Lösungen

zur

Technischen Mechanik

- Dynamik -

Ausgabe 2016

Lösungen zur Aufgabensammlung Dynamik (Ausgabe 2001, 2016)

Kinematik des Punktes

Lösung 1.1

$$\ddot{x}(t) = a_x \qquad \dot{x}(t) = a_x t + C_1 \qquad x(t) = \frac{1}{2} a_x t^2 + C_1 t + C_2$$

AB: $t = 0 \quad \dot{x}(t=0) = v_{x0} \quad \Rightarrow \quad C_1 = v_{x0}$

$x(t=0) = x_0 \quad \Rightarrow \quad C_2 = x_0$

$$\dot{x}(t) = a_x t + v_{x0} \qquad x(t) = \frac{1}{2} a_x t^2 + v_{x0} t + x_0$$

$$x_1 = x(t = 3s) = 0\,m \qquad v_{x1} = \dot{x}_1 = \dot{x}(t = 3s) = 1\,\frac{m}{s}$$

Umkehrpunkt der Bewegung:

$$\dot{x}(t = t_u) = 0 \quad \Rightarrow t_u = -\frac{v_{x0}}{a_x} = 2{,}5s \qquad x_2 = x(t = t_u) = -0{,}25m$$

Lösung 1.2

$$\ddot{x}(t) = a_x \qquad \dot{x}(t) = a_x t + C_1 \qquad x(t) = \frac{1}{2} a_x t^2 + C_1 t + C_2$$

AB: $t = 0 \quad \dot{x}(t=0) = v \quad \Rightarrow \quad C_1 = v$

$x(t=0) = 0 \quad \Rightarrow \quad C_2 = 0$

$$\dot{x}(t) = a_x t + v \qquad x(t) = \frac{1}{2} a_x t^2 + vt$$

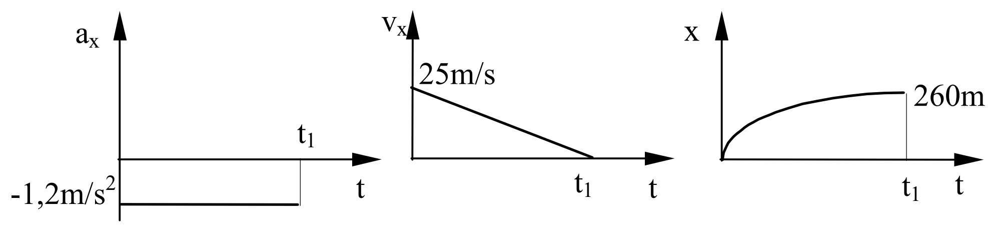
Endb.: $t = t_1 \quad \dot{x}(t = t_1) = 0 \quad 0 = a_x t_1 + v \qquad (1)$

$x(t = t_1) = s \quad s = \frac{1}{2} a_x t_1^2 + v t_1 \qquad (2)$

$$a_x = -\frac{v}{t_1} \quad \Rightarrow \quad s = -\frac{v}{2} t_1 + v t_1 = \frac{v}{2} t_1 \quad t_1 = \frac{2s}{v} \quad \Rightarrow \quad a_x = -\frac{v^2}{2s}$$

Zahlenwerte: $t_1 = 20{,}8s \qquad a_x = -1{,}2\,\frac{m}{s^2}$

Diagramme:



Lösung 1.3

1. $s(t) = bt^3 \quad v(t) = \frac{ds}{dt} = 3bt^2 \quad a(t) = \frac{dv}{dt} = 6bt$

$$s(t) = bt^3 \quad \Rightarrow t(s) = \sqrt[3]{\frac{s}{b}} \qquad a(s) = 6b\sqrt[3]{\frac{s}{b}} = 6\sqrt[3]{b^2 s}$$

2. $v(t)=ct^3 \quad a(t)=\frac{dv}{dt}=3ct^2 \quad v(t)=\frac{ds}{dt} \quad \Rightarrow \quad ds=v(t)dt \quad s(t)=\frac{ct^4}{4}+C_1$

