$\sigma_z(y) = -\frac{ql^2}{I_{xx}} y$$

$$\sigma_z(y = e_1) = -0{,}0894 \frac{ql^2}{h^3} \Rightarrow \text{Druckspannung}$$

$$\sigma_z(y = -e_2) = 0{,}0482 \frac{ql^2}{h^3} \Rightarrow \text{Zugspannung}$$



Lösung 4.2

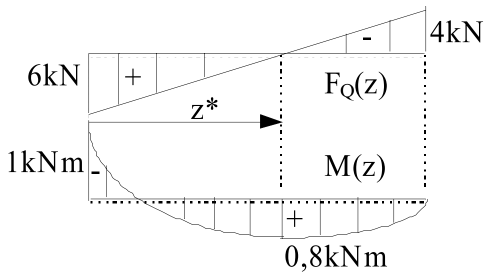


$F_Q(z) = q(l-z) - F \quad F_Q(0) = ql - F = 6kN; \;\; F_Q(l) = -F = -4kN$

$F_Q(z^*) = 0 \qquad z^* = l - \frac{F}{q} = 0{,}6m$

$$M(z) = F(l-z) - \frac{1}{2}q(l-z)^2; \quad M(0) = -1\mathrm{kNm}; \quad M(l) = 0$$

$$M(z^*) = 0{,}8\mathrm{kNm}$$



$$|M_{max}| = 1\mathrm{kNm}; \quad |\sigma_{max}| = \frac{|M_{max}|}{W} \quad W = \frac{ab^2}{6} = \frac{b^3}{12}$$

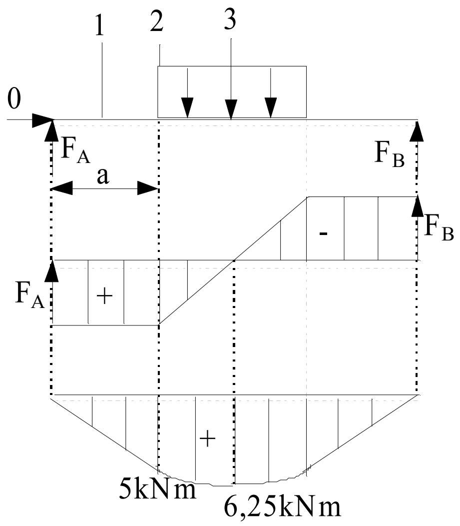
$$|\sigma_{max}| = \frac{12|M_{max}|}{b^3} = \frac{3|M_{max}|}{2a^3} \le \sigma_{zul}$$

$$b_{erf} = \sqrt[3]{\frac{12|M_{max}|}{\sigma_{zul}}} = \sqrt[3]{\frac{12\cdot 10^6\mathrm{mm}^3}{200}} = 39{,}15\mathrm{mm}$$

$$b_{gew} = 40\mathrm{mm} \quad \Rightarrow \quad a = 20\mathrm{mm}$$

$$\sigma_{vorh} = 187{,}5\,\frac{\mathrm{N}}{\mathrm{mm}^2} < \sigma_{zul} = 200\,\frac{\mathrm{N}}{\mathrm{mm}^2}$$

Lösung 4.3



Symmetrie

a = 500mm

$$F_A = F_B = \frac{1}{2}qa = 10\mathrm{kN}$$

$$M_1 = F_A\cdot\frac{a}{2} = 2{,}5\cdot 10^6\mathrm{Nmm}$$

$$M_2 = F_A\cdot a = 5\cdot 10^6\mathrm{Nmm}$$

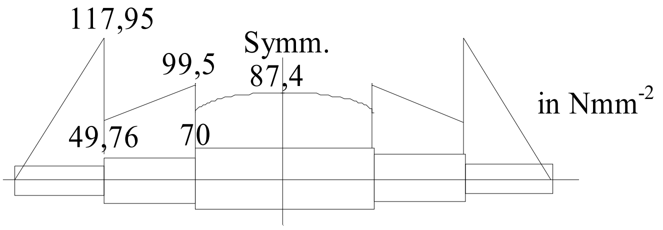
$$M_3 = F_A\cdot\frac{3a}{2} - \frac{1}{8}qa^2 = 6{,}25\cdot 10^6\mathrm{Nmm}$$

$$\sigma = \frac{M}{W}; \quad W = \frac{\pi d^3}{32}$$

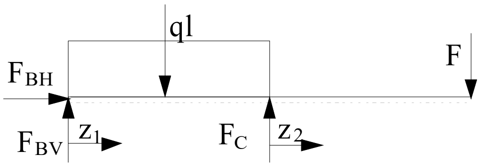
$$\sigma_{1max} = \frac{M_1}{W_1} = 117{,}95\,\frac{\mathrm{N}}{\mathrm{mm}^2} \quad \left(\sigma_1 = \frac{M_1}{W_2} = 49{,}76\,\frac{\mathrm{N}}{\mathrm{mm}^2}\right)$$

$$\sigma_{2max} = \frac{M_2}{W_2} = \frac{2M_1}{W_2} = 99{,}52\,\frac{\mathrm{N}}{\mathrm{mm}^2} \quad \left(\sigma_2 = \frac{M_2}{W_3} = 69{,}9\,\frac{\mathrm{N}}{\mathrm{mm}^2}\right)$$

$$\sigma_{3max} = \frac{M_3}{W_3} = 87{,}37\,\frac{\mathrm{N}}{\mathrm{mm}^2}$$



Lösung 4.4



$\rightarrow$: $F_{BH} = 0$

$\uparrow$: $F_{BV} + F_C - ql - F = 0$

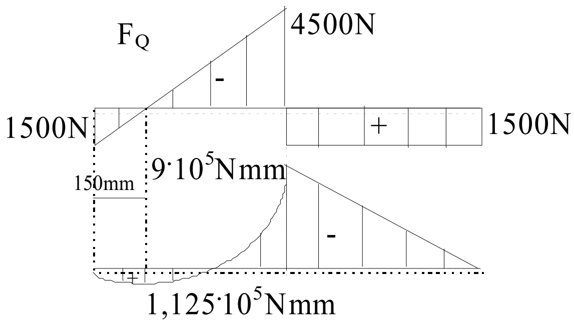
$\curvearrowleft B$: $F_C \cdot l - \frac{1}{2} ql^2 - 2Fl = 0$

$F_C = 6000\,\text{N} \qquad F_{BV} = 1500\,\text{N}$

$0 \le z_1 \le l$ $\qquad$ $0 \le z_2 \le l$

$F_Q(z_1) = F_{BV} - q \cdot z_1$ $\qquad$ $F_Q(z_2) = F$

$M(z_1) = F_{BV} \cdot z_1 - \frac{1}{2} q \cdot z_1^2$ $\qquad$ $M(z_2) = -F \cdot (l - z_2)$



$|M_{max}| = 9 \cdot 10^5$ Nmm

Lt. Aufgabe 6.1(Statik) ist $I_{xx} = 151{,}25c^4$

Maximalwerte für y:

$y = e_1 = 6{,}5c$ (Unterseite)

$y = -e_2 = -3{,}5c$ (Oberseite)

$$W_{bxerf} = \frac{|M_{max}|}{\sigma_{zul}} = 9 \cdot 10^3\, mm^3; \qquad W_{bxerf} = \frac{I_{xxerf}}{e_1} = 23{,}3c^3$$

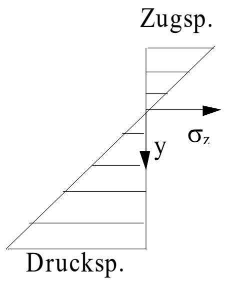
$$23{,}3c^3 = 9 \cdot 10^3\, mm^3 \quad \Rightarrow \quad c = c_{erf} = \sqrt[3]{\frac{9 \cdot 10^3\, mm^3}{23{,}3}} = 7{,}28mm$$

$$gewählt: \quad c = 7{,}5mm$$

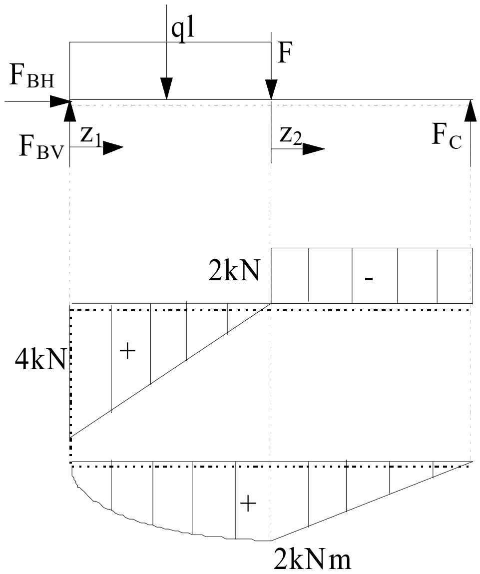
$$\sigma_z(y) = \frac{M_{max}}{I_{xx}} y; I_{xx} = \frac{605}{4} c^4; M_{max} = -9 \cdot 10^5\, Nmm$$

$$\sigma_z(y = 6{,}5c) = -91{,}7 \frac{N}{mm^2} \Rightarrow Druckspannung$$

$$\sigma_z(y = -3{,}5c) = 49{,}4 \frac{N}{mm^2} \Rightarrow Zugspannung$$



Lösung 4.5



$\rightarrow: \quad F_{BH} = 0$

$\uparrow: \quad F_{BV} + F_C - ql - F = 0$

$\curvearrowright B: \quad F_C \cdot 2l - \frac{1}{2} q l^2 - Fl = 0$

$F_C = 2000N \qquad F_{BV} = 4000N$

$0 \le z_1 \le l$ $\qquad$ $0 \le z_2 \le l$

$F_Q(z_1) = F_{BV} - q \cdot z_1$ $\qquad$ $F_Q(z_2) = -F_C = -2kN$

$F_Q(0) = 4kN; F_Q(l) = 0$

$M(z_1) = F_{BV} \cdot z_1 - \frac{1}{2} q \cdot z_1^2$ $\qquad$ $M(z_2) = F_C \cdot (l - z_2)$

$M(0) = 0; M(l) = 2kNm$ $\qquad$ $M(0) = 2kNm; M(l) = 0$

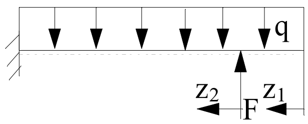
$$\sigma_{max} = \frac{M_{max}}{W_{erf}} \le \sigma_{zul} \qquad W = \frac{\pi D^3}{32}\left[1-\left(\frac{d}{D}\right)^4\right] \qquad \frac{d}{D} = \frac{3}{4}$$

$$\frac{\pi D_{erf}^{\ 3}}{32}\left[1-\left(\frac{3}{4}\right)^4\right] = \frac{M_{max}}{\sigma_{zul}} \quad \Rightarrow \quad D_{erf} = 63{,}6\,\mathrm{mm}$$

$$D_{gew} = 64\,\mathrm{mm} \quad \Rightarrow \quad d = \frac{3}{4}D = 48\mathrm{mm} \quad W_{vorh} = 1{,}759\cdot 10^4\,\mathrm{mm}^3$$

$$\sigma_{vorh} = \frac{M_{max}}{W_{vorh}} = 113{,}68\,\frac{\mathrm{N}}{\mathrm{mm}^2} < \sigma_{zul} = 115\,\frac{\mathrm{N}}{\mathrm{mm}^2}$$

Lösung 4.6



$$0 \le z_1 \le l$$

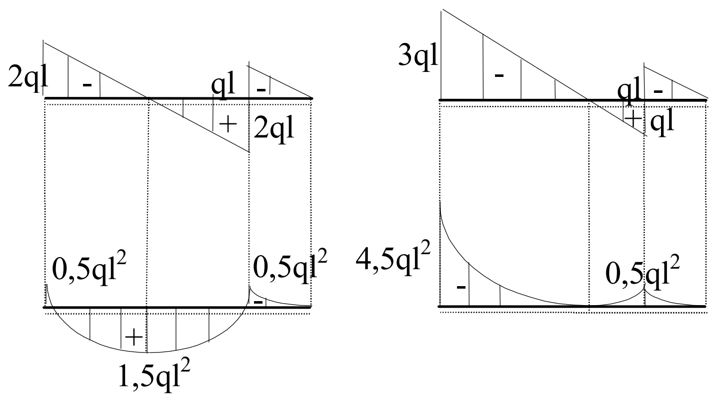
$$F_Q(z_1) = -qz_1 \quad M(z_1) = -\frac{1}{2}qz_1^2$$

$$0 \le z_2 \le 4l$$

$$F_Q(z_2) = -q(l+z_2) + F \quad F_Q(z_2^*) = 0 \quad \Rightarrow \quad z_2^* = \frac{F}{q} - l$$

$$M(z_2) = Fz_2 - \frac{1}{2}q(l+z_2)^2 \;\; M(0) = -\frac{1}{2}ql^2;\; M(4l) = 4Fl - \frac{25}{2}ql^2$$

$1.F = 3ql$	$2.F = 2ql$
$z_2^* = 2l$	$z_2^* = l$
$M(4l) = -\frac{1}{2}ql^2 \quad M(2l) = \frac{3}{2}ql^2$	$M(4l) = -\frac{9}{2}ql^2 \quad M(l) = 0$
$M_{max} = \frac{3}{2}ql^2$	$M_{max} = \frac{9}{2}ql^2$

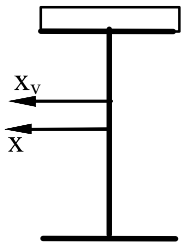


$$M_{max} = W\sigma_{zul}$$

$$W = \frac{\pi d^3}{32}$$

$\frac{3}{2}ql^2 = \frac{\pi d^3}{32}\sigma_{zul}$	$\frac{9}{2}ql^2 = \frac{\pi d^3}{32}\sigma_{zul}$
$q = \frac{2}{3}\frac{\pi}{32}\frac{d^3}{l^2}\sigma_{zul} = 3{,}27\,\frac{\mathrm{N}}{\mathrm{mm}}$	$q = \frac{2}{9}\frac{\pi}{32}\frac{d^3}{l^2}\sigma_{zul} = 1{,}09\,\frac{\mathrm{N}}{\mathrm{mm}}$
$F = 3ql = 4910\,\mathrm{N}$	$F = 2ql = 1090\mathrm{N}$

Lösung 4.7



$$M_{max} = -\frac{1}{2} q l^2$$

für I20: $h = 200\,mm$; $A = 3350\,mm^2$;

$$I_{xx} = 2{,}14 \cdot 10^7\,mm^4; \quad W_x = 2{,}14 \cdot 10^5\,mm^3$$

$$\sigma = \frac{M_{max}}{I_{xx}} \cdot y$$

Verstärkter Träger:

$$y_S = -\frac{b \cdot c \cdot \frac{1}{2}(h+c)}{b \cdot c + A} = -22{,}2\,mm$$

$$I_{xxv} = I_{xx} + A \cdot y_S^2 + b \cdot c\left(\frac{h}{2} + \frac{c}{2} - |y_S|\right)^2 = 2{,}923 \cdot 10^7\,mm^4$$

$$\sigma_v = \frac{M_{max}}{I_{xxv}} y_v$$

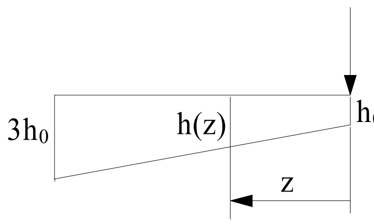
Maximale Zugspannung:

$$\sigma_{max} = \frac{M_{max}}{I_{xx}} \cdot \left(-\frac{h}{2}\right) \qquad \sigma_{vmax} = \frac{M_{max}}{I_{xxv}} \cdot \left[-\left(\frac{h}{2} + c - |y_S|\right)\right]$$

Abweichung in %:

$$\Delta\sigma[\%] = \frac{\sigma_{max} - \sigma_{vmax}}{\sigma_{max}} \cdot 100\% = 35{,}7\%$$

Lösung 4.8



$$|\sigma(z)|_{max} = \frac{|M(z)|}{W_x(z)} \qquad |M(z)| = F \cdot z \quad W_x(z) = \frac{bh^2(z)}{6}$$

$$h(z) = h_0\left(1 + 2\frac{z}{l}\right) \quad W_x(z) = \frac{bh_0^2}{6}\left(1 + 2\frac{z}{l}\right)^2$$

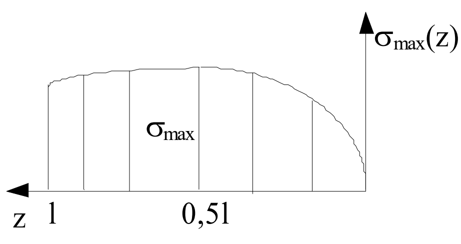
$$|\sigma(z)|_{max} = \frac{6F \cdot z}{bh_0^2\left(1 + 2\frac{z}{l}\right)^2}$$

Ort der max. Biegespannung entweder an der Einspannstelle (max. Moment) oder an der Stelle, an der $\frac{d\sigma_{max}(z)}{dz} = 0$ ist.

$$\frac{d\sigma_{max}(z)}{dz} = \frac{6F}{bh_0^2} \frac{\left(1 + 2\frac{z}{l}\right)^2 - 4\frac{z}{l}\left(1 + 2\frac{z}{l}\right)}{\left(1 + 2\frac{z}{l}\right)^4} = 0$$

$$\left(1 + 2\frac{z}{l}\right) - 4\frac{z}{l} = 0 \qquad \frac{z}{l} = \frac{1}{2}$$

$$\left|\sigma\left(\frac{l}{2}\right)\right|_{max} = \frac{3}{4}\frac{Fl}{bh_0^2} \qquad |\sigma(l)|_{max} = \frac{2}{3}\frac{Fl}{bh_0^2}$$

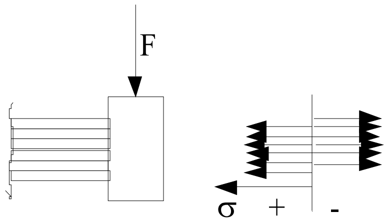


Die absolut größte Spannung beträgt $\sigma_{max} = \frac{3}{4}\frac{Fl}{bh_0^2}$ und tritt an der Stelle $z = \frac{1}{2}$ auf.

Lösung 4.9

$$\sigma_{max} = \frac{M_{max}}{W} \quad M_{max} = \frac{Fl}{4} = 4 \cdot 10^5 \text{Nmm} \quad W = \frac{6 \cdot bh^2}{6} = 1{,}25 \cdot 10^3 \text{mm}^3$$

$$\sigma_{max} = 320 \frac{N}{mm^2}$$



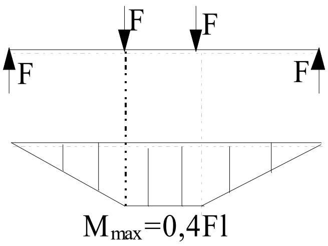
Lösung 4.10

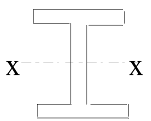
$$M_{max} = F \cdot l \qquad \sigma_{max} = \frac{6M_{max}}{bh^2} = \sigma_{zul} \qquad b = \frac{6Fl}{h^2\sigma_{zul}}$$

$$Trägermasse \quad m = \rho \cdot b \cdot h \cdot l = \frac{\rho 6Fl^2}{h\sigma_{zul}}$$

Die Masse des Trägers wird bei gleicher Tragfähigkeit um so kleiner, je größer die Höhe h wird. Die Höhe h kann jedoch nicht beliebig vergrößert werden, weil sonst die Stabilität verloren geht und der Schubeinfluß berücksichtigt werden müßte (wandartige Träger).

Lösung 4.11



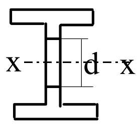


$$M_{max} = 8 \cdot 10^6 \text{Nmm};$$

$$I^*_{xx} = \frac{bh^3}{12} - \frac{(b-s)(h-2t)^3}{12} = 12{,}921 \cdot 10^6 \text{mm}^4$$

$$\sigma^*_{max} = \frac{M_{max}}{W^*} \quad W^* = \frac{I^*_{xx}}{100\text{mm}} = 1{,}292 \cdot 10^5 \text{mm}^3$$

$$\sigma^*_{max} = 61{,}92 \frac{N}{mm^2}$$



$$I_{xx} = I^*_{xx} - \frac{sd^3}{12} = 10{,}671 \cdot 10^6 mm^4 \qquad W = \frac{I_{xx}}{100mm} = 1{,}0671 \cdot 10^5 mm^3$$

$$\sigma_{max} = \frac{M_{max}}{W} = 75 \frac{N}{mm^2}$$

$$A^* = bh - (b-s)(h-2t) = 24{,}4cm^2$$

$$V^* = A^* \cdot l = 2.44 \cdot 10^3 cm^3 m^* = \rho \cdot V^* = 19{,}154kg$$

$$m_L = \frac{\pi}{4} d^2 \cdot s \cdot \rho = 1{,}10976kg \quad m = m^* - 5m_L = 13{,}605kg(\approx 71\% \; von \; m^*)$$

Einsparung an Material von 29% bei einer Spannungserhöhung von 21%.

Lösung 4.12

$$M(z)=\frac{1}{2}F\cdot z \qquad W=\frac{\pi}{32}d^3$$

$$1. z=b_0 \qquad M_A=1{,}25\cdot 10^6\,Nmm \qquad W_{erf}=\frac{M_A}{\sigma_{zul}}=15625mm^3$$

$$d_{erf}=\sqrt[3]{\frac{32W_{erf}}{\pi}}=54{,}2mm=d_0$$

$$2. z=\frac{l}{2} \qquad M_F=10\cdot 10^6\,Nmm \qquad W_{erf}=\frac{M_F}{\sigma_{zul}}=1{,}25\cdot 10^5\,mm^3$$

$$d_{erf}=\sqrt[3]{\frac{32W_{erf}}{\pi}}=108{,}4mm=d_1$$

$$3. b_0\le z\le\frac{l}{2}-\frac{b_1}{2} \qquad M(z)=25\cdot 10^3\,N\cdot z \qquad W_{erf}=\frac{M(z)}{\sigma_{zul}}=312{,}5mm^2\cdot z$$

$$d_{erf}(z)=\sqrt[3]{\frac{32W_{erf}}{\pi}}=\sqrt[3]{\frac{32\cdot 312{,}5mm^2}{\pi}}\cdot\sqrt[3]{z}=14{,}71\cdot\sqrt[3]{z\cdot mm^2}=d(z)$$

Lösung 4.13

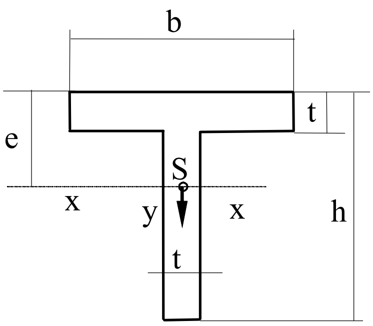
$$M(z)=-Fz$$

$$\left|\sigma_{\max}\right|=\frac{|M(z)|}{W}=\frac{6|M(z)|}{bh^2(z)}=\sigma_{zul} \qquad h(z)=\sqrt{\frac{6|M(z)|}{b\sigma_{zul}}}=12{,}25\sqrt{z\cdot mm}$$

$$h(z=l)=h_0=173{,}21mm$$

Da die Höhe des Trägers in der gleichen Größenordnung wie die Länge liegt, sind die ermittelten Spannungen nur Näherungen.