$t=0 \quad s(0)=0 \quad \Rightarrow \quad C_1=0 \quad \text{und} \quad s(t)=\frac{ct^4}{4} \qquad t=\sqrt[4]{\frac{4s}{c}} \quad v(s)=c\cdot\sqrt[4]{\frac{4s}{c}}^{\,3}=c\cdot\left(\frac{4s}{c}\right)^{\frac{3}{4}}$

3. $a(t)=dt^3 \quad v(t)=\frac{dt^4}{4}+C_1 \quad s(t)=\frac{dt^5}{20}+C_1t+C_2$

$t=0\text{:} \quad v(0)=0 \quad \Rightarrow \quad C_1=0\text{;} \quad s(0)=0 \quad \Rightarrow \quad C_2=0 \quad v(t)=\frac{dt^4}{4} \quad s(t)=\frac{dt^5}{20}$

4. $v(s)=es^2 \quad a=\frac{dv}{dt}=\frac{dv}{ds}\cdot\frac{ds}{dt}=v\cdot\frac{dv}{ds}=es^2\cdot 2es \quad a(s)=2e^2s^3$

$v=\frac{ds}{dt}=es^2 \quad dt=\frac{ds}{es^2} \quad t=\int\frac{ds}{es^2}+C_1=-\frac{1}{es}+C_1 \quad \text{AB:} \quad 0=-\frac{1}{es_0}+C_1 \quad C_1=\frac{1}{es_0}$

$t(s)=\frac{1}{e}\left(\frac{1}{s_0}-\frac{1}{s}\right) \quad \Rightarrow \quad s(t)=\frac{s_0}{1-es_0t} \quad \Rightarrow \quad a(t)=\frac{2e^2s_0^3}{\left(1-es_0t\right)^3}$

5. $a=\frac{dv}{dt}=\frac{dv}{ds}\cdot v \quad \Rightarrow \quad vdv=ads=fs^2ds$

$\frac{1}{2}v^2=\frac{fs^3}{3}+C_1 \quad v^2=\frac{2fs^3}{3}+2C_1 \quad v_0^2=\frac{2fs_0^3}{3}+2C_1 \quad C_1=\frac{1}{2}\left(v_0^2-\frac{2fs_0^3}{3}\right)$

$v(s)=\sqrt{\frac{2}{3}f\left(s^3-s_0^3\right)+v_0^2}$

$v=\frac{ds}{dt} \quad \Rightarrow \quad dt=\frac{ds}{v} \qquad t=\int_{s_0}^{s}\frac{d\bar{s}}{\sqrt{\frac{2}{3}f\left(\bar{s}^3-s_0^3\right)+v_0^2}}$

6. $a=hv^2=\frac{dv}{dt} \quad \Rightarrow \quad dt=\frac{dv}{hv^2} \quad t(v)=\frac{1}{h}\left(\frac{1}{v_0}-\frac{1}{v}\right)$

$\frac{1}{v}=\frac{1}{v_0}-ht=\frac{1-v_0ht}{v_0} \quad \Rightarrow \quad v(t)=\frac{v_0}{1-v_0ht}$

$a=hv^2=\frac{dv}{dt}=\frac{dv}{ds}v \quad \Rightarrow \quad ds=\frac{vdv}{hv^2}=\frac{dv}{hv} \quad s-s_0=\frac{1}{h}\ln\frac{v}{v_0} \quad s(v)=\frac{1}{h}\ln\frac{v}{v_0}+s_0$

Lösung 1.4

1. Annäherung im Bereich I durch ein Polynom 2. Grades ($0 \le t \le 8s$):

$s(t) = K_1 + K_2t + K_3t^2 \qquad v(t)=\frac{ds}{dt}=K_2+2K_3t$

Bestimmumg der K_i aus: $t=0\text{:} \quad v(0)=0\text{;} \quad t=4s\text{:} \quad s=25m\text{;} \quad t=8s\text{:} \quad s=100m$

$v(0)=0=K_2 \quad 25m=K_1+4s\cdot K_2+16s^2\cdot K_3 \quad 100m=K_1+8s\cdot K_2+64s^2\cdot K_3$

$K_1=0 \quad K_2=0 \quad K_3=\frac{25}{16}\frac{m}{s^2} \quad \Rightarrow \quad s(t)=\frac{25}{16}\frac{m}{s^2}\cdot t^2 \quad \text{für} \quad 0\le t\le 8s$

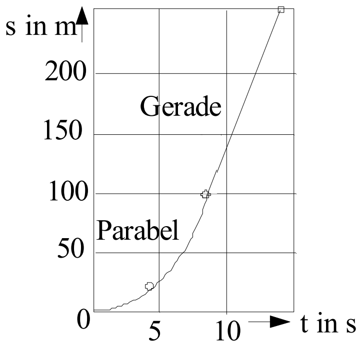
Annäherung im Bereich II durch ein Polynon 1. Grades (Gerade für 8s ≤ t ≤ 14s)

$s(t) = K_4 + K_5 t \mathit{mit} t = 8s : s = 100m t = 14s : s = 250m$

$100m = K_4 + 8s \cdot K_5 250m = K_4 + 14s \cdot K_5$

$\Rightarrow K_5 = 25\frac{m}{s} K_4 = -100m$

$s(t) = -100m + 25\frac{m}{s} \cdot t \mathit{für} 8s \leq t \leq 14s$



2. Beschleunigung:

Bereich I: $v = \frac{ds}{dt} = 2K_3 t = \frac{25}{8}\frac{m}{s^2} \cdot t \quad a = \frac{dv}{dt} = 2K_3 = \frac{25}{8}\frac{m}{s^2}$

Bereich II: $v = \frac{ds}{dt} = K_5 = 25\frac{m}{s} \quad a = \frac{dv}{dt} = 0\frac{m}{s^2}$

Lösung 1.5

1. Bereich: $0 \leq t \leq t_1$

$$a(t) = K_1 t^2 \quad v(t) = \frac{1}{3}K_1 t^3 + C_1 \quad s(t) = \frac{1}{12}K_1 t^4 + C_1 t + C_2$$

$$t = 0: \quad v(0) = 0 \quad s(0) = 0 \quad \Rightarrow \quad C_1 = 0 \quad C_2 = 0$$

$$t = t_1: \quad a(t_1) = a_1 \quad \Rightarrow \quad a_1 = K_1 t_1^2 \quad K_1 = \frac{a_1}{t_1^2} = \frac{1}{100}\frac{m}{s^4}$$

$$v(t_1) = \frac{1}{3}K_1 t_1^3 = \frac{10}{3}\frac{m}{s} \quad s(t_1) = \frac{1}{12}K_1 t_1^4 = \frac{25}{3} m$$

2. Bereich $t_1 \le t \le t_2$

$$a(t) = K_2 + K_3 t \quad v(t) = K_2 t + \frac{1}{2} K_3 t^2 + C_3 \quad s(t) = \frac{1}{2} K_2 t^2 + \frac{1}{6} K_3 t^3 + C_3 t + C_4$$

$$t = t_1\text{:} \quad a(t_1) = a_1 \quad a_1 = K_2 + K_3 t_1 \Big| -$$

$$t = t_2\text{:} \quad a(t_2) = a_2 \quad a_2 = K_2 + K_3 t_2 \Big| +$$

$$a_2 - a_1 = K_3 (t_2 - t_1) \quad \Rightarrow \quad K_3 = \frac{a_2 - a_1}{t_2 - t_1} = \frac{1}{5} \frac{m}{s^3} \quad K_2 = a_1 - K_3 t_1 = -1 \frac{m}{s^2}$$