Lösung 4.14



$$A=bt+(h-t)t$$

$$e=\frac{bt\cdot\frac{t}{2}+(h-t)t\cdot\frac{1}{2}(h+t)}{bt+(h-t)t}=\frac{\frac{1}{2}\left(bt+h^2-t^2\right)}{b+h-t}$$

$$W_{erfz}=\frac{|M_{\max}|}{\sigma_{zul}}=1\cdot 10^6\,mm^3 \qquad W_{erfd}=\frac{|M_{\max}|}{2{,}5\sigma_{zul}}=0{,}4\cdot 10^6\,mm^3$$

$$I_{xx}=\frac{bt^3}{12}+bt\left(e-\frac{t}{2}\right)^2+\frac{t(h-t)^3}{12}+t(h-t)\left(\frac{h}{2}+\frac{t}{2}-e\right)^2$$

$$M_A=-Fl=-60\cdot 10^6\,Nmm \qquad \frac{\sigma_{zul}}{\sigma_{dzul}}=\frac{2}{5} \qquad \sigma_{maxz}=\frac{M_A}{I_{xx}}\cdot(-e) \qquad \sigma_{maxd}=\frac{M_A}{I_{xx}}\cdot(h-e)$$

$$\left|\sigma_{maxz}\right|=\sigma_{zul} \qquad \left|\sigma_{maxd}\right|=2{,}5\sigma_{zul} \qquad \frac{\sigma_{zul}}{\sigma_{dzul}}=\frac{2}{5}=\frac{e}{h-e} \quad\Rightarrow\quad 7e=2h$$

$$\frac{7}{2} \cdot \frac{bt + h^2 - t^2}{b + h - t} = 2h \quad \Rightarrow \quad b = \frac{3h^2 + 4ht - 7t^2}{4h - 7t}$$

Annahme: h = 320mm ⇒ b = 289,474mm; e = 91,4285mm; h - e = 228,5715mm

$W_z = 1{,}3193 \cdot 10^6 mm^3 > 1 \cdot 10^6 mm^3$

$W_d = 0{,}5277 \cdot 10^6 mm^3 > 0{,}4 \cdot 10^6 mm^3$

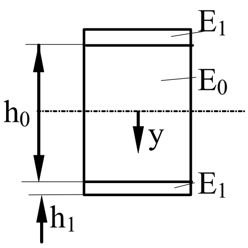
Annahme: h = 280mm ⇒ b = 260mm; e = 80mm; h - e = 200mm ; $\frac{h-e}{e} = 2{,}5$

$W_z = 1{,}0053 \cdot 10^6 mm^3 > 1 \cdot 10^6 mm^3$

$W_d = 0{,}4021 \cdot 10^6 mm^3 > 0{,}4 \cdot 10^6 mm^3$

Verbesserung mit Regula falsi ergibt: h = 279,32mm und b = 259,5mm, d.h. damit gewählt h = 280mm und b = 260mm.

Lösung 4.15



Es gilt:

$$\varepsilon = \frac{y}{\rho} \qquad (Bern.Hypothese)$$

$$\sigma = E_0 \cdot \frac{y}{\rho} \quad für \quad -\frac{h_0}{2} \le y \le \frac{h_0}{2}$$

$$\sigma = E_1 \cdot \frac{y}{\rho} \quad für \quad -\left(h_1 + \frac{h_0}{2}\right) \le y \le -\frac{h_0}{2} \;\; und \;\; \frac{h_0}{2} \le y \le \left(h_1 + \frac{h_0}{2}\right)$$

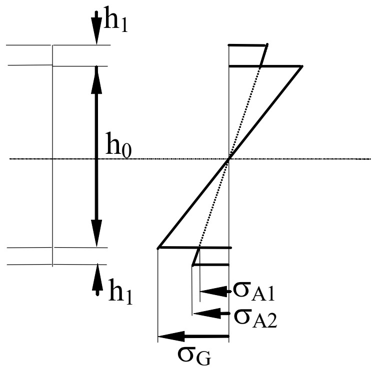
$$M = 2\left[\int_0^{\frac{h_0}{2}} E_0 \frac{y^2}{\rho} b\,dy + \int_{\frac{h_0}{2}}^{\frac{h_0}{2}+h_1} E_1 \frac{y^2}{\rho} b\,dy\right] \qquad \frac{1}{\rho} = \frac{M}{E_0 \frac{bh_0^3}{12}\left\{1 + \frac{E_1}{E_0}\left[\left(1 + \frac{2h_1}{h_0}\right)^3 - 1\right]\right\}}$$

Spannungen im Grundmaterial:

$$\sigma = \frac{M \cdot y}{\frac{bh_0^3}{12}\left\{1 + \frac{E_1}{E_0}\left[\left(1 + \frac{2h_1}{h_0}\right)^3 - 1\right]\right\}}$$

Spannungen in der Auflage:

$$\sigma = \frac{\frac{E_1}{E_0} M \cdot y}{\frac{bh_0^3}{12}\left\{1 + \frac{E_1}{E_0}\left[\left(1 + \frac{2h_1}{h_0}\right)^3 - 1\right]\right\}}$$



Zahlenwerte:

F = 1000N; $M = \frac{F \cdot l}{4} = 50000\,Nmm$

Grundmaterial: $\sigma_G\left(y = \frac{h_0}{2}\right) = 77 \frac{N}{mm^2} = \sigma_{0max}$

Auflage: $\sigma_{A1} = \frac{E_1}{E_0}\sigma_G = 46{,}2 \frac{N}{mm^2}$; $\sigma_{A2} = \sigma_{A1} \frac{\frac{h_0}{2} + h_1}{\frac{h_0}{2}} = 53{,}9 \frac{N}{mm^2} = \sigma_{1max}$

Lösung 4.16
Annahme: Beton überträgt auch Zugspannungen!

$$\varepsilon = \frac{y}{\rho} + \varepsilon_0 (unsymmetrisch)$$

$$\sigma_{Beton} = E_B\left(\frac{y}{\rho} + \varepsilon_0\right) (1); \qquad \sigma_S = E_S\left(\frac{y}{\rho} + \varepsilon_0\right) \cong E_S\left(\frac{a}{\rho} + \varepsilon_0\right) (2)$$

$$F_L = \int_{(A)} \sigma dA \cong E_B \cdot \varepsilon_0 \cdot b \cdot h + n \cdot \frac{\pi}{4} d^2 \cdot E_S\left(\frac{a}{\rho} + \varepsilon_0\right) = 0$$

$$Mit A_S = n \cdot \frac{\pi}{4} d^2 \quad und \quad A_B \cong b \cdot h$$

$$\varepsilon_0 = -\frac{A_S E_S}{A_B E_B + A_S E_S} \cdot \frac{a}{\rho} \quad (3)$$

$$M = \int_{(A)} \sigma \cdot y dA = \int_{A_B} E_B\left(\frac{y}{\rho} + \varepsilon_0\right) y dA + n \cdot \frac{\pi}{4} d^2 \cdot E_S\left(\frac{a}{\rho} + \varepsilon_0\right) \cdot a$$

$$M = \frac{E_B}{\rho}\frac{bh^3}{12} + n \cdot \frac{\pi}{4} d^2 \cdot E_S \frac{a^2}{\rho} + n \cdot \frac{\pi}{4} d^2 \cdot a \cdot \varepsilon_0 \cdot E_S$$

$$= \frac{1}{\rho}\left\{E_B \frac{bh^3}{12} + \frac{A_B E_B \cdot A_S E_S}{A_B E_B + A_S E_S} \cdot a^2\right\}$$

$$\frac{1}{\rho} = \frac{M}{E_B \frac{bh^3}{12} + \frac{A_B E_B \cdot A_S E_S}{A_B E_B + A_S E_S} \cdot a^2} \quad (4)$$

Mit (4) ist ε_0 aus (3) zu bestimmen und aus (1) und (2) ergeben sich σ_{Beton} und σ_S.
Zahlenwerte:

$$M_{max} = \frac{ql^2}{8} = 50 \cdot 10^6 \text{Nmm}; \quad A_S = 314{,}16 \text{mm}^2; \quad A_B \cong 8 \cdot 10^4 \text{mm}^2$$

aus (4): $\frac{1}{\rho} = 1{,}2715 \cdot 10^{-6} \text{mm}^{-1}$; aus (3): $\varepsilon_0 = -5{,}023 \cdot 10^{-6}$

aus (1): $\sigma_{Beton}\left(y = \frac{h}{2}\right) = E_B \cdot \left(\frac{h}{2\rho} + \varepsilon_0\right) = 8{,}72 \frac{\text{N}}{\text{mm}^2}$

$$\sigma_{Beton}\left(y = -\frac{h}{2}\right) = E_B \cdot \left(-\frac{h}{2\rho} + \varepsilon_0\right) = -9{,}08 \frac{\text{N}}{\text{mm}^2}$$

aus (2): $\sigma_S = E_S \cdot \left(\frac{a}{\rho} + \varepsilon_0\right) = 44{,}8 \frac{\text{N}}{\text{mm}^2}$

Lösung 4.17
x und y sind Hauptachsen, M_b wirkt nicht in Richtung einer Hauptachse und muß zerlegt werden, d.h. schiefe Biegung.

$$\sigma(x,y) = \frac{M_{bx}}{I_{xx}} \cdot y + \frac{M_{by}}{I_{yy}} \cdot x$$

Mit dem Koordinatensystem aus der Skizze erhält man:

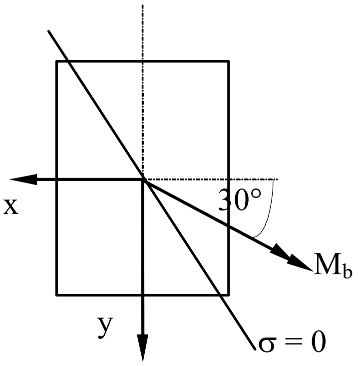
M_{bx} = - M_b cos30° = -866 Nm und M_{by} = - M_b sin30° = -500 Nm

$$I_{xx} = \frac{bh^3}{12} = 0{,}72 \cdot 10^6\,\mathrm{mm}^4; \quad I_{yy} = \frac{hb^3}{12} = 0{,}32 \cdot 10^6\,\mathrm{mm}^4$$

$$\sigma(x,y) = -1{,}194\,\frac{\mathrm{N}}{\mathrm{mm}^3} \cdot y - 1{,}563\,\frac{\mathrm{N}}{\mathrm{mm}^3} \cdot x$$

$$\sigma(x,y) = 0 \quad \Rightarrow \quad y = -1{,}31x$$

$$\sigma_{max} = \sigma\left(x = -\frac{b}{2}, y = -\frac{h}{2}\right) = 67{,}1\,\frac{\mathrm{N}}{\mathrm{mm}^2}$$



Lösung 4.18
Größte Beanspruchung an der Einspannstelle! Max. Spannung an einem Eckpunkt des Querschnittes, d.h.:

$$|\sigma_{max}| = \frac{|M_{xmax}|}{I_{xx}}|y_{max}| + \frac{|M_{ymax}|}{I_{yy}}|x_{max}| = \frac{|M_{xmax}|}{W_x} + \frac{|M_{ymax}|}{W_y}$$

$$|M_{xmax}| = 2F \cdot 2l \qquad W_x = \frac{bh^2}{6}$$

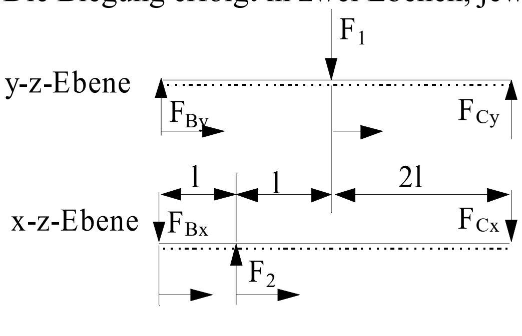
$$|M_{ymax}| = F \cdot l \qquad W_y = \frac{hb^2}{6}$$

$$|\sigma_{max}| = \frac{24Fl}{bh^2} + \frac{6Fl}{hb^2} = \sigma_{zul} \quad \Rightarrow \quad b^2 - \frac{24Fl}{\sigma_{zul}h^2} \cdot b - \frac{6Fl}{\sigma_{zul}h} = 0$$

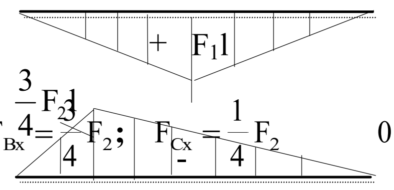
$$b = \frac{12Fl}{\sigma_{zul}h^2}\left(1 \pm \sqrt{1 + \frac{\sigma_{zul} \cdot h^3}{24Fl}}\right) \qquad b = 58{,}7mm \qquad b_{gew} = 60mm$$

Nur das positive Vorzeichen ist physikalisch sinnvoll.

Lösung 4.19
Die Biegung erfolgt in zwei Ebenen, jeweils um eine Hauptachse.



$$F_{By} = F_{Cy} = \frac{F_1}{2}; \quad F_{Bx} = \frac{3}{4}F_2; \quad F_{Cx} = \frac{1}{4}F_2$$



$0 \leq z_1 \leq 2l \quad M$

$0 \leq z_2 \leq 2l \quad M$

$0 \leq z_1 \leq l \quad M_y($

$0 \leq z_2 \leq 3l \quad M$

Angriffsstelle von F_1:

$$\sigma(x, y, z = \text{konst.}) = \frac{M_x(z_1 = 2l)}{I_{xx}} \cdot y + \frac{M_y(z_2 = l)}{I_{yy}} \cdot x$$

$$M_x(z_1 = 2l) = F_1 l = 7000\text{Nm} \qquad M_y(z_2 = l) = -\frac{1}{2} F_2 l = -1750\text{Nm}$$

Aus der Tabelle für I 16: $I_{xx} = 9{,}35 \cdot 10^6$ mm^4, $I_{yy} = 0{,}547 \cdot 10^6$ mm^4

$$\sigma(x, y) = 0{,}75 \frac{N}{mm^3} \cdot y - 3{,}2 \frac{N}{mm^3} \cdot x$$

$$\sigma_{max} = \sigma(x = -37\text{mm}, y = 80\text{mm}) = 178{,}4 \frac{N}{mm^2}$$

Angriffsstelle von F_1:

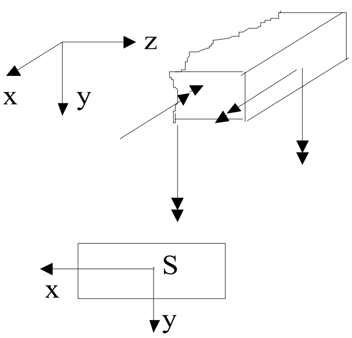
$$M_x(z_1 = l) = \frac{1}{2} F_1 l = 3500\text{Nm} \qquad M_y(z_1 = l) = -\frac{3}{4} F_2 l = -2625\text{Nm}$$

$$\sigma(x, y) = 0{,}375 \frac{N}{mm^3} \cdot y - 4{,}8 \frac{N}{mm^3} \cdot x$$

$$\sigma_{max} = \sigma(x = -37\text{mm}, y = 80\text{mm}) = 207{,}6 \frac{N}{mm^2}$$

Lösung 4.20

$$\sigma = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x \qquad M_{bx} = M_x \quad M_{by} = - M_y$$

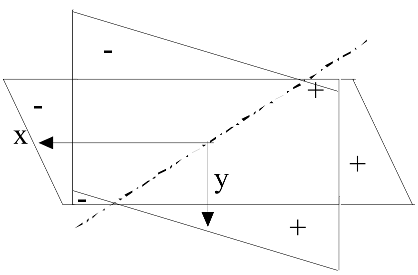


$$I_{xx} = 2 \cdot \frac{th^3}{12} + 2bt\left(\frac{h}{2}\right)^2 = \frac{1}{6} th^2 (h + 3b)$$

$$I_{yy} = 2 \cdot \frac{tb^3}{12} + 2ht\left(\frac{b}{2}\right)^2 = \frac{1}{6} tb^2 (b + 3h)$$

$$\sigma = \frac{6Fl}{th^3}\left(\frac{y}{7} - \frac{x}{10}\right)$$

Spannungsnullinie : $\sigma = 0$ oder $y = \frac{7}{10} x$



Lösung 4.21

Sind x und y keine Hauptachsen, so gilt für Schwerpunktskoordinaten bei schiefer Biegung:

$$\sigma(x,y,z)=\frac{M_x(z)\cdot I_{yy}+M_y(z)\cdot I_{xy}}{I_{xx}\cdot I_{yy}-I_{xy}^2}\cdot y+\frac{M_x(z)\cdot I_{xy}+M_y(z)\cdot I_{xx}}{I_{xx}\cdot I_{yy}-I_{xy}^2}\cdot x$$

mit

$$M_x=M_b \quad M_y=0$$

$$I_{xx}=2\left[\frac{2c\cdot c^3}{12}+2c^2\left(\frac{3}{2}c\right)^2\right]+\frac{c\cdot(2c)^3}{12}=10c^4$$

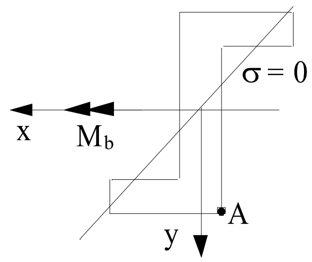
$$I_{yy}=2\left[\frac{c\cdot(2c)^3}{12}+2c^2\left(\frac{1}{2}c\right)^2\right]+\frac{2c\cdot c^3}{12}=\frac{5}{2}c^4$$

$$I_{xy}=2\left[0-2c^2\cdot\frac{1}{2}c\cdot\frac{3}{2}c\right]+0=-3c^4$$

$$\sigma(x,y)=\frac{5}{32}\cdot\frac{M_b}{c^4}\cdot y-\frac{3}{16}\cdot\frac{M_b}{c^4}\cdot x$$

$$\sigma(x,y)=0 \quad\Rightarrow\quad y=\frac{6}{5}x$$

$$\sigma_{max}\left(x=-\frac{c}{2},y=2c\right)=\frac{13}{32}\cdot\frac{M_b}{c^3}=0{,}40625\frac{M_b}{c^3}$$



Lösung 4.22

$$\sigma(x,y,z=l)=\frac{M_x(l)\cdot I_{yy}+M_y(l)\cdot I_{xy}}{I_{xx}\cdot I_{yy}-I_{xy}^2}\cdot y+\frac{M_x(l)\cdot I_{xy}+M_y(l)\cdot I_{xx}}{I_{xx}\cdot I_{yy}-I_{xy}^2}\cdot x$$

mit

$$M_x(l)=-\frac{1}{2}ql^2=-2000Nm \qquad M_y=0$$

$$I_{xx}=\left[\frac{3c\cdot c^3}{12}+3c^2c^2\right]+\left[\frac{c\cdot(3c)^3}{12}+3c^2c^2\right]=\frac{17}{2}c^4=1{,}36\cdot10^6mm^4$$

$$I_{yy}=\left[\frac{c\cdot(3c)^3}{12}+3c^2\left(\frac{1}{2}c\right)^2\right]+\left[\frac{3c\cdot c^3}{12}+3c^2\left(\frac{1}{2}c\right)^2\right]=4c^4=0{,}64\cdot10^6mm^4$$

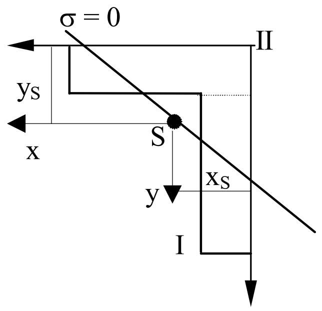
$$I_{xy}=\left[0-3c^2\cdot\frac{1}{2}c\cdot(-c)\right]+\left[0-3c^2\cdot\left(-\frac{1}{2}c\right)\cdot c\right]=3c^4=0{,}48\cdot10^6mm^4$$

$$\sigma(x,y)=-2\cdot\frac{N}{mm^3}\cdot y-1.5\cdot\frac{N}{mm^3}\cdot x$$

$$\sigma(x,y)=0 \quad\Rightarrow\quad y=-\frac{3}{4}x$$

$$\sigma_{\mathrm{I}}\left(x=0,y=\frac{5}{2}c=50mm\right)=-100\frac{N}{mm^2}=|\sigma_{max}|$$

$$\sigma_{\mathrm{II}}\left(x=-c=-20mm,y=-\frac{3}{2}c=-30mm\right)=90\frac{N}{mm^2}$$



$$y_S = \frac{\frac{3}{2}c^3 + \frac{15}{2}c^3}{6c^2} = \frac{3}{2}c$$

$$x_S = \frac{\frac{9}{2}c^3 + \frac{3}{2}c^3}{6c^2} = c$$

Lösung 4.23

Statik Aufgabe 3.7: $A = 13cm^2$; $x_S = 1{,}6538cm$; $y_S = 2{,}6638cm$; $I_{xx} = 80{,}78cm^4$; $I_{yy} = 38{,}78cm^4$; $I_{xy} = -32{,}31cm^4$

z = 0: $M_x = -F_1 l = -10^6 Nmm$; $M_y = F_2 l = 10^6 Nmm$; $F_L = -F_3 = -10^4 N$

$$\sigma(x,y,z=0) = \frac{F_L}{A} + \frac{M_x(0)\cdot I_{yy} + M_y(0)\cdot I_{xy}}{I_{xx}\cdot I_{yy} - I_{xy}^2}\cdot y + \frac{M_x(0)\cdot I_{xy} + M_y(0)\cdot I_{xx}}{I_{xx}\cdot I_{yy} - I_{xy}^2}\cdot x$$

$$\sigma_1(x=-43{,}5mm, y=-26{,}5mm) = -153\frac{N}{mm^2}$$

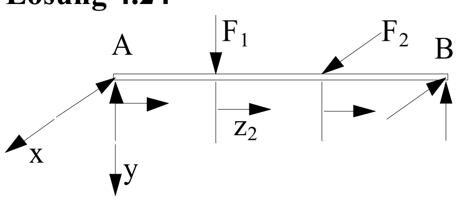
$$\sigma_2(x=16{,}5mm, y=-26{,}5mm) = 171{,}9\frac{N}{mm^2}$$

$$\sigma_3(x=16{,}5mm, y=53{,}5mm) = -100{,}5\frac{N}{mm^2}$$

$$\sigma_4(x=6{,}5mm, y=53{,}5mm) = -154{,}6\frac{N}{mm^2}$$

$$\sigma_5(x=-43{,}5mm, y=-16{,}5mm) = -187{,}1\frac{N}{mm^2}$$

Lösung 4.24



$$F_{AV} = \frac{2}{3}F_1 = 667N; \quad F_{AH} = \frac{1}{3}F_2 = 400N$$

$$F_{BV} = \frac{1}{3}F_1 = 333N; \quad F_{BH} = \frac{2}{3}F_2 = 800N$$

$$0 \le z_2 \le \frac{l}{3}$$

$$M_x(z_2) = F_{AV}\left(\frac{l}{3} + z_2\right) - F_1 z_2 \qquad M_x(0) = F_{AV}\frac{l}{3} = 6{,}67\cdot 10^4 Nmm$$

$$M_x\left(\frac{l}{3}\right) = F_{AV}\frac{2}{3}l - F_1\frac{l}{3} = 3{,}33\cdot 10^4 Nmm$$

$$M_y(z_2) = F_{AH}\left(\frac{l}{3} + z_2\right) \qquad M_y(0) = F_{AH}\frac{l}{3} = 4\cdot 10^4 Nmm$$

$$M_y\left(\frac{l}{3}\right) = F_{AH}\frac{2}{3}l = 8\cdot 10^4 Nmm$$

$$M_{res}(z_2)=\sqrt{M_x^2(z_2)+M_y^2(z_2)} \qquad W=\frac{\pi d^3}{32}=6283mm^3$$

$$M_{res}(0)=\sqrt{M_x^2(0)+M_y^2(0)}=7{,}78\cdot 10^4\,Nmm \qquad \sigma_1=\frac{M_{res}(0)}{W}=12{,}4\frac{N}{mm^2}$$

$$M_{res}\left(\frac{l}{3}\right)=\sqrt{M_x^2(\tfrac{l}{3})+M_y^2(\tfrac{l}{3})}=8{,}67\cdot 10^4\,Nmm \qquad \sigma_2=\frac{M_{res}(\frac{l}{3})}{W}=13{,}8\frac{N}{mm^2}$$

$$\frac{dM_{res}(z_2)}{dz_2}=\frac{2M_x\frac{\partial M_x}{\partial z_2}+2M_y\frac{\partial M_y}{\partial z_2}}{2\sqrt{M_x^2+M_y^2}}=0 \qquad \frac{\partial M_x}{\partial z_2}=F_{AV}-F_1=-\frac{1}{3}F_1 \qquad \frac{\partial M_y}{\partial z_2}=F_{AH}=\frac{1}{3}F_2$$

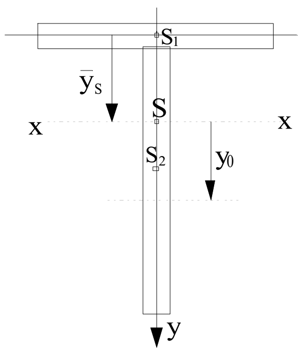
$$-\frac{2}{9}F_1^2 l-\frac{2}{3}F_1^2 z_2+F_1^2 z_2+\frac{1}{9}F_2^2 l+\frac{1}{3}F_2^2 z_2=0 \qquad z_2=\frac{2F_1^2-F_2^2}{F_1^2+F_2^2}\cdot\frac{l}{3}=z_{20}=22{,}95mm$$

$$M_x(z_{20})=5{,}906\cdot 10^4\,Nmm \qquad M_y(z_{20})=4{,}918\cdot 10^4\,Nmm$$

$$M_{res}(z_{20})=\sqrt{M_x^2(z_{20})+M_y^2(z_{20})}=7{,}69\cdot 10^4\,Nmm \qquad \sigma(z_{20})=\frac{M_{res}(z_{20})}{W}=12{,}2\frac{N}{mm^2}$$

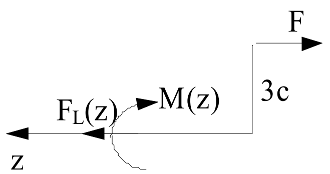
Für $z_2 = z_{20} = 22{,}95$mm nimmt die Biegespannung im Bereich 2 ein Minimum an.