$$\text{ÜB:} \quad t = t_1 \quad v_2(t_1) = v_1(t_1) = \frac{10}{3} \frac{m}{s} \quad \Rightarrow \quad \frac{10}{3} \frac{m}{s} = K_2 t_1 + \frac{1}{2} K_3 t_1^2 + C_3 = -10 \frac{m}{s} + 10 \frac{m}{s} + C_3$$

$$s_2(t_1) = s_1(t_1) = \frac{25}{3} m \quad \Rightarrow \quad \frac{25}{3} m = -50m + \frac{100}{3} m + C_3 t_1 + C_4$$

$$C_3 = \frac{10}{3} \frac{m}{s} \quad C_4 = \left(50 - \frac{175}{3}\right) m = -\frac{25}{3} m$$

$$v_{End} = v_2(t_2) = K_2 t_2 + \frac{1}{2} K_3 t_2^2 + C_3 = -20 \frac{m}{s} + 40 \frac{m}{s} + \frac{10}{3} \frac{m}{s} = 23{,}33 \frac{m}{s}$$

$$s_{ges} = s_2(t_2) = \frac{1}{2} K_2 t_2^2 + \frac{1}{6} K_3 t_2^3 + C_3 t_2 + C_4 = -200m + \frac{800}{3} m + \frac{200}{3} m - \frac{25}{3} m = 125m$$

Lösung 1.6

Bereich I: $\ddot{x} = a_A \quad \dot{x} = a_A t + C_1 \quad x = \frac{1}{2} a_A t^2 + C_1 t + C_2$

AB: $t = 0 \quad \dot{x} = 0, \quad x = 0 \quad \Rightarrow \quad C_1 = 0, \quad C_2 = 0$

$t = t_1$: $\dot{x}(t_1) = v, \quad x(t_1) = s_1 \quad \Rightarrow \quad t_1 = \frac{v}{a_A} = 20s \quad s_1 = \frac{1}{2} a_A t_1^2 = 160m$

Bereich II: $\ddot{x} = 0 \quad \dot{x} = C_3 \quad x = C_3 t + C_4$

ÜB: $t = t_1 \quad \dot{x} = v, \quad x = s_1 \quad \Rightarrow \quad C_3 = v = 16\frac{m}{s}, \quad C_4 = s_1 - vt_1 = -160m$

$t = t_2$: $\dot{x}(t_2) = v, \quad x(t_2) = s_2 = vt_2 + s_1 - vt_1 = v(t_2 - t_1) + s_1$

Bereich III: $\ddot{x} = a_B \quad \dot{x} = a_B t + C_5 \quad x = \frac{1}{2} a_B t^2 + C_5 t + C_6$

ÜB: $t = t_2 \quad \dot{x} = v, \quad x = s_2 \quad \Rightarrow \quad C_5 = v - a_B t_2 \quad C_6 = s_2 - vt_2 + \frac{1}{2} a_B t_2^2$

Endbedingung: $t = t_3 \quad \dot{x} = 0, \quad x = s_3 = S \quad \Rightarrow \quad a_B(t_3 - t_2) + v = 0 \quad (1)$ und

$$S = \frac{1}{2} a_B t_3^2 + (v - a_B t_2) t_3 + s_2 - vt_2 + \frac{1}{2} a_B t_2^2 = \frac{1}{2} a_B (t_3 - t_2)^2 + v(t_3 - t_2) + s_2 \quad (2)$$

$t_2 = t_3 + \frac{v}{a_B}$ in $(2) \quad \Rightarrow \quad t_3 = t_1 + \frac{S - s_1}{v} - \frac{1}{2}\frac{v}{a_B} = (20 + 23 + 8)s = 51s = t_{ges}$

$t_2 = (51 - 16)s = 35s \quad \Rightarrow \quad s_2 = S - \frac{1}{2} a_B (t_3 - t_2)^2 - v(t_3 - t_2) = 400m$