Lösung 4.25



$$\bar{y}_S=\frac{9c^2\cdot 5c}{18c^2}=\frac{5}{2}c$$

$$I_{xx}=\frac{9c\cdot c^3}{12}+9c^2\cdot\left(\frac{5}{2}c\right)^2+\frac{c\cdot(9c)^2}{12}+9c^2\cdot\left(\frac{5}{2}c\right)^2=174c^4$$



$$F_L(z)=F \qquad M(z)=-3Fc$$

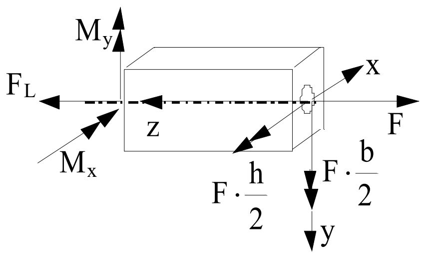
$$\sigma_z=\frac{F_L}{A}+\frac{M(z)}{I_{xx}}y=\frac{F}{18c^2}-\frac{3Fc}{174c^4}y$$

$\sigma_z=0$ **(**Spannungsnullinie**)**

$$\frac{F}{18c^2}-\frac{F}{58c^3}y_0=0 \quad\Rightarrow\quad y_0=\frac{29}{9}c=3{,}22c$$

$$\sigma_{max}\big|_{y=-3c}=\frac{F}{18c^2}+\frac{3F}{58c^2}=\frac{56}{522}\frac{F}{c^2}=42{,}9\frac{N}{mm^2}$$

Lösung 4.26



$$\sigma_z = \frac{F_L}{A} + \frac{M_x}{I_{xx}} \cdot y + \frac{M_y}{I_{yy}} \cdot x$$

$$F_L = F \quad M_x = F \cdot \frac{h}{2} \quad M_y = F \cdot \frac{b}{2}$$

Maximale Spannung tritt an der Lastangriffsstelle auf.

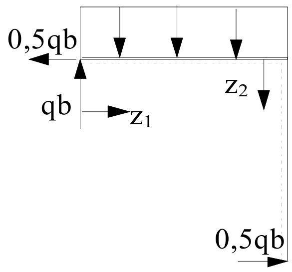
$$\sigma_{z\max} = \frac{F_L}{A} + \frac{M_x}{I_{xx}} \cdot \frac{h}{2} + \frac{M_y}{I_{yy}} \cdot \frac{b}{2}$$

$$A = b \cdot h - \frac{\pi}{4} d^2 = 3{,}544 \cdot 10^3 \, mm^2$$

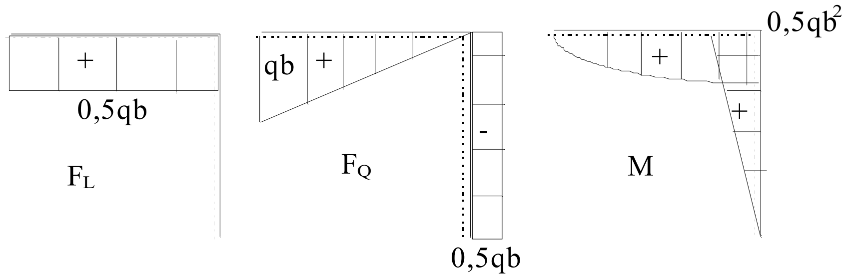
$$I_{xx} = \frac{bh^3}{12} - \frac{\pi d^4}{64} = 2{,}434 \cdot 10^6 mm^4 \quad I_{yy} = \frac{hb^3}{12} - \frac{\pi d^4}{64} = 1{,}314 \cdot 10^6 mm^4$$

$$\sigma_{z\max} = 1{,}625 \cdot 10^{-3} F \cdot \frac{1}{mm^2} \le \sigma_{zul} \quad \Rightarrow \quad F \le \frac{10^3}{1{,}625} mm^2 \cdot \sigma_{zul} = 86 \cdot 10^3 N$$

Lösung 4.27



$0 \le z_1 \le b$	$0 \le z_2 \le b$
$F_L = \frac{1}{2} qb$	$F_L = 0$
$F_Q = qb - qz_1$	$F_Q = -\frac{1}{2} qb$
$M = qbz_1 - \frac{1}{2} qz_1^2$	$M = \frac{1}{2} qb(b - z_2)$



$$|\sigma_{\max}| = \frac{M_{\max}}{W_x} \le \sigma_{zul} \quad \Rightarrow \quad W_{xerf} = \frac{M_{\max}}{\sigma_{zul}} = \frac{qb^2}{2\sigma_{zul}} = 31{,}2 \cdot 10^3 mm^3$$

Gewählt: I 10 mit $W_x = 34{,}2 \cdot 10^3 mm^3$ und $A = 1{,}06 \cdot 10^3 mm^2$.

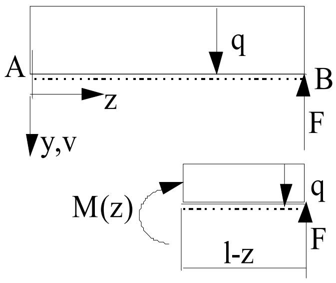
Spannungsnachweis:

$$\left|\sigma_{\max}\right| = \frac{\left|F_L\right|}{A} + \frac{\left|M_{\max}\right|}{\mathrm{W_x}} = \frac{qb}{2A} + \frac{qb^2}{2W_x} = 4{,}7\frac{N}{mm^2} + 146{,}5\frac{N}{mm^2}$$

$$\sigma_{vorh} = 151{,}2\frac{N}{mm^2} < \sigma_{zul} = 160\frac{N}{mm^2}$$

Die Zugspannung ist wesentlich kleiner als die Biegespannung.

Lösungen 4.31 und 4.32



$$EIv''(z) = -M(z)$$

$$M(z) = F(l-z) - \frac{1}{2}q(l-z)^2$$

$$EIv''(z) = \frac{1}{2}q(l-z)^2 - F(l-z)$$

$$EIv'(z) = -\frac{1}{6}q(l-z)^3 + \frac{1}{2}F(l-z)^2 + C_1$$

$$EIv(z) = \frac{1}{24}q(l-z)^4 - \frac{1}{6}F(l-z)^3 + C_1 z + C_2$$

RB: $v'(0) = 0 \quad 0 = -\frac{1}{6}ql^3 + \frac{1}{2}Fl^2 + C_1 \quad \Rightarrow \quad C_1 = \frac{1}{6}ql^3 - \frac{1}{2}Fl^2$

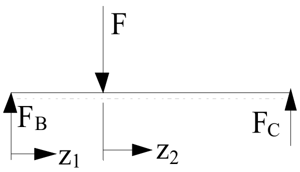
$v(0) = 0 \quad 0 = \frac{1}{24}ql^4 - \frac{1}{6}Fl^3 + C_2 \quad \Rightarrow \quad C_2 = -\frac{1}{24}ql^4 + \frac{1}{6}Fl^3$

$$EIv(z) = \frac{1}{24}ql^4\left[\left(1-\frac{z}{l}\right)^4 + 4\frac{z}{l} - 1\right] - \frac{1}{6}Fl^3\left[\left(1-\frac{z}{l}\right)^3 + 3\frac{z}{l} - 1\right]$$

3.28: $F = 0 \quad \Rightarrow \quad EIv(z) = \frac{1}{24}ql^4\left[\left(1-\frac{z}{l}\right)^4 + 4\frac{z}{l} - 1\right] \quad v(z=l) = v_E = \frac{ql^4}{8EI}$

3.29: $v(z=l) = v_B = 0 \quad \Rightarrow \quad 0 = \frac{1}{8}ql^4 - \frac{1}{3}Fl^3 \quad F_B = \frac{3}{8}ql$

Lösung 4.33



$F_B = \frac{2}{3}F; \qquad F_C = \frac{1}{3}F$

$0 \le z_1 \le a \qquad\qquad 0 \le z_2 \le 2a$

$M(z_1) = F_B z_1 = \frac{2}{3}Fz_1 \quad M(z_2) = F_C(2a - z_2) = \frac{1}{3}F(2a - z_2)$

$$EIv''(z_1) = -\frac{2}{3} F z_1 \qquad EIv''(z_2) = -\frac{1}{3} F(2a - z_2)$$

$$EIv'(z_1) = -\frac{1}{3} F z_1^2 + C_1 \qquad EIv'(z_2) = \frac{1}{6} F(2a - z_2)^2 + C_3$$

$$EIv(z_1) = -\frac{1}{9} F z_1^3 + C_1 z_1 + C_2 \qquad EIv(z_2) = -\frac{1}{18} F(2a - z_2)^3 + C_3 z_2 + C_4$$

RB / ÜB: $v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$

$$v(z_2 = 2a) = 0 \quad \Rightarrow \quad 2C_3 a + C_4 = 0$$

$$v(z_1 = a) = v(z_2 = 0) \quad \Rightarrow \quad -\frac{1}{9} F a^3 + C_1 a = -\frac{4}{9} F a^3 + C_4$$

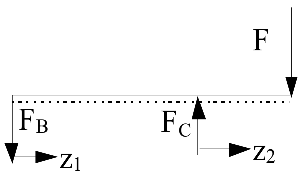
$$v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad -\frac{1}{3} F a^2 + C_1 = \frac{2}{3} F a^2 + C_3$$

$$C_1 = \frac{5}{9} F a^2 \qquad C_2 = 0 \qquad C_3 = -\frac{4}{9} F a^2 \qquad C_4 = \frac{8}{9} F a^3$$

$$v(z_1) = \frac{Fa^3}{9EI}\left[5\frac{z_1}{a} - \left(\frac{z_1}{a}\right)^3\right] \qquad v(z_2) = \frac{Fa^3}{18EI}\left[8 + 4\frac{z_2}{a} - 6\left(\frac{z_2}{a}\right)^2 + \left(\frac{z_2}{a}\right)^3\right]$$

$$v_F = v(z_1 = a) = v(z_2 = 0) = \frac{4Fa^3}{9EI}$$

Lösung 4.34



$$F_B = \frac{1}{2}F; \qquad F_C = \frac{3}{2}F$$

$$0 \le z_1 \le 2a \qquad\qquad 0 \le z_2 \le a$$

$$M(z_1) = -F_B z_1 = -\frac{1}{2}Fz_1 \quad M(z_2) = -F(a - z_2)$$

$$EIv''(z_1) = \frac{1}{2}Fz_1 \qquad\qquad EIv''(z_2) = F(a - z_2)$$

$$EIv'(z_1) = \frac{1}{4}Fz_1^2 + C_1 \qquad\qquad EIv'(z_2) = F\left(az_2 - \frac{z_2^2}{2}\right) + C_3$$

$$EIv(z_1) = \frac{1}{12}Fz_1^3 + C_1 z_1 + C_2 \qquad\qquad EIv(z_2) = F\left(\frac{az_2^2}{2} - \frac{z_2^3}{6}\right) + C_3 z_2 + C_4$$

RB / ÜB: $v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$

$$v(z_2 = 0) = 0 \quad \Rightarrow \quad C_4 = 0$$

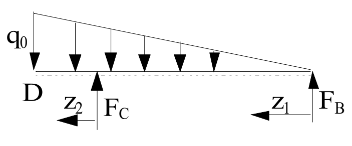
$$v(z_1 = 2a) = 0 \quad \Rightarrow \quad \frac{2}{3}Fa^3 + 2C_1 a = 0 \qquad C_1 = -\frac{1}{3}Fa^2$$

$$v'(z_1 = 2a) = v'(z_2 = 0) \quad \Rightarrow \quad Fa^2 + C_1 = C_3 \qquad C_3 = \frac{2}{3}Fa^2$$

$$v(z_1) = \frac{Fa^3}{12EI}\left[-4\frac{z_1}{a} + \left(\frac{z_1}{a}\right)^3\right] \qquad v(z_2) = \frac{Fa^3}{6EI}\left[4\frac{z_2}{a} + 3\left(\frac{z_2}{a}\right)^2 - \left(\frac{z_2}{a}\right)^3\right]$$

$$v_F = v(z_2 = a) = \frac{Fa^3}{EI} \quad \text{mit} \quad I = 151{,}25\,c^4 \quad \text{folgt} \qquad v_F = 26{,}45\,\text{mm}$$

Lösung 4.35



$$q(z_1) = q_0 \cdot \frac{z_1}{a+b} \quad M(z_1) = F_B z_1 - \frac{1}{6}q(z_1)\cdot z_1^2$$

$$q(z_2) = q_0 \cdot \frac{a + z_2}{a+b} \quad M(z_2) = F_B(a + z_2) + F_C z_2 - \frac{1}{6}q(z_2)\cdot(a + z_2)^2$$

$$EIv''(z_1) = \frac{1}{6}q_0\frac{z_1^3}{a+b} - F_B z_1$$

$$EIv'(z_1) = \frac{1}{24}q_0\frac{z_1^4}{a+b} - \frac{1}{2}F_B z_1^2 + C_1$$

$$EIv(z_1) = \frac{1}{120}q_0\frac{z_1^5}{a+b} - \frac{1}{6}F_B z_1^3 + C_1 z_1 + C_2$$

$$EIv''(z_2) = \frac{1}{6} q_0 \frac{(a+z_2)^3}{a+b} - F_B(a+z_2) - F_C z_2$$

$$EIv'(z_2) = \frac{1}{24} q_0 \frac{(a+z_2)^4}{a+b} - \frac{1}{2} F_B(a+z_2)^2 - \frac{1}{2} F_C z_2^2 + C_3$$

$$EIv(z_2) = \frac{1}{120} q_0 \frac{(a+z_2)^5}{a+b} - \frac{1}{6} F_B(a+z_2)^3 - \frac{1}{6} F_C z_2^3 + C_3 z_2 + C_4$$

RB/ÜB:

$1.\ v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$

$2.\ v(z_1 = a) = 0 \quad \Rightarrow \quad C_1 = \frac{1}{6} F_B a^2 - \frac{1}{120} q_0 \frac{a^4}{a+b}$

$3.\ v(z_2 = 0) = 0 \quad \Rightarrow \quad C_4 = C_1 a = \frac{1}{6} F_B a^3 - \frac{1}{120} q_0 \frac{a^5}{a+b}$

$4.\ v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad C_3 = C_1 = \frac{1}{6} F_B a^2 - \frac{1}{120} q_0 \frac{a^4}{a+b}$

Verschiebu ng des Punktes D:

$$EIv_D = EIv(z_2 = b) = \frac{1}{120} q_0 \left[(a+b)^4 - a^4\right] - \frac{1}{6} F_C b^3 - \frac{1}{6} F_B(a+b)\left[(a+b)^2 - a^2\right]$$

Ermittlung der Auflagerreaktionen:

$\leftarrow:\ F_{BH} = 0 \qquad \uparrow:\ F_B + F_C - \frac{1}{2} q_0(a+b) = 0 \qquad \circlearrowright B:\ F_C a - \frac{1}{3} q_0 (a+b)^2 = 0$

$F_C = \frac{1}{3} q_0 \frac{(a+b)^2}{a} = 81{,}667\,\text{kN} \qquad F_B = \frac{1}{6} q_0 \frac{(a+b)(a-2b)}{a} = 5{,}833\,\text{kN}$

$EIv_D = 12{,}223\,\text{kNm}^3$

Ermittlung von I:

Im 1.Bereich ist $F_Q = 0$ möglich.

$-F_B + \frac{1}{2} q_0 \frac{z_1^{*2}}{a+b} = 0 \quad \Rightarrow \quad z_1^* = \sqrt{2 \frac{F_B}{q_0}(a+b)} = 0{,}904\,\text{m}$

$M(z_1^*) = 3{,}516\,\text{kNm}$ (relat. Extremwert)

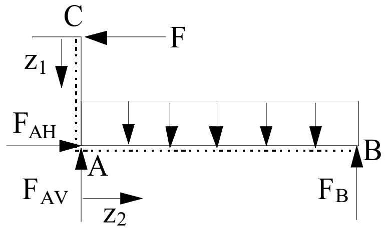
$M_{max} = M(z_1 = a) = F_B a - \frac{1}{6} q_0 \frac{a^3}{a+b} = -22{,}62 \cdot 10^6\,\text{Nmm}$

$\left|\sigma_{max}\right| = \frac{\left|M_{max}\right|}{W} \le \sigma_{zul} \quad \Rightarrow \quad W_{erf} = \frac{\left|M_{max}\right|}{\sigma_{zul}} = 188{,}5 \cdot 10^3\,\text{mm}^3$

I 20 gewählt mit $W = 2{,}14 \cdot 10^5\,\text{mm}^3$ und $I = 2{,}14 \cdot 10^7\,\text{mm}^4$

$$v_D = \frac{12{,}223 \cdot 10^{12}}{2{,}1 \cdot 2{,}14 \cdot 10^{12}}\,\text{mm} = 2{,}72\,\text{mm}$$

Lösung 4.36



RB/ÜB:

1. $v\left(z_1 = \frac{l}{2}\right) = 0$

2. $v'\left(z_1 = \frac{l}{2}\right) = v'(z_2 = 0)$

3. $v(z_2 = 0) = 0$

4. $v(z_2 = l) = 0$

$0 \le z_1 \le \frac{l}{2}$ 0 $\quad\quad \le z_2 \le l$

$M(z_1) = -Fz_1 \quad\quad M(z_2) = F_{AV} z_2 - \frac{1}{2} q z_2^2 - \frac{1}{2} Fl$

$EIv''(z_1) = Fz_1 \quad\quad EIv''(z_2) = \frac{1}{2} q z_2^2 - F_{AV} z_2 + \frac{1}{2} Fl$

$EIv'(z_1) = \frac{1}{2} F z_1^2 + C_1 \quad\quad EIv'(z_2) = \frac{1}{6} q z_2^3 - \frac{1}{2} F_{AV} z_2^2 + \frac{1}{2} Flz_2 + C_3$

$EIv(z_1) = \frac{1}{6} F z_1^3 + C_1 z_1 + C_2 \quad\quad EIv(z_2) = \frac{1}{24} q z_2^4 - \frac{1}{6} F_{AV} z_2^3 + \frac{1}{4} Flz_2^2 + C_3 z_2 + C_4$

GG: $\curvearrowright B$: $\quad F_{AV} l - \frac{1}{2} ql^2 - \frac{1}{2} Fl = 0 \quad \Rightarrow \quad F_{AV} = \frac{1}{2}(ql + F)$

aus 1.: $\quad \frac{1}{48} Fl^3 + \frac{1}{2} C_1 l + C_2 = 0$

aus 2.: $\quad \frac{1}{8} Fl^2 + C_1 = C_3$

aus 3.: $\quad C_4 = 0$

aus 4.: $\quad \frac{1}{24} ql^4 - \frac{1}{6} F_{AV} l^3 + \frac{1}{4} Fl^3 + C_3 l = 0 \quad \Rightarrow \quad C_3 = \frac{1}{24} ql^3 - \frac{1}{6} Fl^2$

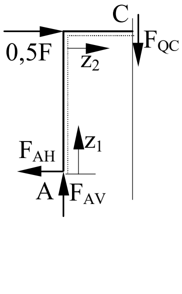
$C_1 = \frac{1}{24} ql^3 - \frac{7}{24} Fl^2 \quad\quad C_2 = \frac{1}{8} Fl^3 - \frac{1}{48} ql^4$

$$EIv(z_1) = \frac{Fl^3}{24}\left[4\left(\frac{z_1}{l}\right)^3 - 7\left(\frac{z_1}{l}\right) + 3\right] + \frac{ql^4}{48}\left[2\left(\frac{z_1}{l}\right) - 1\right]$$

$$EIv(z_2) = \frac{ql^4}{24}\left[\left(\frac{z_2}{l}\right)^4 - 2\left(\frac{z_2}{l}\right)^3 + \left(\frac{z_2}{l}\right)\right] - \frac{Fl^3}{12}\left[\left(\frac{z_2}{l}\right)^3 - 3\left(\frac{z_2}{l}\right)^2 + 2\left(\frac{z_2}{l}\right)\right]$$

$$v_C = v(z_1 = 0) = \frac{C_2}{EI} = \frac{ql^4}{48EI}\left[6\frac{F}{ql} - 1\right] = \frac{ql^4}{24EI} = \frac{1}{3}\text{mm} = 0{,}333\text{mm}$$

Lösung 4.37



Symm. Rahmen mit antimetr. Belastung, Antimetrieschnitt

$\rightarrow$: $\quad F_{AH} = \frac{1}{2} F \quad \uparrow$: $\quad F_{AV} - F_{QC} = 0 \quad \curvearrowleft M_C = 0 = F_{AV} \cdot \frac{l}{2} + F_{AH} l$

$F_{AV} = -2F_{AH} = -F$

Aus dem Gesamtsystem: $F_{AV}+F_{BV}=0 \quad F_{BV}=F \quad F_{AH}+F_{BH}-F=0 \quad F_{BH}=\frac{1}{2}F$

$$M(z_1)=F_{AH}z_1 \qquad M(z_2)=F_{AH}l+F_{AV}z_2$$

$$EIv''(z_1)=-F_{AH}z_1 \qquad EIv''(z_2)=-F_{AH}l-F_{AV}z_2$$

$$EIv'(z_1)=-\frac{1}{2}F_{AH}z_1^2+C_1 \qquad EIv'(z_2)=-F_{AH}lz_2-\frac{1}{2}F_{AV}z_2^2+C_3$$

$$EIv'(z_1)=-\frac{1}{6}F_{AH}z_1^3+C_1z_1+C_2 \qquad EIv(z_2)=-\frac{1}{2}F_{AH}lz_2^2-\frac{1}{6}F_{AV}z_2^3+C_3z_2+C_4$$

RB / ÜB: $v(z_1=0)=0 \quad \Rightarrow \quad C_2=0$

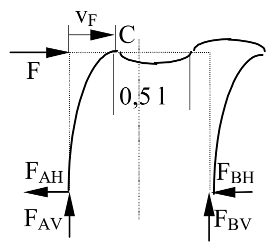
$v(z_2=0)=0 \quad \Rightarrow \quad C_4=0$

$v'(z_1=l)=v'(z_2=0) \quad \Rightarrow \quad -\frac{1}{2}F_{AH}l^2+C_1=C_3$

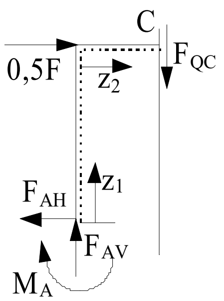
$v\left(z_2=\frac{l}{2}\right)=0 \quad \Rightarrow \quad -\frac{1}{8}F_{AH}l^3-\frac{1}{48}F_{AV}l^3+\frac{1}{2}C_3l=0 \quad C_3=\frac{1}{12}Fl^2$

$$C_1=\frac{1}{3}Fl^2$$

$$v_F=v(z_1=l)=-\frac{Fl^3}{12EI}+\frac{Fl^3}{3EI}=\frac{Fl^3}{4EI}$$



Lösung 4.38



$\rightarrow: \quad F_{AH}=\frac{1}{2}F \qquad \uparrow: \quad F_{AV}=F_{QC}$

A: $\quad M_A=-F\cdot\frac{l}{2}-F_{QC}\cdot\frac{l}{2}$

$$M(z_1)=M_A+F_{AH}z_1 \qquad M(z_2)=M_A+F_{AH}l+F_{AV}z_2$$

$$EIv''(z_1)=-M_A-F_{AH}z_1 \qquad EIv''(z_2)=-M_A-F_{AH}l-F_{AV}z_2$$

$$EIv'(z_1)=-M_Az_1-\frac{1}{2}F_{AH}z_1^2+C_1 \qquad EIv'(z_2)=-M_Az_2-F_{AH}lz_2-\frac{1}{2}F_{AV}z_2^2+C_3$$

$$EIv(z_1)=-\frac{1}{2}M_Az_1^2-\frac{1}{6}F_{AH}z_1^3+C_1z_1+C_2 \qquad EIv(z_2)=-\frac{1}{2}M_Az_2^2-\frac{1}{2}F_{AH}lz_2^2-\frac{1}{6}F_{AV}z_2^3+C_3z_2+C_4$$

$RB/ÜB$: $v(z_1=0)=0 \quad \Rightarrow \quad C_2=0$

$v(z_2=0)=0 \quad \Rightarrow \quad C_4=0$

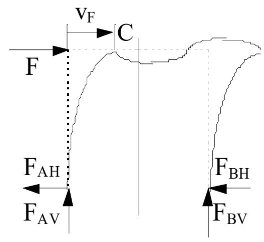
$v'(z_1=l)=v'(z_2=0) \quad \Rightarrow \quad -M_Al-\frac{1}{2}F_{AH}l^2+C_1=C_3$

$v\left(z_2=\frac{l}{2}\right)=0 \quad \Rightarrow \quad -\frac{1}{8}M_Al^2-\frac{1}{8}F_{AH}l^3-\frac{1}{48}F_{AV}l^3+\frac{1}{2}C_3l=0$

$v'(z_1=0)=0 \quad \Rightarrow \quad C_1=0$

$$-M_A l - \frac{1}{2} F_{AH} l^2 = C_3$$

$$\frac{1}{4} M_A l + \frac{1}{4} F_{AH} l^2 + \frac{1}{24} F_{AV} l^2 = C_3$$



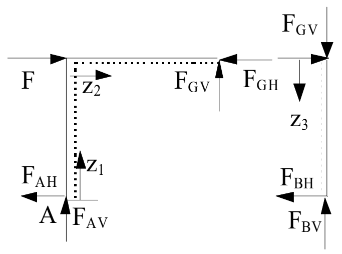
GG. einsetzen liefert:

$$F_{QC} = -\frac{3}{7} F; \quad F_{AV} = -\frac{3}{7} F; \quad M_A = -\frac{2}{7} Fl$$

$$F_{BH} = \frac{1}{2} F; \quad F_{BV} = \frac{3}{7} F; \quad M_B = \frac{2}{7} Fl$$

$$v_F = v(z_1 = l) = -\frac{Fl^3}{12EI} + \frac{Fl^2}{7EI} = \frac{5Fl^2}{84EI}$$

Lösung 4.39



Auflagerreaktionen aus den Gleichgewichtsbedingungen:

$$F_{GH} = F_{BH} = 0; \quad F_{GV} = F_{BV} = F;$$
$$F_{AH} = F; \quad F_{AV} = -F$$

$M(z_1) = Fz_1$	$M(z_2) = Fl - Fz_2$	$M(z_3) = 0$
$EIv''(z_1) = -Fz_1$	$EIv''(z_2) = -Fl + Fz_2$	$EIv''(z_3) = 0$
$EIv'(z_1) = -\frac{1}{2}Fz_1^2 + C_1$	$EIv'(z_2) = -Flz_2 + \frac{1}{2}Fz_2^2 + C_3$	$EIv'(z_3) = C_5$
$EIv(z_1) = -\frac{1}{6}Fz_1^3 + C_1 z_1 + C_2$	$EIv(z_2) = -\frac{1}{2}Flz_2^2 + \frac{1}{6}Fz_2^3 + C_3 z_2 + C_4$	$EIv(z_3) = C_5 z_3 + C_6$

Rand- und Übergangsbedingungen:

$$1. v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$$

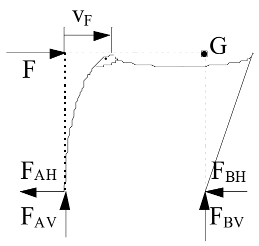
$$2. v(z_2 = 0) = 0 \quad \Rightarrow \quad C_4 = 0$$

$$3. v'(z_1 = l) = v'(z_2 = 0) \quad \Rightarrow \quad -\frac{1}{2}Fl^2 + C_1 = C_3$$

$$4. v(z_2 = l) = 0 \quad \Rightarrow \quad -\frac{1}{2}Fl^3 + \frac{1}{6}Fl^3 + C_3 l = 0 \quad C_3 = \frac{1}{3}Fl^2$$

$$5. v(z_3 = l) = 0 \quad \Rightarrow \quad C_5 l + C_6 = 0$$

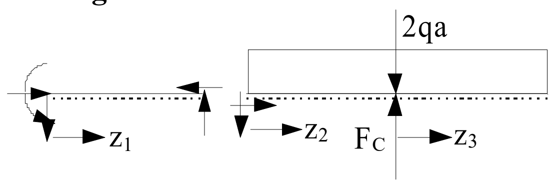
$$6. v(z_1 = l) = -v(z_3 = 0) \quad \Rightarrow \quad -\frac{1}{6}Fl^3 + C_1 l = -C_6$$



$$C_1 = \frac{5}{6} Fl^2 \qquad C_6 = -\frac{2}{3} Fl^3 \qquad C_5 = \frac{2}{3} Fl^2$$

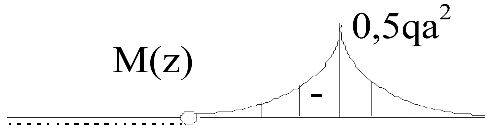
$$v_F = v(z_1 = l) = -v(z_3 = 0) = -\frac{C_6}{EI} = \frac{2Fl^3}{3EI}$$

Lösung 4.40



Die 6 Gleichgewichtsbedingungen liefern:

$F_{BH} = 0 \quad F_{BV} = 0 \quad M_B = 0 \quad F_{GH} = 0$
$F_{GV} = 0 \quad F_C = 2qa$



$$M(z_1) = 0 \qquad M(z_2) = -\frac{1}{2}qz_2^2 \qquad M(z_3) = -\frac{1}{2}q(a - z_3)^2$$

$$EIv''(z_1) = 0 \qquad EIv''(z_2) = \frac{1}{2}qz_2^2 \qquad EIv''(z_3) = \frac{1}{2}qz_3^2 - qaz_3 + \frac{1}{2}qa^2$$

$$EIv'(z_1) = C_1 \qquad EIv'(z_2) = \frac{1}{6}qz_2^3 + C_3 \qquad EIv'(z_3) = \frac{1}{6}qz_3^3 - \frac{1}{2}qaz_3^2 + \frac{1}{2}qa^2z_3 + C_5$$

$$EIv(z_1) = C_1z_1 + C_2 \quad EIv(z_2) = \frac{1}{24}qz_2^4 + C_3z_2 + C_4 \quad EIv'(z_3) = \frac{1}{24}qz_3^4 - \frac{1}{6}qaz_3^3 + \frac{1}{4}qa^2z_3^2 + C_5z_3 + C_6$$

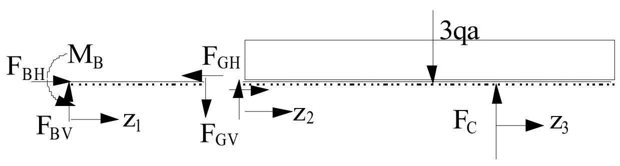
1. $v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$
2. $v'(z_1 = 0) = 0 \quad \Rightarrow \quad C_1 = 0$
3. $v(z_2 = a) = 0 \quad \Rightarrow \quad \frac{1}{24}qa^4 + C_3a + C_4 = 0 \qquad C_3 = -\frac{1}{24}qa^3$
4. $v(z_3 = 0) = 0 \quad \Rightarrow \quad C_6 = 0$
5. $v(z_1 = a) = v(z_2 = 0) \quad \Rightarrow \quad C_4 = 0$
6. $v'(z_2 = a) = v'(z_3 = 0) \quad \Rightarrow \quad \frac{1}{6}qa^3 + C_3 = C_5 \qquad C_5 = \frac{1}{8}qa^3$

$$v(z_1) = 0$$

$$v(z_2) = \frac{qa^4}{24EI}\left[\left(\frac{z_2}{a}\right)^4 - \left(\frac{z_2}{a}\right)\right] \qquad v_G = v(z_1 = a) = v(z_2 = 0) = 0$$

$$v(z_3) = \frac{qa^4}{24EI}\left[\left(\frac{z_3}{a}\right)^4 - 4\left(\frac{z_3}{a}\right)^3 + 6\left(\frac{z_3}{a}\right)^2 + 3\left(\frac{z_3}{a}\right)\right] \qquad v_D = v(z_3 = a) = \frac{qa^4}{4EI}$$

Lösung 4.41



Die 6 Gleichgewichtsbedingungen liefern:

$$F_{BH} = 0 \quad F_{BV} = \frac{3}{4}qa \quad M_B = \frac{3}{4}qa^2$$

$$F_{GH} = 0 \quad F_{GV} = \frac{3}{4}qa \quad F_C = \frac{9}{4}qa$$

$$M(z_1) = -\frac{3}{4}qa(a - z_1) \qquad M(z_2) = \frac{3}{4}qaz_2 - \frac{1}{2}qz_2^2$$

$$EIv''(z_1) = \frac{3}{4}qa(a - z_1) \qquad EIv''(z_2) = \frac{1}{2}qz_2^2 - \frac{3}{4}qaz_2$$

$$EIv'(z_1) = \frac{3}{4}qa\left(az_1 - \frac{1}{2}z_1^2\right) + C_1 \qquad EIv'(z_2) = \frac{1}{6}qz_2^3 - \frac{3}{8}qaz_2^2 + C_3$$

$$EIv(z_1) = \frac{3}{4}qa\left(\frac{1}{2}az_1^2 - \frac{1}{6}z_1^3\right) + C_1z_1 + C_2 \qquad EIv(z_2) = \frac{1}{24}qz_2^4 - \frac{1}{8}qaz_2^3 + C_3z_2 + C_4$$

$$M(z_3) = -\frac{1}{2}q(a - z_3)^2$$

$$EIv''(z_3) = \frac{1}{2}qz_3^2 - qaz_3 + \frac{1}{2}qa^2$$

$$EIv'(z_3) = \frac{1}{6}qz_3^3 - \frac{1}{2}qaz_3^2 + \frac{1}{2}qa^2z_3 + C_5$$

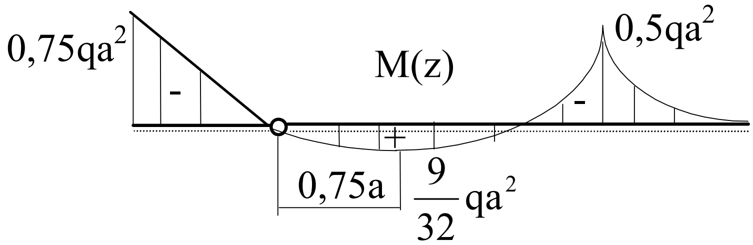
$$EIv(z_3) = \frac{1}{24}qz_3^4 - \frac{1}{6}qaz_3^3 + \frac{1}{4}qa^2z_3^2 + C_5z_3 + C_6$$

1. $\mathrm{v}(\mathrm{z}_1 = 0) = 0 \quad \Rightarrow \quad \mathrm{C}_2 = 0$
2. $\mathrm{v}'(\mathrm{z}_1 = 0) = 0 \quad \Rightarrow \quad \mathrm{C}_1 = 0$
3. $\mathrm{v}(\mathrm{z}_2 = 2\mathrm{a}) = 0 \quad \Rightarrow \quad -\frac{1}{3}\mathrm{qa}^4 + 2\mathrm{C}_3\mathrm{a} + \mathrm{C}_4 = 0 \qquad \mathrm{C}_3 = \frac{1}{24}\mathrm{qa}^3$
4. $\mathrm{v}(\mathrm{z}_3 = 0) = 0 \quad \Rightarrow \quad \mathrm{C}_6 = 0$
5. $\mathrm{v}(\mathrm{z}_1 = \mathrm{a}) = \mathrm{v}(\mathrm{z}_2 = 0) \quad \Rightarrow \quad \mathrm{C}_4 = \frac{1}{4}\mathrm{qa}^4$
6. $\mathrm{v}'(\mathrm{z}_2 = 2\mathrm{a}) = \mathrm{v}'(\mathrm{z}_3 = 0) \quad \Rightarrow \quad -\frac{1}{6}\mathrm{qa}^3 + \mathrm{C}_3 = \mathrm{C}_5 \qquad \mathrm{C}_5 = -\frac{1}{8}\mathrm{qa}^3$

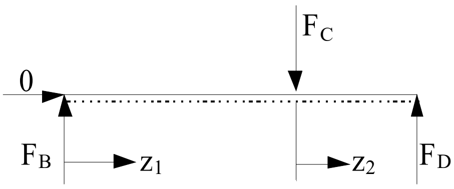
$$v(z_1) = \frac{qa^4}{8EI}\left[3\left(\frac{z_1}{a}\right)^2 - \left(\frac{z_1}{a}\right)^3\right]$$

$$v(z_2) = \frac{qa^4}{24EI}\left[\left(\frac{z_2}{a}\right)^4 - 3\left(\frac{z_2}{a}\right)^3 + \left(\frac{z_2}{a}\right) + 6\right] \qquad v_G = v(z_1 = a) = v(z_2 = 0) = \frac{qa^4}{4EI}$$

$$v(z_3) = \frac{qa^4}{24EI}\left[\left(\frac{z_3}{a}\right)^4 - 4\left(\frac{z_3}{a}\right)^3 + 6\left(\frac{z_3}{a}\right)^2 - 3\left(\frac{z_3}{a}\right)\right] \qquad v_D = v(z_3 = a) = 0$$



Lösung 4.42



$$F_B = \frac{1}{3}F_C \quad F_D = \frac{2}{3}F_C$$

$$M(z_1) = \frac{1}{3}F_C z_1 \quad M(z_2) = \frac{2}{3}F_C(a - z_2)$$

$$EIv''(z_1) = -\frac{1}{3}F_C z_1 \qquad EIv''(z_2) = -\frac{2}{3}F_C(a - z_2)$$

$$EIv'(z_1) = -\frac{1}{6}F_C z_1^2 + C_1 \qquad EIv'(z_2) = -\frac{2}{3}F_C\left(az_2 - \frac{1}{2}z_2^2\right) + C_3$$

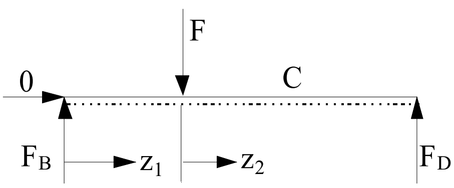
$$EIv(z_1) = -\frac{1}{18}F_C z_1^3 + C_1 z_1 + C_2 \qquad EIv(z_2) = -\frac{2}{3}F_C\left(\frac{1}{2}az_2^2 - \frac{1}{6}z_2^3\right) + C_3 z_2 + C_4$$

RB/ÜB:

1. $v(z_1 = 0) = 0$ $\qquad C_2 = 0$
2. $v(z_1 = 2a) = \Delta h$ $\qquad C_1 = \frac{2}{9}F_C a^2 + \frac{EI\Delta h}{2a}$
3. $v(z_2 = 0) = \Delta h$ $\qquad C_4 = EI\Delta h$
4. $v(z_2 = a) = 0$ $\qquad C_3 = \frac{2}{9}F_C a^2 - \frac{EI\Delta h}{a}$
5. $v'(z_1 = 2a) = v'(z_2 = 0)$ $\qquad -\frac{2}{3}F_C a^2 + C_1 = C_3$

$$-\frac{2}{3}F_C a^2 + \frac{3}{2}\frac{EI\Delta h}{a} = 0 \qquad F_C = \frac{9}{4}\frac{EI\Delta h}{a^3} = 111{,}8\,\mathrm{N}$$

Lösung 4.43



$$F_B = \frac{2}{3}F \quad F_D = \frac{1}{3}F$$

$$M(z_1) = \frac{2}{3}F z_1 \quad M(z_2) = \frac{1}{3}F(2a - z_2)$$

$$EIv''(z_1) = -\frac{2}{3}F z_1 \qquad EIv''(z_2) = -\frac{1}{3}F(2a - z_2)$$

$$EIv'(z_1) = -\frac{1}{3}F z_1^2 + C_1 \qquad EIv'(z_2) = -\frac{1}{3}F\left(2az_2 - \frac{1}{2}z_2^2\right) + C_3$$

$$EIv(z_1) = -\frac{1}{9}F z_1^3 + C_1 z_1 + C_2 \qquad EIv(z_2) = -\frac{1}{3}F\left(az_2^2 - \frac{1}{6}z_2^3\right) + C_3 z_2 + C_4$$

RB/ÜB:

$$1.\, v(z_1=0)=0 \qquad C_2=0$$

$$2.\, v(z_1=a)=v(z_2=0) \qquad -\frac{1}{9}Fa^3+C_1a=C_4$$

$$3.\, v(z_2=a)=\Delta h \qquad EI\Delta h=-\frac{5}{18}Fa^3+C_3a+C_4$$

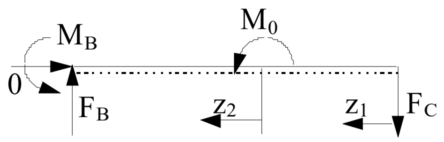
$$4.\, v(z_2=2a)=0 \qquad -\frac{8}{9}Fa^3+2C_3a+C_4=0$$

$$5.\, v'(z_1=a)=v'(z_2=0) \qquad -\frac{1}{3}Fa^2+C_1=C_3$$

$$C_3=\frac{2}{9}Fa^2 \qquad C_1=\frac{5}{9}Fa^2 \qquad C_4=\frac{4}{9}Fa^3$$

$$EI\Delta h=\frac{7}{18}Fa^3 \qquad F=\frac{18}{7}\frac{EI\Delta h}{a^3}=127{,}8N$$

Lösung 4.44



$M(z_1) = - F_C\, z_1$

$M(z_2) = M_0 - F_C\, (a+z_2)$

$$EIv''(z_1)=F_Cz_1 \qquad EIv''(z_2)=F_C(a+z_2)-M_0$$

$$EIv'(z_1)=\frac{1}{2}F_Cz_1^2+C_1 \qquad EIv'(z_2)=F_C\left(az_2+\frac{1}{2}z_2^2\right)-M_0z_2+C_3$$

$$EIv(z_1)=\frac{1}{6}F_Cz_1^3+C_1z_1+C_2 \qquad EIv(z_2)=F_C\left(\frac{1}{2}az_2^2+\frac{1}{6}z_2^3\right)-\frac{1}{2}M_0z_2^2+C_3z_2+C_4$$

RB/ÜB:

$$1.\, v(z_1=0)=0 \qquad C_2=0$$

$$2.\, v(z_1=a)=v(z_2=0) \qquad \frac{1}{6}F_Ca^3+C_1a=C_4$$

$$3.\, v'(z_1=a)=v'(z_2=0) \qquad \frac{1}{2}F_Ca^2+C_1=C_3$$

$$4.\, v(z_2=2a)=0 \qquad \frac{10}{3}F_Ca^3-2M_0a^2+2C_3a+C_4=0$$

$$5.\, v'(z_2=2a)=0 \qquad 4F_Ca^2-2M_0a+C_3=0$$

aus 5: $\quad C_3 = -4F_C a^2 + 2M_0 a$

aus 3: $\quad C_1 = -\frac{9}{2}F_C a^2 + 2M_0 a$

aus 2: $\quad C_4 = -\frac{13}{3}F_C a^3 + 2M_0 a^2$

in 4: $\quad 4M_0 - 9F_C a = 0 F_C = \frac{4}{9}\frac{M_0}{a}$

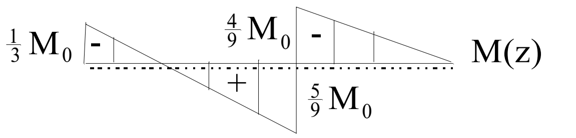
$C_3 = \frac{2}{9}M_0 a \quad C_4 = \frac{2}{27}M_0 a^2 \quad C_1 = 0 \quad C_2 = 0$

Die Gleichgewichtsbedingungen liefern:

$F_B = F_C = \frac{4}{9}\frac{M_0}{a} \qquad F_{BH} = 0 \qquad M_B = \frac{1}{3}M_0$

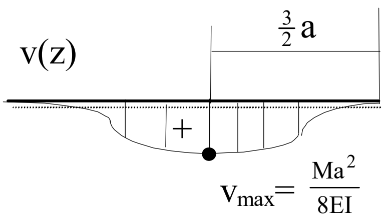
$M(z_1) = -\frac{4}{9}M_0\frac{z_1}{a} \qquad M(0) = 0 \qquad M(a) = -\frac{4}{9}M_0$

$M(z_2) = M_0 - \frac{4}{9}M_0\left(1 + \frac{z_2}{a}\right) \qquad M(0) = \frac{5}{9}M_0 \qquad M(2a) = -\frac{1}{3}M_0$



$$EIv(z_1) = \frac{2}{27}M_0 a^2\left(\frac{z_1}{a}\right)^3$$

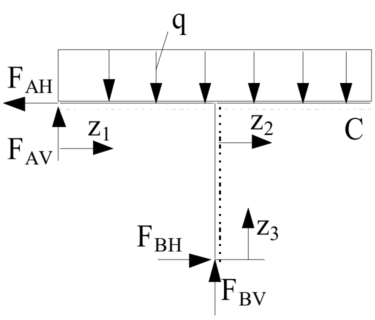
$$EIv(z_2) = \frac{2}{27}M_0 a^2\left[\left(\frac{z_2}{a}\right)^3 - \frac{15}{4}\left(\frac{z_2}{a}\right)^2 + 3\left(\frac{z_2}{a}\right) + 1\right]$$



$$\frac{dv(z_2)}{d\left(\frac{z_2}{a}\right)} = 0 = 3 - \frac{15}{2}\left(\frac{z_2}{a}\right) + 3\left(\frac{z_2}{a}\right)^2$$

$$\left(\frac{z_2}{a}\right)_{1,2} = \frac{5}{4} \pm \frac{3}{4} \qquad \left(\frac{z_2}{a}\right)_1 = 2 \qquad \left(\frac{z_2}{a}\right)_2 = \frac{1}{2}$$

Lösung 4.45



Die Aufgabe ist einfach statisch unbestimmt.
Gleichgewichtsbedingungen:

$\leftarrow: \; F_{AH} - F_{BH} = 0$

$\uparrow \; : \; F_{AV} + F_{BV} - 2qa = 0$

$B: \; F_{AV} \cdot a - F_{AH} \cdot a = 0$

$$M(z_1) = F_{AV} \cdot z_1 - \frac{1}{2} q z_1^2 \qquad M(z_2) = -\frac{1}{2} q (a - z_2)^2$$

$$EIv''(z_1) = \frac{1}{2} q z_1^2 - F_{AV} \cdot z_1 \qquad EIv''(z_2) = \frac{1}{2} q (a - z_2)^2$$

$$EIv'(z_1) = \frac{1}{6} q z_1^3 - \frac{1}{2} F_{AV} \cdot z_1^2 + C_1 \qquad EIv'(z_2) = -\frac{1}{6} q (a - z_2)^3 + C_3$$

$$EIv(z_1) = \frac{1}{24} q z_1^4 - \frac{1}{6} F_{AV} \cdot z_1^3 + C_1 z_1 + C_2 \qquad EIv(z_2) = \frac{1}{24} q (a - z_2)^4 + C_3 z_2 + C_4$$

$$M(z_3) = -F_{BH} \cdot z_3$$

$$EIv''(z_3) = F_{BH} \cdot z_3$$

$$EIv'(z_3) = \frac{1}{2} F_{BH} \cdot z_3^2 + C_5$$

$$EIv(z_3) = \frac{1}{6} F_{BH} \cdot z_3^3 + C_5 z_3 + C_6$$

RB/ÜB:

1. $v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$
2. $v(z_3 = 0) = 0 \quad \Rightarrow \quad C_6 = 0$
3. $v(z_1 = a) = 0 \quad \Rightarrow \quad \frac{1}{24} q a^4 - \frac{1}{6} F_{AV} a^3 + C_1 a = 0 \quad C_1 = \frac{1}{6} F_{AV} a^2 - \frac{1}{24} q a^3$
4. $v(z_2 = 0) = 0 \quad \Rightarrow \quad \frac{1}{24} q a^4 + C_4 = 0 \qquad C_4 = -\frac{1}{24} q a^4$
5. $v(z_3 = a) = 0 \quad \Rightarrow \quad \frac{1}{6} F_{BH} a^3 + C_5 a = 0 \qquad C_5 = -\frac{1}{6} F_{BH} a^2$
6. $v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad \frac{1}{6} q a^3 - \frac{1}{2} F_{AV} a^2 + C_1 = -\frac{1}{6} q a^3 + C_3$
7. $v'(z_3 = a) = v'(z_2 = 0) \quad \Rightarrow \quad \frac{1}{2} F_{BH} a^2 + C_5 = -\frac{1}{6} q a^3 + C_3 \quad C_3 = \frac{1}{3} F_{BH} a^2 + \frac{1}{6} q a^3$

Ergebnisse aus GGW und RB/ÜB:

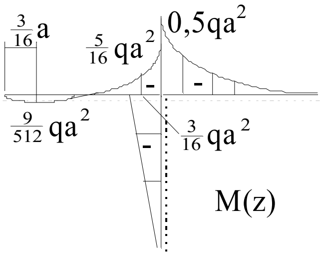
aus 6. folgt mit den GGW $F_{AV} = \frac{3}{16} qa \quad \Rightarrow \quad F_{AH} = F_{BH} = \frac{3}{16} qa$ und $F_{BV} = \frac{29}{16} qa$

Momentenverlauf:

$$M(z_1) = \frac{1}{2} q z_1 \left(\frac{3}{8} a - z_1 \right) \quad F_Q(z_1^*) = \frac{3}{16} qa - q z_1^* = 0 \quad z_1^* = \frac{3}{16} a \quad M(z_1^*) = \frac{9}{512} q a^2$$

$$M(z_2) = -\frac{1}{2} q (a - z_2)^2$$

$$M(z_3) = -\frac{3}{16} q a z_3$$

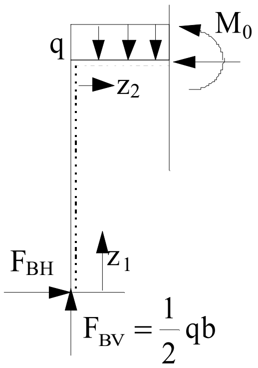


Verschiebung des Punktes C:

$$v_C = v(z_2 = a) = \frac{1}{EI} (C_3 a + C_4) = \frac{3 q a^4}{16 EI}$$

Lösung 4.46

Das System ist einfach statisch unbestimmt und symmetrisch.
Symmetrieschnitt:



$$F_{BV} = \frac{1}{2} qb$$

$$M(z_1) = -F_{BH} \cdot z_1$$

$$EI_1 v''(z_1) = F_{BH} \cdot z_1$$

$$EI_1 v'(z_1) = \frac{1}{2} F_{BH} \cdot z_1^2 + C_1$$

$$EI_1 v(z_1) = \frac{1}{6} F_{BH} \cdot z_1^3 + C_1 z_1 + C_2$$

$$M(z_2) = -F_{BH} a + \frac{1}{2} qbz_2 - \frac{1}{2} qz_2^2$$

$$EI_2 v''(z_2) = \frac{1}{2} qz_2^2 - \frac{1}{2} qbz_2 + F_{BH} a$$

$$EI_2 v'(z_2) = \frac{1}{6} qz_2^3 - \frac{1}{4} qbz_2^2 + F_{BH} az_2 + C_3$$

$$EI_2 v(z_2) = \frac{1}{24} qz_2^4 - \frac{1}{12} qbz_2^3 + \frac{1}{2} F_{BH} az_2^2 + C_3 z_2 + C_4$$

RB/ÜB:

$$v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$$

$$v(z_1 = a) = 0 \quad \Rightarrow \quad \frac{1}{6} F_{BH} a^3 + C_1 a = 0$$

$$v(z_2 = 0) = 0 \quad \Rightarrow \quad C_4 = 0$$

$$v'\left(z_2 = \frac{b}{2}\right) = 0 \quad \Rightarrow \quad \frac{1}{2} F_{BH} ab - \frac{1}{24} qb^3 + C_3 = 0$$

$$v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad \frac{1}{I_1}\left(\frac{1}{2} F_{BH} a^2 + C_1\right) = \frac{C_3}{I_2}$$

Daraus folgt:

$$F_{BH} = \frac{qb^2}{4a} \cdot \frac{1}{2\frac{a}{b} \cdot \frac{I_2}{I_1} + 3} \qquad C_1 = -\frac{qab^2}{24} \cdot \frac{1}{2\frac{a}{b} \cdot \frac{I_2}{I_1} + 3} \qquad C_3 = \frac{qb^3}{12} \cdot \frac{1}{2 + 3\frac{b}{a} \cdot \frac{I_1}{I_2}}$$

Zahlenwerte: $F_{BH} = 2{,}778$ kN $\quad F_{BV} = 20$ kN

Maximale Verschiebung im Bereich 1:

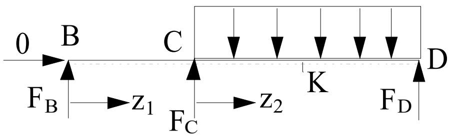
$$v'(z_1^*) = 0 \quad \Rightarrow \frac{1}{2} F_{BH} z_1^{*2} + C_1 = 0 \quad z_1^* = \sqrt{-\frac{2C_1}{F_{BH}}} = \frac{a\sqrt{3}}{3}$$

$$v_{1\max} = v(z_1^*) = -\frac{qa^2 b^2}{36\sqrt{3} \cdot EI_1} \cdot \frac{1}{2\frac{a}{b}\frac{I_2}{I_1} + 3} = -0{,}5728 mm$$

Maximale Verschiebung im Bereich 2:

$$v_{2\max} = v\left(z_2 = \frac{b}{2}\right) = \frac{qb^4}{384EI_2}\left\{\frac{20\frac{a}{b}\frac{I_2}{I_1} + 36 + 9\frac{b}{a}\frac{I_1}{I_2}}{\left(2\frac{a}{b}\frac{I_2}{I_1} + 3\right)\left(2 + 3\frac{b}{a}\frac{I_1}{I_2}\right)}\right\} = \frac{qb^4}{384EI_2}\cdot\frac{3 + 10\frac{a}{b}\frac{I_2}{I_1}}{2\frac{a}{b}\frac{I_2}{I_1} + 3} = 1{,}653\text{mm}$$

Lösung 4.47



$0 \le z_1 \le a \quad M(z_1) = F_B z_1$

$0 \le z_2 \le 2a \quad M(z_2) = F_C z_2 + F_B(a + z_2) - \frac{1}{2}qz_2^2$

$EIv''(z_1) = -F_B \cdot z_1 \qquad EIv''(z_2) = \frac{1}{2}qz_2^2 - (F_B + F_C)z_2 - F_B a$

$EIv'(z_1) = -\frac{1}{2}F_B \cdot z_1^2 + C_1 \qquad EIv'(z_2) = \frac{1}{6}qz_2^3 - \frac{1}{2}(F_B + F_C)z_2^2 - F_B a z_2 + C_3$

$EIv(z_1) = -\frac{1}{6}F_B \cdot z_1^3 + C_1 z_1 + C_2 \qquad EIv(z_2) = \frac{1}{24}qz_2^4 - \frac{1}{6}(F_B + F_C)z_2^3 - \frac{1}{2}F_B a z_2^2 + C_3 z_2 + C_4$

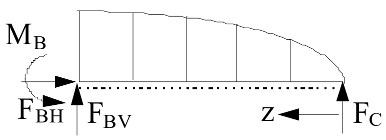
RB/ÜB:

1. $v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$
2. $v(z_1 = a) = 0 \quad \Rightarrow \quad C_1 = \frac{1}{6}F_B a^2$
3. $v(z_2 = 0) = 0 \quad \Rightarrow \quad C_4 = 0$
4. $v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad C_3 = -\frac{1}{3}F_B a^2$
5. $v(z_2 = 2a) = 0 \quad \Rightarrow \quad \frac{2}{3}qa^4 - \frac{4}{3}(F_B + F_C)a^3 - 2F_B a^3 - \frac{2}{3}F_B a^3 = 0$

Mit der Gleichgewichtsbedingung $D:\ 3F_B a + 2F_C a - 2qa^2 = 0 \quad F_C = qa - \frac{3}{2}F_B$ erhält man $F_B = -\frac{1}{3}qa$ und $F_C = \frac{3}{2}qa$ und aus den restlichen Gleichgewichtsbedingungen $F_{BH} = 0$ und $F_D = \frac{5}{6}qa$

$$v_K = v(z_2 = a) = \frac{1}{EI}\left\{\frac{1}{24}qa^4 - \frac{7}{36}qa^4 + \frac{1}{6}qa^4 + \frac{1}{9}qa^4\right\} = \frac{qa^4}{8EI}$$

Lösung 4.48



$q(z) = A + Bz + Cz^2$

$q'(z) = B + 2Cz$

$$q(0)=0 \quad \Rightarrow \quad A=0$$

$$q'(l)=0 \quad \Rightarrow \quad 0=B+2Cl \;\big|\cdot l \quad \big|-$$

$$q(l)=q_0 \quad \Rightarrow \quad q_0=Bl+Cl^2 \qquad \big|+ \quad C=-\frac{q_0}{l^2} \quad B=2\frac{q_0}{l}$$

$$q(z)=2q_0\frac{z}{l}-q_0\left(\frac{z}{l}\right)^2=q_0\frac{z}{l}\left(2-\frac{z}{l}\right)$$

$$EIv''''(z)=q(z)=2q_0\frac{z}{l}-q_0\left(\frac{z}{l}\right)^2$$

$$EIv'''(z)=-F_Q(z)=q_0\frac{z^2}{l}-\frac{1}{3}q_0\frac{z^3}{l^2}+C_1$$

$$EIv''(z)=-M(z)=\frac{1}{3}q_0\frac{z^3}{l}-\frac{1}{12}q_0\frac{z^4}{l^2}+C_1z+C_2$$

$$EIv'(z)=\frac{1}{12}q_0\frac{z^4}{l}-\frac{1}{60}q_0\frac{z^5}{l^2}+\frac{1}{2}C_1z^2+C_2z+C_3$$

$$EIv(z)=\frac{1}{60}q_0\frac{z^5}{l}-\frac{1}{360}q_0\frac{z^6}{l^2}+\frac{1}{6}C_1z^3+\frac{1}{2}C_2z^2+C_3z+C_4$$

RB:

$$1.\,v(0)=0 \qquad \Rightarrow \qquad C_4=0$$

$$2.\,v(l)=0 \qquad \Rightarrow \qquad \frac{1}{60}q_0l^4-\frac{1}{360}q_0l^4+\frac{1}{6}C_1l^3+\frac{1}{2}C_2l^2+C_3l=0$$

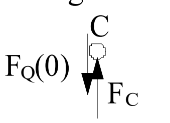
$$3.\,v'(l)=0 \qquad \Rightarrow \qquad \frac{1}{12}q_0l^3-\frac{1}{60}q_0l^3+\frac{1}{2}C_1l^2+C_2l+C_3=0$$

$$4.\,v''(0)=0 \qquad \Rightarrow \qquad C_2=0$$

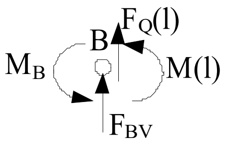
Aus (2) und (3):

$$C_1=-\frac{19}{120}q_0l \qquad C_3=\frac{1}{80}q_0l^3$$

Auflagerreaktionen:



$$\uparrow F_C-F_Q(0)=0 \quad F_C=F_Q(0)=-EIv'''(0)=-C_1 \quad F_C=\frac{19}{120}q_0l$$



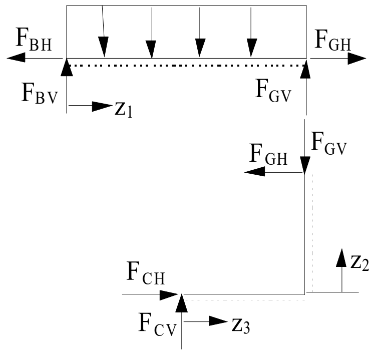
$$F_{BV}=-F_Q(l)=EIv'''(l)=q_0l-\frac{1}{3}q_0l-\frac{19}{120}q_0l$$

$$F_{BV}=F_B=\frac{61}{120}q_0l$$

$$M_B=-M(l)=EIv''(l)=\frac{1}{3}q_0l^2-\frac{1}{12}q_0l^2-\frac{19}{120}q_0l^2$$

$$M_B=\frac{11}{120}q_0l^2$$

Lösung 4.49



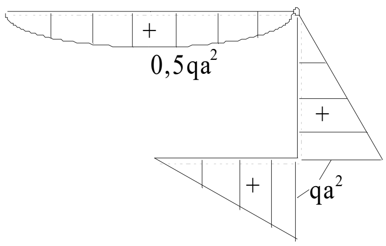
Die Gleichgewichtsbedingungen liefern:
$F_{BV} = qa \quad F_{GV} = qa \quad F_{CV} = qa$
$F_{GH} = qa \quad F_{CH} = qa \quad F_{BH} = qa$

$$M(z_1) = F_{BV} z_1 - \frac{1}{2} q z_1^2 = qa\left(z_1 - \frac{1}{2}\frac{z_1^2}{a}\right)$$

$$M(z_2) = F_{GH}(a - z_2) = qa(a - z_2)$$

$$M(z_3) = F_{CV} z_3 = qaz_3$$

Grafische Darstellung des Biegemomentes:



$$EIv''(z_1) = \frac{1}{2} q z_1^2 - qaz_1 \qquad EIv''(z_2) = qaz_2 - qa^2$$

$$EIv'(z_1) = \frac{1}{6} q z_1^3 - \frac{1}{2} qaz_1^2 + C_1 \qquad EIv'(z_2) = \frac{1}{2} qaz_2^2 - qa^2 z_2 + C_3$$

$$EIv(z_1) = \frac{1}{24} q z_1^4 - \frac{1}{6} qaz_1^3 + C_1 z_1 + C_2 \qquad EIv(z_2) = \frac{1}{6} qaz_2^3 - \frac{1}{2} qa^2 z_2^2 + C_3 z_2 + C_4$$

$$EIv''(z_3) = -qaz_3$$

$$EIv'(z_3) = -\frac{1}{2} qaz_3^2 + C_5$$

$$EIv(z_3) = -\frac{1}{6} qaz_3^3 + C_5 z_3 + C_6$$

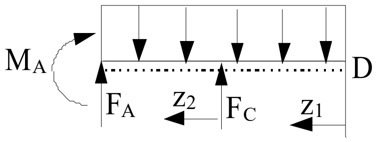
RB/ÜB:

1. $v(z_1 = 0) = 0 \qquad \Rightarrow \quad C_2 = 0$
2. $v(z_2 = 0) = 0 \qquad \Rightarrow \quad C_4 = 0$
3. $v(z_3 = 0) = 0 \qquad \Rightarrow \quad C_6 = 0$
4. $v(z_2 = a) = 0 \qquad \Rightarrow \quad C_3 = \frac{1}{3} qa^3$
5. $v(z_1 = 2a) = v(z_3 = a) \quad \Rightarrow \quad C_1 = \frac{1}{4} qa^3 + \frac{1}{2} C_5$
6. $v'(z_3 = a) = v'(z_2 = 0) \quad \Rightarrow \quad C_5 = \frac{1}{2} qa^3 + C_3 = \frac{5}{6} qa^3 \qquad C_1 = \frac{2}{3} qa^3$

$$v(z_1) = \frac{qa^4}{24EI}\left[\left(\frac{z_1}{a}\right)^4 - 4\left(\frac{z_1}{a}\right)^3 + 16\left(\frac{z_1}{a}\right)\right] \qquad v(z_2) = \frac{qa^4}{6EI}\left[\left(\frac{z_2}{a}\right)^3 - 3\left(\frac{z_2}{a}\right)^2 + 2\left(\frac{z_2}{a}\right)\right]$$

$$v(z_3) = \frac{qa^4}{6EI}\left[-\left(\frac{z_3}{a}\right)^3 + 5\left(\frac{z_3}{a}\right)\right] \qquad v_G = v(z_1 = 2a) = v(z_3 = a) = \frac{2qa^4}{3EI}$$

Lösung 4.50



$F_C = v(z_2 = 0) \cdot c$

$$M(z_1) = -\frac{1}{2}qz_1^2 \qquad M(z_2) = -\frac{1}{2}q(a+z_2)^2 + F_C z_2$$

$$EIv''(z_1) = \frac{1}{2}qz_1^2 \qquad EIv''(z_2) = \frac{1}{2}q(a+z_2)^2 - F_C z_2$$

$$EIv'(z_1) = \frac{1}{6}qz_1^3 + C_1 \qquad EIv'(z_2) = \frac{1}{6}q(a+z_2)^3 - \frac{1}{2}F_C z_2^2 + C_3$$

$$EIv(z_1) = \frac{1}{24}qz_1^4 + C_1 z_1 + C_2 \quad EIv(z_2) = \frac{1}{24}q(a+z_2)^4 - \frac{1}{6}F_C z_2^3 + C_3 z_2 + C_4$$

RB/ÜB:

1. $v'(z_2 = a) = 0 \quad \Rightarrow \quad C_3 = \frac{1}{2}F_C a^2 - \frac{4}{3}qa^3$

2. $v(z_2 = a) = 0 \quad \Rightarrow \quad C_4 = \frac{2}{3}qa^4 - \frac{1}{3}F_C a^3$

3. $v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad C_1 = C_3 = \frac{1}{2}F_C a^2 - \frac{4}{3}qa^3$

4. $v(z_1 = a) = v(z_2 = 0) \quad \Rightarrow \quad C_2 = C_4 - C_1 a = 2qa^4 - \frac{5}{6}F_C a^3$

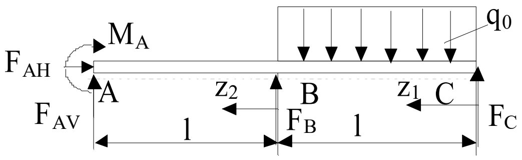
$$F_C = v(z_2 = 0)\cdot c = \left(\frac{17qa^4}{24EI} - \frac{F_C a^3}{3EI}\right)\cdot c \quad \Rightarrow \quad F_C = \frac{17qac}{8\left(\frac{3EI}{a^3} + c\right)}$$

$$F_C(c \to \infty) = \frac{17}{8}qa$$

$$v_D = v(z_1 = 0) = \frac{C_2}{EI} = \frac{2qa^4}{EI} - \frac{5F_C a^3}{6EI}$$

$$v_D(c \to 0) = \frac{2qa^4}{EI} \qquad v_D(c \to \infty) = \frac{2qa^4}{EI} - \frac{85qa^4}{48EI} = \frac{11qa^4}{48EI}$$

Lösung 4.51



$$M(z_1) = F_C z_1 - \frac{1}{2}qz_1^2$$

$$M(z_2) = F_C(l + z_2) + F_B z_2 - ql\left(\frac{1}{2}l + z_2\right)$$

$$= F_C l - \frac{1}{2}ql^2 + (F_B + F_C - ql)z_2$$

$$EIv''(z_1) = \frac{1}{2} q z_1^2 - F_C z_1$$

$$EIv'(z_1) = \frac{1}{6} q z_1^3 - \frac{1}{2} F_C z_1^2 + C_1$$

$$EIv(z_1) = \frac{1}{24} q z_1^4 - \frac{1}{6} F_C z_1^3 + C_1 z_1 + C_2$$

$$EIv''(z_2) = (ql - F_B - F_C) z_2 - \left(F_C l - \frac{1}{2} ql^2 \right)$$

$$EIv'(z_2) = \frac{1}{2}(ql - F_B - F_C) z_2^2 - \left(F_C l - \frac{1}{2} ql^2 \right) z_2 + C_3$$

$$EIv(z_2) = \frac{1}{6}(ql - F_B - F_C) z_2^3 - \frac{1}{2}\left(F_C l - \frac{1}{2} ql^2 \right) z_2^2 + C_3 z_2 + C_4$$

RB/ÜB :

1.) $v_1(0) = 0$ $\quad C_2 = 0$

2.) $v_1(l) = 0$ $\quad \frac{1}{24} ql^4 - \frac{1}{6} F_C l^3 + C_1 l = 0$

3.) $v_2(0) = 0$ $\quad C_4 = 0$

4.) $v_2(l) = 0$ $\quad \frac{1}{6}(ql - F_B - F_C) l^3 + \frac{1}{2}\left(\frac{1}{2} ql - F_C \right) l^3 + C_3 l = 0$

5.) $v_2'(l) = 0$ $\quad \frac{1}{2}(ql - F_B - F_C) l^2 + \left(\frac{1}{2} ql - F_C \right) l^2 + C_3 = 0$

6.) $v_1'(l) = v_2'(0)$ $\quad \frac{1}{6} ql^3 - \frac{1}{2} F_C l^2 + C_1 = C_3$

2′: $C_1 = \frac{1}{6} F_C l^2 - \frac{1}{24} ql^3$

6′: $C_3 = \frac{1}{8} ql^3 - \frac{1}{3} F_C l^2$

4′: $13ql - 4F_B - 24F_C = 0$

5′: $9ql - 4F_B - \frac{44}{3} F_C = 0$

(4′)−(5′): $4ql - \frac{28}{3} F_C = 0$ $\quad \boxed{F_C = \frac{3}{7} ql}$

(4′): $\boxed{F_B = \frac{19}{28} ql}$

Gleichgewichtsbedingungen :

$\rightarrow$: $F_{AH} = 0$

$\uparrow$: $F_{AV} + F_B + F_C - ql = 0$

A : $M_A + \frac{3}{2} ql^2 - F_B l - 2F_C l = 0$

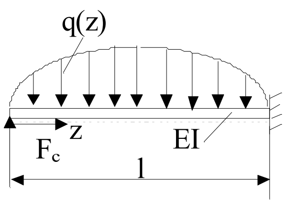
$$\boxed{F_{AV} = -\frac{3}{28} ql \qquad M_A = \frac{1}{28} ql^2}$$

Biegelinie :

$0 \leq z_1 \leq l$ $$v(z_1) = \frac{ql^4}{168EI}\left\{7\left(\frac{z_1}{l}\right)^4 - 12\left(\frac{z_1}{l}\right)^3 + 5\left(\frac{z_1}{l}\right)\right\}$$

$0 \leq z_2 \leq l$ $$v(z_2) = \frac{ql^4}{56EI}\left\{-\left(\frac{z_2}{l}\right)^3 + 2\left(\frac{z_2}{l}\right)^2 - \left(\frac{z_2}{l}\right)\right\}$$

Lösung 4.52



Ermittlung der Intensität der Streckenlast :