Lösung 1.7

1. Bereich: $0 \le t \le t_1$

$a(t) = K_1 t^2 \quad v(t) = \frac{1}{3} K_1 t^3 + C_1 \quad s(t) = \frac{1}{12} K_1 t^4 + C_1 t + C_2$

$t = 0$: $v(0) = 0 \quad s(0) = 0 \quad \Rightarrow \quad C_1 = 0 \quad C_2 = 0$

$t = t_1$: $a(t_1) = a_1 \quad \Rightarrow \quad a_1 = K_1 t_1^2 \quad K_1 = \frac{a_1}{t_1^2} = 1\frac{m}{s^4}$

$$v(t_1) = \frac{1}{3} K_1 t_1^3 = \frac{1}{3}\frac{m}{s} \quad s(t_1) = \frac{1}{12} K_1 t_1^4 = \frac{1}{12} m$$

2. Bereich $t_1 \le t \le t_2$

$a(t) = a_1 \quad v(t) = a_1 t + C_3 \quad s(t) = \frac{1}{2} a_1 t^2 + C_3 t + C_4$

ÜB: $t = t_1 \quad v_2(t_1) = v_1(t_1) = \frac{1}{3}\frac{m}{s} \quad \Rightarrow \quad \frac{1}{3}\frac{m}{s} = a_1 t_1 + C_3 = 1\frac{m}{s} + C_3 \quad C_3 = -\frac{2}{3}\frac{m}{s}$

$$s_2(t_1) = s_1(t_1) = \frac{1}{12} m \quad \Rightarrow \quad \frac{1}{12} m = \frac{1}{2} m - \frac{2}{3} m + C_4 \qquad C_4 = \frac{1}{4} m$$

$t = t_2 \quad v_2(t_2) = a_1 t_2 + C_3 = 2\frac{m}{s} - \frac{2}{3}\frac{m}{s} = \frac{4}{3}\frac{m}{s}$

$$s_2(t_2) = \frac{1}{2} a_1 t_2^2 + C_3 t_2 + C_4 = 2m - \frac{4}{3} m + \frac{1}{4} m = \frac{11}{12} m$$

$3.\,Bereich \quad t_2 \le t \le t_3$

$a(t) = K_2 + K_3 t \quad v(t) = K_2 t + \frac{1}{2} K_3 t^2 + C_5 \quad s(t) = \frac{1}{2} K_2 t^2 + \frac{1}{6} K_3 t^3 + C_5 t + C_6$

$t = t_2: \quad a(t_2) = a_1 \quad a_1 = K_2 + K_3 t_2 \big| -$

$t = t_3: \quad a(t_3) = 0 \quad 0 = K_2 + K_3 t_3 \big| +$

$-a_1 = K_3 (t_3 - t_2) \quad \Rightarrow \quad K_3 = \frac{-a_1}{t_3 - t_2} = -1 \frac{m}{s^3} K_2 = -K_3 t_3 = 3 \frac{m}{s^2}$

$ÜB: \quad t = t_2 \quad v_2(t_2) = v_3(t_2) = \frac{4}{3} \frac{m}{s} \quad \Rightarrow \quad \frac{4}{3} \frac{m}{s} = K_2 t_2 + \frac{1}{2} K_3 t_2^2 + C_5 = 6 \frac{m}{s} - 2 \frac{m}{s} + C_5$

$s_2(t_2) = s_3(t_2) = \frac{11}{12} m \quad \Rightarrow \quad \frac{11}{12} m = -\frac{4}{3} m + 6m + C_5 t_2 + C_6$

$C_5 = -\frac{8}{3} \frac{m}{s} \quad C_6 = \frac{19}{12} m$

Lösung 1.8

$\ddot{x}(x) = a_0 \left(1 - \frac{x}{l} \right) \quad oder \quad \ddot{x} + \frac{a_0}{l} x = a_0$ inhomogene DGL 2.Ordnung mit konstanten Koeffizienten.