$q(z) = A + B\,z + C\,z^2$

$q(0) = 0 \qquad A = 0$

$$q\left(\frac{l}{2}\right) = q_0 \qquad q_0 = \frac{1}{2}Bl + \frac{1}{4}Cl^2$$

$$q(l) = 0 \qquad 0 = Bl + Cl^2$$

$$q(z) = 4q_0\left[\frac{z}{l} - \left(\frac{z}{l}\right)^2\right]$$

$$F_Q(z) = -\int q(z)dz = -4q_0\left(\frac{z^2}{2l} - \frac{z^3}{3l^2}\right) + K_1$$

$$-EIv'' = M(z) = \int F_Q(z)dz = -4q_0\left(\frac{z^3}{6l} - \frac{z^4}{12l^2}\right) + K_1 z + K_2$$

$$M(0) = 0 \qquad K_2 = 0; \qquad F_Q(0) = F_C \qquad K_1 = F_C$$

$$EIv''(z) = 4q_0\left(\frac{z^3}{6l} - \frac{z^4}{12l^2}\right) - F_C z$$

$$EIv'(z) = 4q_0\left(\frac{z^4}{24l} - \frac{z^5}{60l^2}\right) - \frac{1}{2}F_C z^2 + C_1$$

$$EIv(z) = 4q_0\left(\frac{z^5}{120l} - \frac{z^6}{360l^2}\right) - \frac{1}{6}F_C z^3 + C_1 z + C_2$$

RB :

$$v'(l)=0 \qquad \frac{1}{10}q_0l^3-\frac{1}{2}F_Cl^2+C_1=0$$

$$v(l)=0 \qquad \frac{1}{45}q_0l^4-\frac{1}{6}F_Cl^3+C_1l+C_2=0$$

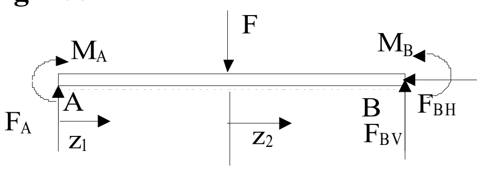
$$F_Q(0)=F_C=c\cdot v(0) \qquad F_C=c\cdot\frac{C_2}{EI}$$

$$C_1=-\frac{q_0l^3}{10}\left[\frac{1}{\Delta}-\frac{1}{18}\frac{cl^3}{EI\Delta}\right] \quad , \qquad C_2=\frac{7}{90}\frac{q_0l^4}{\Delta}$$

Lösung : $F_C=\dfrac{7q_0l^4c}{90\cdot EI\left(1+\dfrac{cl^3}{3EI}\right)}$ und mit $\Delta=\left(1+\dfrac{cl^3}{3EI}\right)$

$$\boxed{v(z)=\frac{q_0l^4}{10EI}\left\{\frac{1}{3}\left(\frac{z}{l}\right)^5-\frac{1}{9}\left(\frac{z}{l}\right)^6-\frac{7}{54}\frac{cl^3}{EI\Delta}\left(\frac{z}{l}\right)^3-\frac{1}{\Delta}\left(1-\frac{1}{18}\frac{cl^3}{EI}\right)\left(\frac{z}{l}\right)+\frac{7}{9\Delta}\right\}}$$

Lösung 4.53



$$M_A=c_A\cdot\varphi=-c_Av_1'(0) \qquad \textbf{(5)}$$

$$F_{BV}=c_B\cdot f=\ \ c_Bv_2(a) \qquad \textbf{(6)}$$

$$M(z_1)=M_A+F_Az_1$$

$$M(z_2)=M_A+F_A(a+z_2)-Fz_2$$

$$EIv''(z_1)=-M_A-F_Az_1$$

$$EIv'(z_1)=-M_Az_1-\frac{1}{2}F_Az_1^2+C_1$$

$$EIv(z_1)=-\frac{1}{2}M_Az_1^2-\frac{1}{6}F_Az_1^3+C_1z_1+C_2$$

$$EIv''(z_2)=(-F_A+F)z_2-(M_A+F_Aa)$$

$$EIv'(z_2)=\frac{1}{2}(-F_A+F)z_2^2-(M_A+F_Aa)z_2+C_3$$

$$EIv(z_2)=\frac{1}{6}(-F_A+F)z_2^3-\frac{1}{2}(M_A+F_Aa)z_2^2+C_3z_2+C_4$$

RB/ÜB :

$$(1) \quad v_1(0)=0 \qquad C_2=0$$

$$(2) \quad v_1(a)=v_2(0) \quad -\frac{1}{2}M_Aa^2-\frac{1}{6}F_Aa^3+C_1a=C_4$$

$$(3) \quad v_1'(a)=v_2'(0) \quad -M_Aa-\frac{1}{2}F_Aa^2+C_1=C_3$$

$$(4) \quad v_2'(a)=0 \qquad \frac{1}{2}(-F_A+F)a^2-(M_A+F_Aa)a+C_3=0$$

GGW:

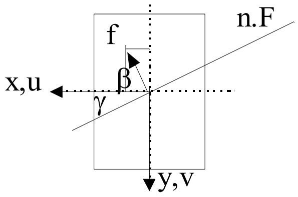
$$\uparrow: \quad F_A-F+F_{BV}=0$$

$$\rightarrow: \quad F_{BH}=0$$

$$A: \quad M_A+Fa-2F_{BV}a-M_B=0$$

Lösungen : $\boxed{F_{BV} = \frac{5}{14}F \quad M_A = -\frac{11}{42}Fa \quad F_A = \frac{9}{14}F \quad M_B = \frac{1}{42}Fa}$

Lösung 4.54



$$M_x = M_0 \cos\alpha \qquad M_y = -M_0 \sin\alpha$$

$$v'' = -\frac{M_x}{EI_x} \qquad u'' = -\frac{M_y}{EI_y}$$

$$v(l) = -\frac{M_0 \cos\alpha}{EI_x}\frac{l^2}{2} \qquad u(l) = \frac{M_0 \sin\alpha}{EI_y}\frac{l^2}{2}$$

$$f = \sqrt{u^2 + v^2} = \frac{M_0 l^2}{2E}\sqrt{\frac{\cos^2\alpha}{I_x^2} + \frac{\sin^2\alpha}{I_y^2}} \qquad \tan\beta = -\frac{I_y}{I_x}\cot\alpha$$

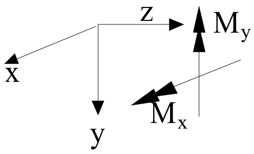
$$\sigma = \frac{M_0 \cos\alpha}{I_x} y - \frac{M_0 \sin\alpha}{I_y} x$$

Neutrale Faser : $\sigma = 0 \qquad y = \frac{I_x}{I_y}\tan\alpha \cdot x = \tan\gamma \cdot x$

Mit $\tan\beta = -\frac{I_y}{I_x}\cot\alpha$ und $\tan\gamma = \frac{I_x}{I_y}\tan\alpha$ folgt $\tan\gamma = -\cot\beta$ oder $\gamma = \beta + 90°$, d.h. f ist senkrecht zur neutralen Faser.

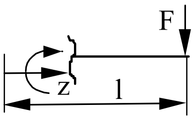
Lösung 4.55

x, y sind keine Hauptachsen:



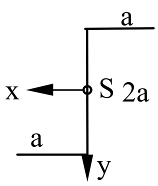
$$Ev'' = \frac{1}{\Delta}\left(-M_x I_y - M_y I_{xy}\right)$$

$$Eu'' = \frac{1}{\Delta}\left(-M_y I_x - M_x I_{xy}\right) \qquad \Delta = I_x I_y - I_{xy}^2$$



$$M_x = -F(l - z) \qquad M_y = 0$$

$$I_x = \frac{t8a^3}{12} + 2ata^2 = \frac{8}{3}ta^3$$



$$I_y = 2\left\{\frac{ta^3}{12} + at\left(\frac{a}{2}\right)^2\right\} = \frac{2}{3}ta^3 \qquad \Delta = \frac{7}{9}t^2a^6$$

$$I_{xy} = -at\left(\frac{a^2}{2} + \frac{a^2}{2}\right) = -ta^3 \qquad \frac{I_y}{\Delta} = \frac{6}{7}\frac{1}{ta^3} \qquad \frac{I_{xy}}{\Delta} = -\frac{9}{7}\frac{1}{ta^3}$$

$$Ev'' = -\frac{I_y}{\Delta} M_x = \frac{6}{7}\frac{F}{ta^3}(l - z)$$

$$Ev' = \frac{6}{7}\frac{F}{ta^3}\left(lz - \frac{1}{2}z^2\right) + C_1 \qquad \textbf{RB:} \quad v'(0) = 0$$

$$Ev = \frac{6}{7}\frac{F}{ta^3}\left(\frac{1}{2}lz^2 - \frac{1}{6}z^3\right) + C_1 z + C_2 \qquad v(0) = 0$$

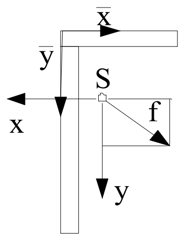
$$Eu'' = -\frac{I_{xy}}{\Delta} M_x = -\frac{9}{7}\frac{F}{ta^3}(l - z) \qquad u'(0) = 0$$

$$Eu' = -\frac{9}{7}\frac{F}{ta^3}\left(lz - \frac{1}{2}z^2\right) + C_3 \qquad u(0) = 0$$

$$Eu = -\frac{9}{7}\frac{F}{ta^3}\left(\frac{1}{2}lz^2 - \frac{1}{6}z^3\right) + C_3 z + C_4 \qquad C_1 = 0; \quad C_2 = 0; \quad C_3 = 0; \quad C_4 = 0$$

$$z = l: \quad \boxed{v(l) = \frac{2}{7}\frac{Fl^3}{Eta^3} \qquad u(l) = -\frac{3}{7}\frac{Fl^3}{Eta^3}}$$

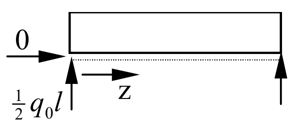
Lösung 4.56



$$A = 1850\,mm; \qquad \bar{y}_S = 47{,}16\,mm; \qquad \bar{x}_S = 14{,}66\,mm$$

$$I_{xx} = 322{,}68 \cdot 10^4\,mm^4 \quad I_{yy} = 55{,}77 \cdot 10^4\,mm^4 \quad I_{xy} = -75{,}37 \cdot 10^4\,mm^4$$

$$\Delta = I_{xx}I_{yy} - I_{xy}^2 = 12315{,}23 \cdot 10^8\,mm^8$$



$$M_x(z) = \frac{1}{2}q_0 lz - \frac{1}{2}q_0 z^2 \qquad Ev'' = -\frac{M_x I_{yy}}{\Delta} \qquad Eu'' = -\frac{M_x I_{xy}}{\Delta}$$

$$Ev'' = \frac{I_{yy}}{\Delta}\frac{q_0}{2}\left(z^2 - lz\right) \qquad \textbf{RB:} \quad v(0) = 0 \quad v(l) = 0$$

$$Ev' = \frac{I_{yy}}{\Delta}\frac{q_0}{2}\left(\frac{1}{3}z^3 - \frac{1}{2}lz^2\right) + C_1 \qquad u(0) = 0 \quad u(l) = 0$$

$$Ev = \frac{I_{yy}}{\Delta}\frac{q_0}{2}\left(\frac{1}{12}z^4 - \frac{1}{6}lz^3\right) + C_1 z + C_2 \qquad C_2 = 0 \quad C_4 = 0$$

$$Eu'' = \frac{I_{xy}}{\Delta}\frac{q_0}{2}\left(z^2 - lz\right) \qquad C_1 = \frac{I_{yy}}{\Delta}\frac{q_0 l^3}{24}$$

$$Eu' = \frac{I_{xy}}{\Delta}\frac{q_0}{2}\left(\frac{1}{3}z^3 - \frac{1}{2}lz^2\right) + C_3 \qquad C_3 = \frac{I_{xy}}{\Delta}\frac{q_0 l^3}{24}$$

$$Eu = \frac{I_{xy}}{\Delta}\frac{q_0}{2}\left(\frac{1}{12}z^4 - \frac{1}{6}lz^3\right) + C_3 z + C_4$$

Lösung :
$$\boxed{\begin{aligned} Ev &= \frac{I_{yy}}{\Delta}\frac{q_0 l^4}{24}\left[\left(\frac{z}{l}\right)^4 - 2\left(\frac{z}{l}\right)^3 + \left(\frac{z}{l}\right)\right] \\ Eu &= \frac{I_{xy}}{\Delta}\frac{q_0 l^4}{24}\left[\left(\frac{z}{l}\right)^4 - 2\left(\frac{z}{l}\right)^3 + \left(\frac{z}{l}\right)\right] \end{aligned}}$$

Speziell für $\frac{z}{l} = \frac{1}{2}$**:** $v = 4{,}5\,\text{mm} \quad u = -6{,}07\,\text{mm} \quad f = \sqrt{u^2 + v^2} = 7{,}56\,\text{mm}$

5. Torsion und Scherung

Lösung 5.1

Mit $M_t = F \cdot b =$ konst. folgt

$$\varphi = \frac{M_t}{GI_t} \cdot l \quad \Rightarrow \quad M_t = \frac{\varphi}{l} GI_t \quad \text{mit} \quad I_t = \frac{\pi d^4}{32}$$

$$\frac{\varphi^\circ}{360^\circ} = \frac{\widehat{\varphi}}{2\pi} \quad \Rightarrow \quad M_t = \frac{\varphi^\circ}{l} \cdot \frac{2\pi}{360^\circ} \cdot G \cdot \frac{\pi d^4}{32} = 0{,}271\,\text{Nm}$$

$$\tau = \frac{M_t}{W_t} = \frac{16 M_t}{\pi d^3} = 172{,}79 \frac{\text{N}}{\text{mm}^2}$$

Lösung 5.2

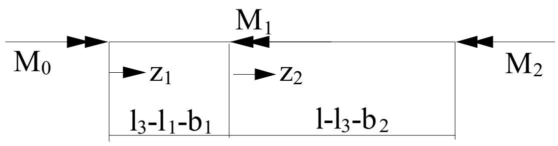
Leistung $\quad P = M_t \cdot \omega = M_t \cdot 2\pi n = F \cdot b \cdot 2\pi n = 150{,}8 \frac{\text{Nm}}{\text{s}} = 150{,}8\,\text{W}$

Spannung $\quad \tau = \frac{16 M_t}{\pi d^3} = \frac{16 F \cdot b}{\pi d^3} = 11{,}32 \frac{\text{N}}{\text{mm}^2}$

Verdrehwinkel $\quad \varphi = \frac{M_t l}{GI_t} = \frac{32 F \cdot b \cdot l}{G \pi d^4} = \frac{2\tau l}{Gd} = 0{,}953 \cdot 10^{-2}$

$$\widehat{\varphi} = 0{,}953 \cdot 10^{-2} \quad \Rightarrow \quad \varphi^\circ = 0{,}546^\circ$$

Lösung 5.3



$$M_1 = F_1 \cdot \frac{d_1}{2} = 2 \cdot 10^3\,\text{Nm}$$

$$M_2 = F_2 \cdot \frac{d_2}{2} = 1 \cdot 10^3\,\text{Nm}$$

$$M_0 = M_1 + M_2 = 3 \cdot 10^3\,\text{Nm}$$

$$M_t(z_1) = -M_0 \quad M_t(z_2) = -M_0 + M_1 = -M_2$$

$$\left|\tau_{max}\right|_1 = \frac{M_0}{W_{t1}} < \tau_{zul} \quad \Rightarrow \quad d_3 \geq \sqrt[3]{\frac{16 M_0}{\pi \tau_{zul}}} = \sqrt[3]{\frac{16 \cdot 3 \cdot 10^6\,\text{mm}^3}{\pi 100}} = 10\,\text{mm} \sqrt[3]{\frac{16 \cdot 30}{\pi}} = 53{,}5\,\text{mm}$$

$$d_3 \geq 53{,}5\,\text{mm} \quad d_{3gew} = 55\,\text{mm}$$

$$\left|\tau_{max}\right|_2 = \frac{M_2}{W_{t2}} < \tau_{zul} \quad \Rightarrow \quad d_4 \geq \sqrt[3]{\frac{16 M_2}{\pi \tau_{zul}}} = 10\,\text{mm} \sqrt[3]{\frac{16 \cdot 10}{\pi}} = 37\,\text{mm}$$

$$d_4 \geq 37\,\text{mm} \quad d_{4gew} = 40\,\text{mm}$$

Verformung der Zahnräder wird vernachlässigt.

$$\varphi = -\frac{M_0}{GI_{t1}}\left(l_3 - l_1 - b_1\right) - \frac{M_2}{GI_{t1}} l_1 - \frac{M_2}{GI_{t2}}\left(l - l_3 - b_2\right) = -3{,}87 \cdot 10^{-2}$$

φ mit d_{erf}: $\quad \varphi = -5 \cdot 10^{-2}$

Lösung 5.5

1. Relative Spannungserhöhung:
$M_t = konst.$

$$(1)\quad \tau_1 = \frac{16M_t}{\pi D_1^3} \quad und \quad (2)\quad \tau_2 = \frac{16M_t}{\pi D_2^3}$$

$$\tau_2 = \left(\frac{D_1}{D_2}\right)^3 \tau_1 = 1{,}13\tau_1$$

Relative Spannungserhöhung: 13%

Relative Drillungszunahme:
$M_t = konst. \quad und \quad l_1 = l_2 = l$

$$(3)\quad \varphi_1 = \frac{32M_t l}{\pi D_1^4} \quad und \quad (4)\quad \varphi_2 = \frac{32M_t l}{\pi D_2^4}$$

$$\varphi_2 = \left(\frac{D_1}{D_2}\right)^4 \varphi_1 = 1{,}177\varphi_1$$

Relative Drillungszunahme: 17,7%

Lösung 5.6

1. $$m_v = A_v \cdot \rho \cdot l = \frac{\pi D^2}{4} \cdot \rho \cdot l \quad m_h = A_h \cdot \rho \cdot l = \frac{\pi}{4}\left(D^2 - d^2\right) \cdot \rho \cdot l$$

$$\frac{m_v - m_h}{m_v} \cdot 100\% = \left(\frac{d}{D}\right)^2 \cdot 100\% = 25\%$$

2. $$\tau_v = \frac{M_t}{W_{tv}} \quad \tau_h = \frac{M_t}{W_{th}} \qquad W_{tv} = \frac{\pi D^3}{16} \quad W_{th} = W_{tv}\left(1 - \left(\frac{d}{D}\right)^4\right)$$

$$\left(\frac{\tau_h - \tau_v}{\tau_v}\right) \cdot 100\% = \left(\frac{W_{tv}}{W_{th}} - 1\right) \cdot 100\% = \left(\frac{1}{1 - \left(\frac{d}{D}\right)^4} - 1\right) \cdot 100\% = \left(\frac{16}{15} - 1\right) \cdot 100\% = 6{,}7\%$$

3. $$\left(\frac{\vartheta_h - \vartheta_v}{\vartheta_v}\right) \cdot 100\% = \left(\frac{I_{tv}}{I_{th}} - 1\right) \cdot 100\% = \left(\frac{1}{1 - \left(\frac{d}{D}\right)^4} - 1\right) \cdot 100\% = \left(\frac{16}{15} - 1\right) \cdot 100\% = 6{,}7\%$$

Lösung 5.7

$$M_t = c_t \cdot \varphi \quad \Rightarrow \quad c_t = \frac{M_t}{\varphi}$$

$$\varphi = \frac{M_t(l-z)}{GI_{t1}} + \frac{M_t z}{GI_{t2}} = \frac{M_t}{G}\left(\frac{l-z}{I_{t1}} + \frac{z}{I_{t2}}\right) \quad I_{t1} = \frac{\pi D^4}{32} \quad I_{t2} = I_{t1}\left(1-\left(\frac{d}{D}\right)^4\right)$$

$$\frac{\varphi}{M_t} = \frac{1}{c_t} = \frac{32}{G\pi D^4}\left[(l-z) + \frac{z}{1-\left(\frac{d}{D}\right)^4}\right] \quad \Rightarrow \quad z = \left(\frac{G\pi D^4}{32 c_t} - l\right)\cdot\left(\frac{D^4}{d^4} - 1\right)$$

Lösung 5.8

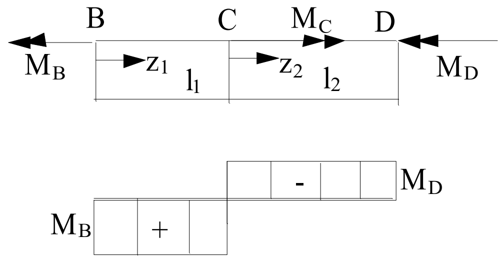
Leistungsbilanz: $P_C = P_B + P_D \quad \rightarrow \quad P_D = 70\text{kW} \qquad P = M \cdot \omega = M \cdot 2\pi n$

$M_B + M_D - M_C = 0$

$$M_C = \frac{P_C}{2\pi n} = \frac{150 \cdot 10^3 \frac{\text{Nm}}{\text{s}} \cdot 60\text{s}}{2\pi \cdot 200} = 7162\text{Nm}$$

$$M_B = \frac{P_B}{2\pi n} = \frac{80 \cdot 10^3 \frac{\text{Nm}}{\text{s}} \cdot 60\text{s}}{2\pi \cdot 200} = 3820\text{Nm}$$

$$M_D = \frac{P_D}{2\pi n} = \frac{70 \cdot 10^3 \frac{\text{Nm}}{\text{s}} \cdot 60\text{s}}{2\pi \cdot 200} = 3342\text{Nm}$$



$$\tau_{\max} = \frac{M_t}{W_t} \le \tau_{zul} \quad \Rightarrow \quad d_i \ge \sqrt[3]{\frac{16 M_{ti}}{\pi \tau_{zul}}} \quad d_1 \ge 86{,}6mm \quad d_2 \ge 82{,}8mm$$

$$\vartheta_{\max} = \frac{M_t}{G I_t} \le \vartheta_{zul} \quad \Rightarrow \quad d_i \ge \sqrt[4]{\frac{32 M_{ti}}{\pi G \vartheta_{zul}}} \quad d_1 \ge 103mm \quad d_2 \ge 99{,}5mm$$

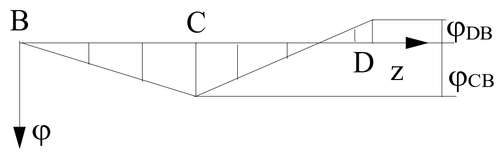
gewählte Durchmesser: $d_1 = 110\text{mm}$, $d_2 = 100\text{mm}$

relative Verdrehung des Punktes C gegenüber B:

$$\varphi_{CB} = \frac{M_B l_1}{G I_{t1}} = \frac{32 M_B l_1}{G \pi d_1^4} = 3{,}691 \cdot 10^{-3}$$

relative Verdrehung des Punktes D gegenüber C:

$$\varphi_{DC} = -\frac{M_D l_2}{G I_{t2}} = -\frac{32 M_D l_2}{G \pi d_2^4} = -5{,}158 \cdot 10^{-3}$$



relative Verdrehung des Punktes D gegenüber B:

$$\varphi_{DB} = \varphi_{CB} + \varphi_{DC} = -1{,}4667 \cdot 10^{-3}$$

Lösung 5.9

reiner Schubspannungszustand, $\alpha = 45°$, d.h. in Richtung der größten Normalspannung,

$$\frac{\gamma}{2} = \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\sin 2\alpha \qquad \varepsilon_1 = -\varepsilon_2 = |\varepsilon| \qquad \alpha = 45° \quad \Rightarrow \quad \gamma = 2|\varepsilon|$$

$$\tau = \frac{M_t}{W_t} = G \cdot \gamma = \frac{E}{2(1+\nu)} \cdot \gamma = \frac{E \cdot |\varepsilon|}{(1+\nu)} \qquad \Rightarrow \qquad M_t = \frac{E \cdot |\varepsilon|}{(1+\nu)} \cdot \frac{\pi d^3}{16}$$

Lösung 5.11

Anordnung I: Stahlwelle

$$\tau_{max} = \frac{M_t}{W_{tmin}} = \frac{16M_t}{\pi d^3} = 65{,}2\,\frac{N}{mm^2}$$

$$\varphi = \frac{M_t \cdot l}{GI_t} = \frac{32M_t l}{G\pi d^4} = 2{,}61\cdot 10^{-2} \quad [\varphi° = 1{,}5°]$$

Anordnung II: Stahlwelle und Kupferrohr über Scheiben starr miteinander verbunden

$$\vartheta_{st} = \vartheta_{cu} \quad \Rightarrow \frac{M_{tst}}{G_{st}I_{tst}} = \frac{M_{tcu}}{G_{cu}I_{tcu}} \qquad M_t = M_{tst} + M_{tcu}$$

$$M_{tst} = M_{tcu}\frac{G_{st}}{G_{cu}}\cdot\frac{I_{tst}}{I_{tcu}} \qquad M_t = M_{tcu}\frac{G_{st}}{G_{cu}}\cdot\frac{I_{tst}}{I_{tcu}} + M_{tcu}$$

$$M_t = M_{tcu}\left(\frac{G_{st}}{G_{cu}}\cdot\frac{I_{tst}}{I_{tcu}} + 1\right) = 2{,}44M_{tcu} \quad M_{tcu} = 0{,}41M_t \quad M_{tst} = 0{,}59M_t$$

$$\tau_{stmax} = \frac{M_{tst}}{W_{tst}} = \frac{16M_{tst}}{\pi d^3} = 38{,}4\,\frac{N}{mm^2} \quad \tau_{cumax} = \frac{M_{tcu}}{W_{tcu}} = \frac{16M_{tcu}}{\pi d_1^3\left(1 - \frac{(d_1-2s)^4}{d_1^4}\right)} = 32{,}34\,\frac{N}{mm^2}$$

$$\varphi_{st} = \varphi_{cu} = \frac{M_{tst}l}{G_{st}I_{st}} = 1{,}54\cdot 10^{-2} \qquad [\varphi° = 0{,}883°]$$

Lösung 5.12

$$\varphi = \frac{M_t l}{GI_t} \qquad M_t = -2Fr_a \quad I_t = I_p = \frac{\pi}{2}\left(r_a^4 - r_i^4\right)$$

$$\varphi = -\frac{4r_a F\cdot l}{G\pi\left(r_a^4 - r_i^4\right)}$$

$$\tau_{max} = \frac{|M_t|}{W_t} = \frac{4r_a^2 F}{\pi\left(r_a^4 - r_i^4\right)}$$

Lösung 5.13

$$\left|M_t\right| = M_0 \qquad I_{ti} = \frac{\pi}{32} d_i^4 \qquad W_{ti} = \frac{\pi}{16} d_i^3$$

$$\tau_{max_i} = \frac{M_0}{W_{ti}} \quad \tau_{max} = \frac{M_0}{W_{tmin}} = \frac{16M_0}{\pi d_1^3}$$

$$\varphi_{AB} = -\left(\frac{32M_0 \cdot 3a}{G\pi d_1^4} + \frac{32M_0 \cdot a}{G\pi d_2^4} + \frac{32M_0 \cdot 2a}{G\pi d_3^4} + \frac{16M_0 \cdot a}{G\pi d_2^4} + \frac{32M_0 \cdot a}{G\pi d_1^4}\right)$$

$$\varphi_{AB} = -\left(\frac{128M_0 \cdot a}{G\pi d_1^4} + \frac{48M_0 \cdot a}{G\pi d_2^4} + \frac{64M_0 \cdot a}{G\pi d_3^4}\right) = -\frac{2355M_0 \cdot a}{16G\pi d_1^4} = -147{,}19\frac{M_0 \cdot a}{G\pi d_1^4}$$

Lösung 5.14

$$\tau_{max} = \frac{M_t}{W_t} \qquad W_t = \eta_2 bh^2 \quad \text{für} \quad \frac{b}{h} = 2 \quad \eta_2 = 0{,}246$$

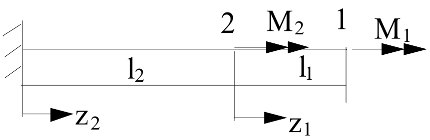
$$\tau_{max} = \frac{M_t}{\eta_2 bh^2} = \frac{M_0}{\eta_2 2h^3} \le \tau_{zul} \quad \Rightarrow h \ge \sqrt[3]{\frac{M_0}{2\eta_2\tau_{zul}}} = 15\text{mm}$$

$$h = 15\text{mm} \quad \Rightarrow \quad b = 30\text{mm} \quad \text{und} \quad d = \sqrt{b^2 + h^2} = h\sqrt{5} = 33{,}54\text{mm}$$

$$\varphi = \frac{M_0 l_1}{GI_{t1}} + \frac{M_0 l_2}{GI_{t2}} \qquad I_{t1} = \eta_3 bh^3 \quad \text{mit} \quad \eta_3 = 0{,}229 \quad \text{und} \quad I_{t2} = \frac{\pi d^4}{32}$$

$$\varphi = \frac{M_0 l_1}{G\eta_3 2h^4} + \frac{32M_0 l_2}{G\pi d^4} = 0{,}010649 + 0{,}0039748 = 1{,}46238 \cdot 10^{-2}$$

Lösung 5.15



$$0 \le z_1 \le l_1 \quad M_t(z_1) = M_1$$
$$0 \le z_2 \le l_2 \quad M_t(z_2) = M_1 + M_2 = 3M_1$$

$$\tau_{1max} = \frac{2M_1}{\pi r^3} \quad \tau_{2max} = \frac{2(M_1 + M_2)}{\pi r^3} = \frac{6M_1}{\pi r^3} \quad \tau_{max} = \tau_{2max}$$

$$\int_0^{\varphi_2} d\varphi = \frac{3M_1}{GI_t}\int_0^{l_2} dz \quad \varphi_2 = \frac{3M_1 l_2}{GI_t} = \frac{12M_1 l_1}{G\pi r^4}$$

$$\int_{\varphi_2}^{\varphi_1} d\varphi = \frac{M_1}{GI_t}\int_0^{l_1} dz \quad \varphi_1 - \varphi_2 = \frac{M_1 l_1}{GI_t} \quad \varphi_1 = \frac{14M_1 l_1}{G\pi r^4}$$

oder

$$\varphi_1 = \varphi_{02} + \varphi_{21} = \frac{12M_1 l_1}{G\pi r^4} + \frac{2M_1 l_1}{G\pi r^4} = \frac{14M_1 l_1}{G\pi r^4}$$

Lösung 5.18

Schubspannung:

$$\tau = \frac{M_t}{W_t} \quad \text{mit} \quad W_t = 2A_m \cdot t$$

$$\tau_1 = \frac{M_t}{2 \cdot ah \cdot t} = 22{,}22 \frac{N}{mm^2} \qquad \tau_2 = \frac{M_t}{2 \cdot \left(ah - \frac{ah}{4} \right) \cdot t} = 29{,}63 \frac{N}{mm^2}$$

$$\tau_3 = \frac{M_t}{2 \cdot \frac{ah}{2} \cdot t} = 44{,}44 \frac{N}{mm^2} \qquad \tau_4 = \frac{M_t}{2 \cdot \pi \cdot \frac{a}{2} \cdot \frac{h}{2} \cdot t} = 28{,}29 \frac{N}{mm^2}$$

Drillung:

$$\vartheta = \frac{M_t}{GI_t} \qquad mit \qquad I_t = \frac{4A_m^2}{\frac{1}{t}\oint ds}$$

$$I_{t1} = \frac{4(ah)^2}{\frac{2}{t}(a+h)} = 2{,}88 \cdot 10^4\, mm^4 \qquad I_{t2} = \frac{4\left(ah - \frac{1}{4}ah\right)^2}{\frac{1}{t}\left(a + \frac{a}{2} + 2\sqrt{h^2 + \left(\frac{a}{4}\right)^2}\right)} = 1{,}80 \cdot 10^4\, mm^4$$

$$I_{t3} = \frac{4\left(\frac{ah}{2}\right)^2}{\frac{1}{t}\left(a + 2\sqrt{h^2 + \left(\frac{a}{2}\right)^2}\right)} = 0{,}877 \cdot 10^4\, mm^4 \qquad I_{t4} = \frac{4\left(\pi \frac{a}{2} \cdot \frac{h}{2}\right)^2}{\frac{1}{t} U_{Ellipse}} \approx 2{,}255 \cdot 10^4\, mm^4$$

$$U_{Ellipse} \approx \pi \left[\frac{3}{2}\left(\frac{a}{2} + \frac{h}{2}\right) - \sqrt{\frac{a}{2} \cdot \frac{h}{2}} \right]$$

$$\vartheta_1 = 0{,}2065 \cdot 10^{-4}\, mm^{-1} = 0{,}02065\, m^{-1} \qquad \vartheta_2 = 0{,}33 \cdot 10^{-4}\, mm^{-1} = 0{,}033\, m^{-1}$$

$$\vartheta_3 = 0{,}678 \cdot 10^{-4}\, mm^{-1} = 0{,}0678\, m^{-1} \qquad \vartheta_4 \approx 0{,}263 \cdot 10^{-4}\, mm^{-1} = 0{,}0263\, m^{-1}$$

Lösung 5.19

$$M_t = \tau \cdot W_t$$

$$W_{t1} = \eta_2 h b^2 = \frac{c_1}{c_2} h b^2 \quad n = \frac{h}{b} = 1{,}5 \quad \Rightarrow \quad c_1 = 0{,}196 \quad c_2 = 0{,}852 \quad \eta_2 = 0{,}231$$

$$M_{t1} = 532 \cdot 10^4 \text{Nmm}$$

$$W_{t2} = 2A_m \cdot \delta_{min} = 2bh\delta_1 \qquad W_{t3} = \frac{\frac{1}{3}\sum l_i \delta_i^3}{\delta_{max}} = \frac{\frac{2}{3}\left(b\delta_2^3 + h\delta_1^3\right)}{\delta_2}$$

$$M_{t2} = 345{,}6 \cdot 10^4 \text{Nmm} \qquad M_{t3} = 27{,}36 \cdot 10^4 \text{Nmm}$$

$$M_t = \vartheta \cdot GI_t$$

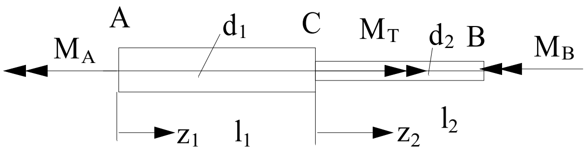
$$I_{t1} = c_1 h b^3 \qquad M_{t1} = 588 \cdot 10^4 \text{Nmm}$$

$$I_{t2} = \frac{4A_m^2}{\oint \frac{ds}{\delta(s)}} = \frac{4b^2h^2}{\frac{2b}{\delta_2} + \frac{2h}{\delta_1}} \quad \frac{b}{h} = \frac{2}{3} \quad \delta_1 = \frac{1}{20}h \quad \delta_2 = \frac{1}{10}h \quad I_{t2} = \frac{h^4}{30} \quad M_{t2} = 337{,}6 \cdot 10^4 \text{Nmm}$$

$$I_{t3} = \frac{1}{3}\sum l_i \delta_i^3 = \frac{2}{3}\left(b\delta_2^3 + h\delta_1^3\right) = \frac{19}{36} \cdot 10^{-3} h^4 \quad M_{t3} = 5{,}345 \cdot 10^4 \text{Nmm}$$

$$I_{t1} : I_{t2} : I_{t3} = 1 : 0{,}574 : 0{,}0091$$

Lösung 5.20



$$M_A + M_B - M_T = 0$$

$$M_t(z_1) = M_A$$
$$M_t(z_2) = M_A - M_T$$

$$\varphi'(z_1) = \frac{M_A}{GI_{p1}} \qquad \varphi(z_1) = \frac{M_A}{GI_{p1}} \cdot z_1 + C_1$$

$$\varphi'(z_2) = \frac{(M_A - M_T)}{GI_{p2}} \quad \varphi(z_2) = \frac{(M_A - M_T)}{GI_{p2}} \cdot z_2 + C_2$$

$$\text{RB / ÜB:}\ \varphi(z_1 = 0) = 0 \quad \Rightarrow \quad C_1 = 0$$

$$\varphi(z_1 = l_1) = \varphi(z_2 = 0) \quad \Rightarrow \quad \frac{M_A l_1}{GI_{p1}} = C_2$$

$$\varphi(z_2 = l_2) = 0 \quad \Rightarrow \quad \frac{(M_A - M_T)l_2}{GI_{p2}} + \frac{M_A l_1}{GI_{p1}} = 0$$

$$M_A = \frac{M_T}{1 + \frac{l_1}{l_2} \cdot \frac{I_{p2}}{I_{p1}}} = \frac{M_T}{1 + \frac{l_1}{l_2} \cdot \left(\frac{d_2}{d_1}\right)^4} \quad M_B = M_T - M_A = \frac{M_T}{1 + \frac{l_2}{l_1} \cdot \left(\frac{d_1}{d_2}\right)^4}$$

$$\varphi_C = \varphi(z_1 = l_1) = \frac{M_A l_1}{GI_{p1}} = \frac{32 M_T}{G\pi\left(\frac{d_1^4}{l_1} + \frac{d_2^4}{l_2}\right)}$$

Lösung 5.21

$$W_{toff} = \frac{\frac{1}{3}\sum l_i d^3}{d} = \frac{1}{3} d^2 \sum l_i = \frac{1}{3} d^2 (2c + 4b) = \frac{2}{3} d^2 (c + 2b)$$

$$I_{toff} = \frac{1}{3}\sum l_i d^3 = \frac{2}{3} d^3 (c + 2b)$$

$$W_{tgeschl} = 2A_m d = 4cbd \qquad I_{tgeschl} = \frac{4A_m^2 \cdot d}{\oint ds} = \frac{8b^2c^2d}{2c + b}$$

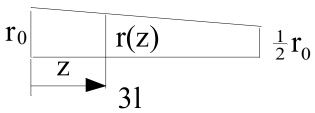
$$\tau_{\max} = \frac{M_t}{W_t} \qquad \tau_{\mathrm{maxoff}} = \frac{3M_t}{2(2b + c)d^2} \qquad \tau_{\mathrm{maxgeschl}} = \frac{M_t}{4cbd}$$

$$\vartheta = \frac{M_t}{GI_t} \qquad \vartheta_{off} = \frac{3M_t}{2(2b + c)Gd^3} \qquad \vartheta_{geschl} = \frac{M_t(2c + b)}{8b^2c^2dG}$$

$$\frac{\tau_{\mathrm{maxoff}}}{\tau_{\mathrm{maxgeschl}}} = \frac{6bc}{d(2b + c)} \qquad \frac{\vartheta_{off}}{\vartheta_{geschl}} = \frac{12b^2c^2}{(2b + c)(b + 2c)d^2}$$

Lösung 5.22

$$\mathrm{I}_{t1} = \frac{1}{2}\pi r_0^4 \qquad \mathrm{I}_{t2} = \frac{1}{2}\pi r^4(z) \qquad \mathrm{I}_{t3} = \frac{1}{32}\pi r_0^4$$



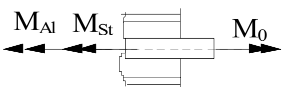
$$r(z) = r_0\left(1 - \frac{z}{6l}\right) \qquad \frac{d\varphi}{dz} = \frac{M_t}{GI_t} \qquad M_t = M_0$$

$$\varphi_E = \varphi_1 + \varphi_2 + \varphi_3$$

$$\varphi_1 = \frac{2M_0 l}{G\pi r_0^4} \qquad \varphi_2 = \frac{2M_0}{G\pi r_0^4}\int_0^{3l} \frac{dz}{\left(1 - \frac{z}{6l}\right)^4} = \frac{28M_0 l}{G\pi r_0^4} \qquad \varphi_3 = \frac{32M_0 l}{G\pi r_0^4}$$

$$\boxed{\varphi_E = \frac{62M_0 l}{G\pi r_0^4}}$$

Lösung 5.23



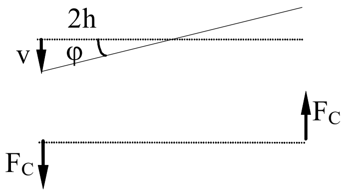
$$\mathrm{I}_{t_{Al}} = 2\pi r_1^3 t = 16\pi r^3 t \qquad \mathrm{I}_{t_{St}} = \frac{1}{2}\pi r^4$$

$$\leftarrow\!\!\leftarrow : \mathrm{M}_{Al} + \mathrm{M}_{St} - \mathrm{M}_0 = 0 \quad (1)$$

$\varphi_{Al} = \varphi_{St}$ $\quad \dfrac{\mathrm{M}_{Al} l}{\mathrm{G}_{Al}\mathrm{I}_{t_{Al}}} = \dfrac{\mathrm{M}_{St} l}{\mathrm{G}_{St}\mathrm{I}_{t_{St}}}$ (2) $\quad \mathrm{M}_{St} = \dfrac{\mathrm{G}_{St}\mathrm{I}_{t_{St}}}{\mathrm{G}_{Al}\mathrm{I}_{t_{Al}}}\mathrm{M}_{Al}$ in (1) einsetzen und nach M_{Al} umst.:

$$\boxed{\begin{array}{l} \mathrm{M}_{Al} = \dfrac{\mathrm{M}_0}{1 + \dfrac{\mathrm{G}_{St}\mathrm{I}_{t_{St}}}{\mathrm{G}_{Al}\mathrm{I}_{t_{Al}}}} = \dfrac{4}{7}\mathrm{M}_0 \\[2ex] \tau_{\mathbf{max}} = \dfrac{\mathrm{M}_{Al}}{\mathrm{I}_{t_{Al}}} r_1 \le \tau_{Alzul} \qquad \mathrm{M}_{0\,\mathbf{max}} = \dfrac{7}{4}\pi r^3 \tau_{Alzul} \end{array}}$$

Lösung 5.24



Δl der Feder: $\Delta l = \frac{F_C}{c}$

$$v = 2h\varphi \qquad M_t = 4F_C h \qquad F_C = \frac{M_t}{4h}$$

$$\varphi = \frac{M_t l}{GI_t}$$

2\. Bredtsche Formel $\quad I_t = \frac{4A_m^2}{\oint \frac{ds}{t}} = \frac{4(2h \cdot h)^2}{2\left(\frac{2h}{2t} + \frac{h}{t}\right)} = 4h^3 t$

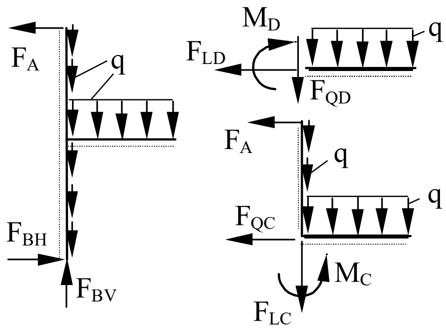
1\. Bredtsche Formel $\quad W_t = 2A_m t_{\min} = 4h^2 t$

$$a = v + \Delta l = 2h\varphi + \frac{F_C}{c} = 2h\frac{M_t l}{GI_t} + \frac{M_t}{4hc} = M_t\left(\frac{2hl}{GI_t} + \frac{1}{4hc}\right)$$

$$a = M_t\left(\frac{2hl}{G4h^3 t} + \frac{1}{4hc}\right) = \frac{M_t}{4ch}\left(1 + \frac{2cl}{Ght}\right)$$

$$\boxed{M_t = \frac{4ach}{\left(1 + \frac{2cl}{Ght}\right)} \qquad \tau_{\max} = \frac{M_t}{W_t} = \frac{4ach}{4h^2 t\left(1 + \frac{2cl}{Ght}\right)} = \frac{acG}{Ght + 2cl}}$$

Lösung 7.1



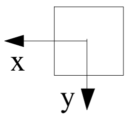
Ermittlung der Lager- und Schnittreaktionen:

$$\leftarrow: \quad F_A - F_{BH} = 0 \quad F_{BH} = \frac{1}{4} q l$$

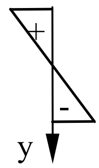
$$\uparrow: \quad F_{BV} - 2ql - ql = 0 \quad F_{BV} = 3ql$$

$$\curvearrowleft B: \quad 2lF_A - \frac{1}{2} q l^2 = 0 \quad F_A = \frac{1}{4} q l$$

$$M_D = -\frac{1}{2} q l^2 \quad F_{LD} = 0\,; \quad M_C = \frac{1}{4} q l^2 \quad F_{LC} = -2ql$$

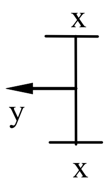


$$A_D = 4at \quad I_{xx} = 2 \cdot \frac{ta^3}{12} + 2 \cdot at \left(\frac{a}{2}\right)^2 = \frac{2}{3} ta^3$$



Querschnitt D : $\sigma = \dfrac{M_D}{I_{xx}} y = -\dfrac{ql^2}{2 \cdot \frac{2}{3} ta^3} y$

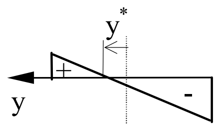
Querschnitt C :



$$A_C = 4at \quad I_{xx} = 2 \cdot \frac{ta^3}{12} = \frac{1}{6} ta^3$$

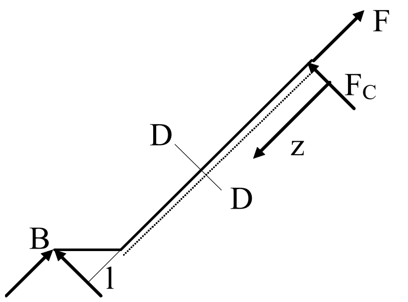
$$\sigma = \frac{F_{LC}}{A_C} + \frac{M_C}{I_{xx}} y = -\frac{2ql}{4at} + \frac{ql^2 \cdot 6}{4ta^3} y$$

$$\sigma = 0 \qquad y^* = \frac{1}{3} \frac{a^2}{l} \quad \textbf{(pos.)}$$



$$\sigma\left(-\tfrac{a}{2}\right) = -\frac{1}{4} \frac{ql}{ta} \left[3 \frac{l}{a} + 2\right] \qquad \sigma\left(\tfrac{a}{2}\right) = \frac{1}{4} \frac{ql}{ta} \left[3 \frac{l}{a} - 2\right]$$

Lösung 7.2



$$\curvearrowleft B: \quad Fl + F_C \cdot 7l = 0 \qquad F_C = -\frac{1}{7} F$$

$$\textbf{D:} \quad F_L(3l) = F \qquad M(3l) = -\frac{3}{7} Fl$$

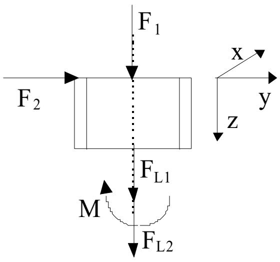
$$A = (16 - \pi) r^2 \qquad I = \left(\frac{64}{3} - \frac{\pi}{4}\right) r^4$$

$$\sigma_D = \frac{F_L}{A} + \frac{M}{I} y$$

$y = -2r$: $\boxed{\sigma_D = \dfrac{F}{r^2} \left[\dfrac{1}{(16-\pi)} + \dfrac{3l \cdot 2}{7\left(\frac{64}{3} - \frac{\pi}{4}\right) r}\right] = \dfrac{F}{r^2} \left(0{,}08 + 0{,}04 \dfrac{l}{r}\right)}$

$y = -2\sqrt{2} r$: $\boxed{\sigma_D = \dfrac{F}{r^2} \left(0{,}08 + 0{,}06 \dfrac{l}{r}\right)}$

Lösung 7.3



$$F_1 + F_{L1} + F_{L2} = 0 \quad \textbf{(a)}$$

$$M_x(z) = -F_2 z \quad \textbf{(b)}$$

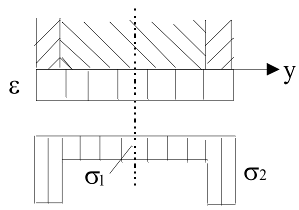
a.) $\sigma = \dfrac{F_L}{A} \qquad F_{L1} = \sigma_1 A_1 \qquad F_{L2} = \sigma_2 A_2$

$$\sigma_i = E_i \varepsilon_i \quad \textit{Verträglichkeitsbedingung}: \quad \varepsilon_1 = \varepsilon_2 = \varepsilon$$

$$E_1 \varepsilon A_1 + E_2 \varepsilon A_2 + F_1 = 0 \qquad A_1 = \frac{2}{3} bh \qquad A_2 = \frac{1}{3} bh$$

$$\varepsilon E\left(\frac{2}{3} bh + 4 \cdot \frac{1}{3} bh\right) = -F_1 \qquad \varepsilon = -\frac{F_1}{2Ebh}$$

$$\boxed{\sigma_1 = -\frac{F_1}{2bh} \qquad \sigma_2 = -2\frac{F_1}{bh}}$$



b.) $z = l \qquad M_x(z) = -F_2 l$

Ebenbleiben der Querschnitte :

$$\varphi = v' \qquad w = \varphi \cdot y$$

$$\varepsilon(y) = \frac{dw}{dz} = \frac{d\varphi}{dz} y = \varphi' y \qquad \sigma(y) = E(y) \cdot \varepsilon(y)$$

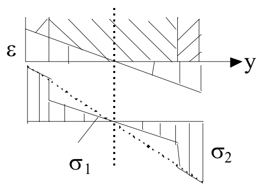
$$M = \int \sigma y dA \qquad dA = bdy$$

$$= 2b\varphi' \left\{ E_1 \int_0^{\frac{h}{3}} y^2 dy + E_2 \int_{\frac{h}{3}}^{\frac{h}{2}} y^2 dy \right\} = 2b\varphi' E \left\{ \frac{1}{3}\left(\frac{h}{3}\right)^3 + \frac{3}{4}\left[\left(\frac{h}{2}\right)^3 - \left(\frac{h}{3}\right)^3\right] \right\}$$

$$M = \frac{7}{27} b\varphi' E h^3 \quad \text{Für} \quad M = -F_2 l \quad \text{folgt} \quad \varphi' = -\frac{27}{7}\frac{F_2 l}{Ebh^3} \quad \text{und damit}$$

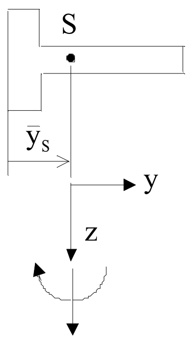
$$\sigma_1 = E_1 \varphi' y = \frac{27}{7}\frac{M}{bh^3} y \qquad \sigma_2 = E_2 \varphi' y = \frac{108}{7}\frac{M}{bh^3} y$$

$$\boxed{\sigma_1\left(\frac{h}{3}\right) = -\frac{9}{7}\frac{F_2 l}{bh^2} \qquad \sigma_2\left(\frac{h}{2}\right) = -\frac{54}{7}\frac{F_2 l}{bh^2}}$$



$$\boxed{\sigma_G = \sigma_{(a)} + \sigma_{(b)}}$$

Lösung 7.4



$$A = 5a^2 \quad \bar{y}_S = \frac{1}{A}\left(2a^2 \cdot \frac{1}{2}a + 3a^2 \cdot \frac{5}{2}a\right) = \frac{17}{10}a$$

$$\sigma = \frac{F_L}{A} + \frac{M}{I}y \quad F_L = -F \quad M = -F \cdot e$$

$$y_B = -\frac{17}{10}a \qquad y_C = \frac{23}{10}a$$

$$\sigma_B = -\frac{F}{A} - \frac{Fe}{I}y_B \qquad \sigma_C = -\frac{F}{A} - \frac{Fe}{I}y_C$$

$$\varepsilon_B = -\frac{F}{EA} - \frac{Fe}{EI}y_B \quad \varepsilon_C = -\frac{F}{EA} - \frac{Fe}{EI}y_C$$

$$\frac{17}{10}a\frac{Fe}{EI} = \varepsilon_B + \frac{F}{EA} \qquad \frac{23}{10}a\frac{Fe}{EI} = -\varepsilon_C - \frac{F}{EA}$$

$$\varepsilon_B + \frac{F}{EA} = -\frac{17}{23}\left(\varepsilon_C + \frac{F}{EA}\right)$$

$$\frac{40}{23}F = -EA\left(\varepsilon_B + \frac{17}{23}\varepsilon_C\right) \qquad \boxed{F = -\frac{a^2E}{8}\left(23\varepsilon_B + 17\varepsilon_C\right)}$$

Lösung 7.5

Spannungen:

Fall a:

$$\sigma_z = -\frac{F}{b^2} = -3{,}75\frac{N}{mm^2}$$

$$\sigma_x = \sigma_y = 0$$

$$\sigma_1 = \sigma_2 = 0 \quad \sigma_3 = -3{,}75\frac{N}{mm^2}$$

Fall b:

$$\sigma_z = -\frac{F}{b^2} = -3{,}75\frac{N}{mm^2}$$

$$\varepsilon_x = 0 = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = 0 = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$\sigma_x = \sigma_y = -\frac{F}{b^2}\frac{\nu}{1-\nu} = -2{,}50\frac{N}{mm^2}$$

$$\sigma_1 = \sigma_2 = -2{,}50\frac{N}{mm^2} \sigma_3 = -3{,}75\frac{N}{mm^2}$$

Vergleichsspannungen:

$$\sigma_{V3} = \sigma_1 - \sigma_3 \qquad \sigma_{V4} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

Fall a:

$$\sigma_{V3} = (0 + 3{,}75)\frac{N}{mm^2} = 3{,}75\frac{N}{mm^2}$$

$$\sigma_{V4} = \sqrt{\frac{1}{2}\left[0 + (0 + 3{,}75)^2 + (-3{,}75)^2\right]}\frac{N}{mm^2}$$

$$\sigma_{V4} = 3{,}75\frac{N}{mm^2}$$

Fall b:

$$\sigma_{V3} = (-2{,}5 + 3{,}75)\frac{N}{mm^2} = 1{,}25\frac{N}{mm^2}$$

$$\sigma_{V4} = \sqrt{\frac{1}{2}\left[0 + (-2{,}5 + 3{,}75)^2 + (-3{,}75 + 2{,}5)^2\right]}\frac{N}{mm^2}$$

$$\sigma_{V4} = 1{,}25\frac{N}{mm^2}$$

Lösung 7.6

$$\begin{vmatrix} \sigma_x - \sigma & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - \sigma & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma \end{vmatrix} = 0 = \begin{vmatrix} -80\dfrac{N}{mm^2} - \sigma & 0 & 50\dfrac{N}{mm^2} \\ 0 & 100\dfrac{N}{mm^2} - \sigma & 0 \\ 50\dfrac{N}{mm^2} & 0 & 80\dfrac{N}{mm^2} - \sigma \end{vmatrix}$$

$$\sigma^3 - 100\frac{N}{mm^2}\sigma^2 - 89 \cdot 10^2 \frac{N^2}{mm^4}\sigma + 89 \cdot 10^4 \frac{N^3}{mm^6} = 0$$

$$Lösung: \quad \sigma_1 = 100\frac{N}{mm^2} \quad \sigma_2 = 94{,}34\frac{N}{mm^2} \quad \sigma_3 = -94{,}34\frac{N}{mm^2}$$

$$\sigma_{V1} = \sigma_1 = 100\frac{N}{mm^2} \qquad \sigma_{V2} = \sigma_1 - \nu(\sigma_2 + \sigma_3) = 100\frac{N}{mm^2}$$

$$\sigma_{V3} = \sigma_1 - \sigma_3 = 194{,}34\frac{N}{mm^2}$$

$$\sigma_{V4} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} = 191{,}6\frac{N}{mm^2}$$

Lösung 7.7

1. Torsionsmoment:

$$\mathrm{P_e = M_t \cdot \omega = M_t \cdot 2\pi n \quad \Rightarrow \quad M_t = \frac{P_e}{2\pi n} = 312{,}52 \cdot 10^3\,Ws = 312{,}52 \cdot 10^3 Nm}$$

2. Erforderlicher Wellendurchmesser:

$$\varphi = \frac{\mathrm{M_t l}}{\mathrm{GI_p}} = \frac{\mathrm{32 M_t l}}{\mathrm{G\pi d^4}} \le \varphi_{\mathrm{zul}} \quad \Rightarrow \quad \mathrm{d} \ge \sqrt[4]{\frac{\mathrm{32 M_t l}}{\mathrm{G\pi\varphi_{zul}}}} = 259{,}2\,\mathrm{mm}$$

gewählt: $\mathrm{d = 260mm}$

3. Spannungen und Sicherheiten:

$$\sigma = \frac{F_L}{A} = -\frac{4F}{\pi d^2} = -18{,}83\frac{N}{mm^2}$$

$$\tau_{\max} = \frac{M_t}{W_t} = \frac{16 M_t}{\pi d^3} = 90{,}56\frac{N}{mm^2}$$

$$\sigma_{V1} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 82\frac{N}{mm^2} \quad \Rightarrow \quad s_1 = \frac{R_e}{\sigma_{V1}} = 4{,}3$$

$$\sigma_{V2} = \frac{1}{2}\left[(1-\nu)\sigma + (1+\nu)\sqrt{\sigma^2 + 4\tau^2}\right] = 111{,}77\frac{N}{mm^2} \quad \Rightarrow \quad s_2 = 3{,}13$$

$$\sigma_{V3} = \sqrt{\sigma^2 + 4\tau^2} = 182{,}096\frac{N}{mm^2} \quad \Rightarrow \quad s_3 = 1{,}92$$

$$\sigma_{V4} = \sqrt{\sigma^2 + 3\tau^2} = 157{,}98\frac{N}{mm^2} \quad \Rightarrow \quad s_4 = 2{,}22$$

Lösung 7.9
Maximales Biegemoment an der Einspannstelle: $M_{max} = F \cdot b = 2 \cdot 10^6$ Nmm
Maximales Torsionsmoment im Bereich: $M_{tmax} = F \cdot a = 1 \cdot 10^6$ Nmm

$$\sigma_{max} = \frac{32 M_{max}}{\pi d^3} = 163{,}0 \frac{N}{mm^2} \qquad \tau_{max} = \frac{16 M_{tmax}}{\pi d^3} = 40{,}7 \frac{N}{mm^2}$$

Hauptspannungen:

$$\sigma_{1,3} = \frac{\sigma_{max}}{2}\left[1 \pm \sqrt{1 + 4\left(\frac{\tau_{max}}{\sigma_{max}}\right)^2}\right] = \begin{cases} 172{,}6 \\ -9{,}6 \end{cases} \frac{N}{mm^2} \qquad \sigma_2 = 0$$

Vergleichsspannungen nach der Hauptspannungshypothese: $\sigma_{V1} = \sigma_1 = 172{,}6 \frac{N}{mm^2}$

Vergleichsspannungen nach der Hauptdehnungshypothese: $\sigma_{V2} = \sigma_1 - \nu\sigma_3 = 175{,}5 \frac{N}{mm^2}$

Vergleichsspannungen nach der Schubspannungshypothese: $\sigma_{V3} = \sigma_1 - \sigma_3 = 182{,}2 \frac{N}{mm^2}$

Vergleichsspannungen nach der Gestaltänderungshypothese:

$$\sigma_{V4} = \sigma_{max}\sqrt{1 + 3\left(\frac{\tau_{max}}{\sigma_{max}}\right)^2} = \sqrt{\frac{1}{2}\left[\sigma_1^2 + \sigma_3^2 + \left(\sigma_3 - \sigma_1\right)^2\right]} = 177{,}6 \frac{N}{mm^2}$$

Anmerkung: Der Einfluß der Querkraftschubspannung wurde vernachlässigt.

Die max. Vergleichsspannung tritt an der Einspannstelle am Querschnittsrand oben und unten auf.

Lösung 7.10

Biegemoment an der Einspannstelle: $|M_{bmax}| = 2 q_0 a^2 = 2 \text{kNm}$

Torsionsmoment an der Einspannstelle: $|M_{tmax}| = \frac{1}{2} q_0 a^2 = 0{,}5 \text{kNm}$

Maximale Biegespannung: $\sigma_{max} = \frac{|M_{bmax}| \cdot 32}{\pi d^3} = 163{,}0 \frac{N}{mm^2}$

Maximale Torsionsspannung: $\tau_{max} = \frac{16 M_{tmax}}{\pi d^3} = 20{,}4 \frac{N}{mm^2}$

Vergleichsspannungen nach der Gestaltänderungshypothese:

$$\sigma_{V4} = \sigma_{max}\sqrt{1 + 3\left(\frac{\tau_{max}}{\sigma_{max}}\right)^2} = 166{,}8 \frac{N}{mm^2}$$

Lösung 7.11

Momente an der Einspannstelle: Biege- und Torsionsmoment erreichen an der Einspannstelle ihr Maximum.

$$\left|M_{bmax}\right| = \left|(2F_1 - F_2)\cdot a\right| = 80\cdot 10^4\,\text{Nmm}$$

$$\left|M_{tmax}\right| = \left|(F_1 - F_2)\cdot b\right| = 60\cdot 10^4\,\text{Nmm}$$

Spannungen: $\sigma_{max} = \dfrac{\left|M_{bmax}\right|\cdot 32\cdot D_a}{\pi\left(D_a^4 - D_i^4\right)}$ $\qquad \tau_{max} = \dfrac{\left|M_{tmax}\right|\cdot 16\cdot D_a}{\pi\left(D_a^4 - D_i^4\right)}$

Der erforderliche Innendurchmesser folgt aus der Beziehung $\sigma_{V4} = \sigma_{max}\sqrt{1+3\left(\dfrac{\tau_{max}}{\sigma_{max}}\right)^2} \le \sigma_{zul}$

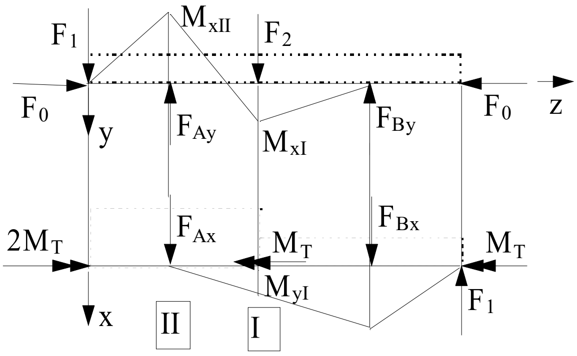
$$\sigma_{zul} \ge \frac{32 D_a \left|M_{bmax}\right|}{\pi\left(D_a^4 - D_i^4\right)}\sqrt{1+3\left(\frac{\left|M_{tmax}\right|}{2\left|M_{bmax}\right|}\right)^2}$$

$$D_i \le \sqrt[4]{D_a^4 - \frac{32 D_a \left|M_{bmax}\right|}{\pi\sigma_{zul}}\sqrt{1+3\left(\frac{\left|M_{tmax}\right|}{2\left|M_{bmax}\right|}\right)^2}} = 42{,}3\,\text{mm}$$

Gewählt: $D_i = 42$ mm

Spannungsnachweis: $\sigma_{Vvorh} = 154{,}8\,\dfrac{N}{mm^2} < \sigma_{zul} = 160\,\dfrac{N}{mm^2}$

Lösung 7.12



Schnittkräfte und -momente:
$F_1 + F_2 - F_{Ay} - F_{By} = 0$
$F_1\,3a - F_{Ay}\,2a + F_2\,a = 0$
$F_{Ay} = 2500\text{N} \quad F_{By} = 500\text{N}$
$F_{Ax} = -500\text{N} \quad F_{Bx} = 1500\text{N}$
$M_{xI} = F_{By}\,a = 25\cdot 10^4$ Nmm
$M_{xII} = -F_1\,a = -50\cdot 10^4$ Nmm
$M_{yI} = 0{,}5\,F_1\,a = 25\cdot 10^4$ Nmm
$M_{tI} = M_{tII} = -2M_T = -20\cdot 10^4$ Nmm
$F_{NI} = F_{NII} = -F_0 = -5000\text{N}$

Resultierendes Biegemoment im Querschnitt I - I:

$$M_{resI} = \sqrt{M_{xI}^2 + M_{yI}^2} = 35{,}355\cdot 10^4\,\text{Nmm}$$

Gesamtnormalspannung im Querschnitt I-I: $\left|\sigma\right|_I = \dfrac{32 M_{res}}{\pi D^3} + \dfrac{4F_0}{\pi D^2} = 31{,}4\,\dfrac{N}{mm^2}$

Torsionsspannung im Querschnitt I-I: $\left|\tau\right|_I = \dfrac{2\cdot 16 M_T}{\pi D^3} = 8{,}1\,\dfrac{N}{mm^2}$

Spannungen im Querschnitt II-II:

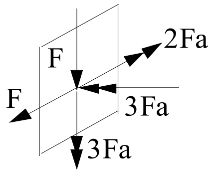
$\left|\sigma\right|_{II} = \dfrac{32 M_{xII}}{\pi d^3} + \dfrac{4F_0}{\pi d^2} = 124\,\dfrac{N}{mm^2}$ $\qquad \left|\tau\right|_{II} = \dfrac{2\cdot 16 M_T}{\pi d^3} = 23{,}8\,\dfrac{N}{mm^2}$

Vergleichsspannungen nach der Gestaltänderungshypothese:

$\sigma_{VI} = \sqrt{\sigma_I^2 + 3\tau_I^2} = 34{,}4\,\dfrac{N}{mm^2}$ $\qquad \sigma_{VII} = \sqrt{\sigma_{II}^2 + 3\tau_{II}^2} = 130{,}6\,\dfrac{N}{mm^2}$

Lösung 7.13

Biege- und Torsionsmomente an der Einspannstelle:



$$F_N = 0$$

$$|M_x| = 2Fa \quad |M_y| = 3Fa \quad |M_t| = 3Fa$$

$$M_{res} = \sqrt{M_x^2 + M_y^2} = Fa\sqrt{4+9} = \sqrt{13}Fa = 1{,}8028 \cdot 10^6 \, \text{Nmm}$$

$$\sigma_{max} = \frac{32 M_{res} \cdot d_a}{\pi\left(d_a^4 - d_i^4\right)} \qquad \tau_{max} = \frac{16 M_t \cdot d_a}{\pi\left(d_a^4 - d_i^4\right)}$$

Spannungen:

$$\sigma_{Vmax} = \sqrt{\sigma_{max}^{\;2} + 3\tau_{max}^{\;2}} = \frac{32}{\pi} \frac{d_a}{\left(d_a^4 - d_i^4\right)} \sqrt{M_{res}^2 + 3\left(\frac{M_t}{2}\right)^2}$$

Erforderlicher Innendurchmesser:

Aus $\sigma_{Vmax} \leq \sigma_{zul}$ folgt

$$d_i \leq \sqrt[4]{d_a^4 - \frac{32}{\pi} \frac{d_a}{\sigma_{zul}} \sqrt{M_{res}^2 + 3\left(\frac{M_t}{2}\right)^2}} = 94{,}9\,\text{mm} \qquad d_{igew} = 94\text{mm}$$

$$\sigma_{vorh} = 103{,}24 \frac{\text{N}}{\text{mm}^2} < \sigma_{zul} = 120 \frac{\text{N}}{\text{mm}^2}$$

Lösung 7.14

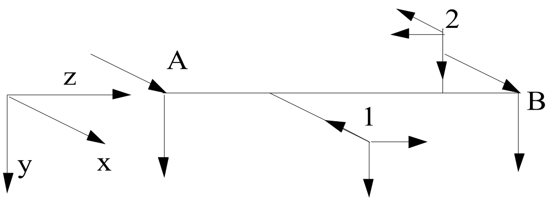
Kräfte:

$$F_{u1} = 4500\text{N}$$

$$F_{a1} = F_{u1}\tan\beta = 1637{,}9\text{N} \qquad F_{r1} = F_{u1}\frac{\tan\alpha}{\tan\beta} = 1743{,}0\text{N}$$

$$F_{u1} \cdot r_1 - F_{u2} \cdot r_2 = 0 \quad \Rightarrow \quad F_{u2} = \frac{r_1}{r_2} F_{u1} = 10800\text{N}$$

$$F_{a2} = F_{u2}\tan\beta = 3930{,}9\text{N} \qquad F_{r2} = F_{u2}\frac{\tan\alpha}{\tan\beta} = 4183{,}2\text{N}$$



Aus den Gleichgewichtsbedingungen folgt:

F_{Ay} = - 5581,3N F_{By} = -3101,9N

F_{Ax} = 3665,1N F_{Bx} = 8877,9N

Biegemomente:

$$M_{x1} = -F_{Ay} \cdot l_1 = 30{,}7 \cdot 10^4 \text{Nmm}$$

$$M_{x21} = -F_{Ay} \cdot \left(l_1 + l_2\right) - F_{u1} \cdot l_2 = 38{,}27 \cdot 10^4 \text{Nmm}$$

$$M_{x2r} = -F_{By} \cdot l_3 = 18{,}61 \cdot 10^4 \text{Nmm}$$

$$M_{y1l} = -F_{Ax} \cdot l_1 = -20{,}16 \cdot 10^4 \text{Nmm}$$

$$M_{y1r} = -F_{Ax} \cdot l_1 - F_{a1} \cdot r_1 = -39{,}81 \cdot 10^4 \text{Nmm}$$

$$M_{y2} = -F_{Bx} \cdot l_3 = -53{,}27 \cdot 10^4 \text{Nmm}$$

Torsionsmoment:
$$M_{t1r} = M_{t21} = -F_{u1} \cdot r_1 = -54{,}0 \cdot 10^4 \,\text{Nmm}$$
Längskräfte:
$$F_{N1l} = F_{a1} - F_{a2} = -2293{,}0\text{N} \qquad F_{N1r} = F_{N2l} = -F_{a2} = -3930{,}9\text{N}$$
Resultierende Biegemomente:
$$M_{1res} = \sqrt{M_{x1}^2 + M_{y1r}^2} = 50{,}27 \cdot 10^4 \,\text{Nmm}$$
$$M_{2res} = \sqrt{M_{x2}^2 + M_{y2}^2} = 65{,}59 \cdot 10^4 \,\text{Nmm}$$
Spannungen:
Normalspannung rechts vom Rad 1:
$$|\sigma|_1 = \frac{32M_{1res}}{\pi d^3} + \frac{|F_{N2}| \cdot 4}{\pi d^2} = 123{,}5 \frac{\text{N}}{\text{mm}^2}$$
Torsionsspannung:
$$|\tau|_1 = \frac{16|M_t|}{\pi d^3} = 64{,}1 \frac{\text{N}}{\text{mm}^2}$$

Normalspannung links vom Rad 2:
$$|\sigma|_2 = \frac{32M_{2res}}{\pi d^3} + \frac{|F_{N2}| \cdot 4}{\pi d^2} = 159{,}9 \frac{\text{N}}{\text{mm}^2}$$
Torsionsspannung:
$$|\tau|_2 = |\tau|_1 = 64{,}1 \frac{\text{N}}{\text{mm}^2}$$

Vergleichsspannung rechts vom Rad 1:
$$\sigma_{V4_1} = \sqrt{\sigma_1^2 + 3\tau_1^2} = 166{,}1 \frac{\text{N}}{\text{mm}^2}$$

Vergleichsspannung links vom Rad 2:
$$\sigma_{V4_2} = \sqrt{\sigma_2^2 + 3\tau_2^2} = 194{,}7 \frac{\text{N}}{\text{mm}^2} = \sigma_{V4max}$$

Vergleichsspannung links vom Rad 1:
$$|\sigma|_{1l} = \frac{32\sqrt{M_{x1}^2 + M_{y1l}^2}}{\pi d^3} + \frac{4|F_{N1l}|}{\pi d^2} = 89{,}6 \frac{N}{mm^2} = \sigma_{V4}$$

Lösung 7.15

Die maximalen Spannungen treten im Querschnitt für x = 0 und $y = \pm\frac{h}{2}$ auf.

Maximale Biegespannung: $|\sigma_{max}| = \frac{M_{bx}}{W_x} = \frac{6M_{bx}}{bh^2} = 240 \frac{\text{N}}{\text{mm}^2}$

Maximale Schubspannung: $\tau_{max} = \kappa\left(\frac{b}{h}\right) \cdot \frac{M_t}{bh^2} = 101 \frac{\text{N}}{\text{mm}^2}$ $\quad \kappa$ für $\frac{b}{h} = 2 \quad \kappa = 4{,}07$

Maximale Vergleichsspannung: $\sigma_{V4} = \sqrt{\sigma_{max}^2 + 3\tau_{max}^2} = 297 \frac{\text{N}}{\text{mm}^2}$