Lösung:

$x = x_{\hom} + x_{part} \quad x_{\hom} = A \sin \omega t + B \cos \omega t \quad mit \quad \omega^2 = \frac{a_0}{l} \quad x_{part} = \frac{a_0}{\omega^2} = l$

$x(t) = A \sin \omega t + B \cos \omega t + l \quad \dot{x}(t) = A \omega \cos \omega t - B \omega \sin \omega t$

$AB: \quad t = 0 \quad \dot{x}(0) = 0 \quad \Rightarrow \quad 0 = A\omega \quad A = 0$

$x(0) = 0 \quad \Rightarrow \quad 0 = B + l \quad B = -l$

$x(t) = l \left\{ 1 - \cos \sqrt{\frac{a_0}{l}} t \right\} \quad \dot{x}(t) = \sqrt{a_0 l} \sin \sqrt{\frac{a_0}{l}} t \quad \ddot{x}(t) = a_0 \cos \sqrt{\frac{a_0}{l}} t$

x(t) nach t umgestellt:

$\frac{x}{l} - 1 = -\cos \sqrt{\frac{a_0}{l}} t \quad \arccos \left(1 - \frac{x}{l} \right) = \sqrt{\frac{a_0}{l}} t \quad t(x) = \sqrt{\frac{l}{a_0}} \arccos \left(1 - \frac{x}{l} \right)$

$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx} \frac{dx}{dt} = \frac{d\dot{x}}{dx} \dot{x} = a_0 \left(1 - \frac{x}{l} \right)$

$\int \dot{x} d\dot{x} = \int a_0 \left(1 - \frac{x}{l} \right) dx \qquad \frac{1}{2} \dot{x}^2 = a_0 \left(x - \frac{1}{2} \frac{x^2}{l} \right) + C_1$

$AB: t = 0 \quad x = 0; \quad \dot{x} = 0 \quad \Rightarrow \quad C_1 = 0$

$\dot{x}^2 = a_0 x \left(2 - \frac{x}{l} \right) \qquad \dot{x}(x) = \sqrt{2 a_0 x \left(1 - \frac{x}{2l} \right)}$

Lösung 1.9

$0 \le t \le t_1$

$\dot{x}_G = v_G = 15\,\frac{m}{s}$ $\qquad \ddot{x}_P(t) = a_{p0}\left(1 - \frac{t}{t_1}\right)$

$x_G(t) = v_G t$ $\qquad \dot{x}_P(t) = a_{p0}\left(t - \frac{t^2}{2t_1}\right) + C_1$

$x_G(t_1) = v_G t_1 = 750\,m$ $\qquad x_P(t) = a_{p0}\left(\frac{1}{2}t^2 - \frac{t^3}{6t_1}\right) + C_1 t + C_2$

$t = 0\text{:}\quad \dot{x}_P(0) = 0 \quad\Rightarrow\quad C_1 = 0$

$\qquad\quad x_P(t) = 0 \quad\Rightarrow\quad C_2 = 0$

$\dot{x}_P(t) = a_{p0} t\left(1 - \frac{1}{2}\frac{t}{t_1}\right)$ $\qquad x_P(t) = \frac{1}{2}a_{p0}t^2\left(1 - \frac{1}{3}\frac{t}{t_1}\right)$

$\dot{x}_P(t_1) = 20\,\frac{m}{s}$ $\qquad x_P(t_1) = 666{,}6\,m$

Kein Treffen der Züge im Intervall $0 \le t \le t_1$.

$t > t_1\text{:}\quad \ddot{x}_P = 0 \quad \dot{x}_P = C_3 \quad x_P = C_3 t + C_4$

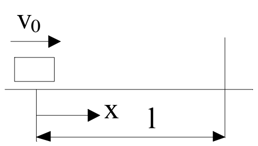
$t = t_1\text{:}\quad \dot{x}_P(t_1) = 20\,\frac{m}{s} \quad\Rightarrow\quad C_3 = 20\,\frac{m}{s}$

$\qquad\quad x_P(t_1) = 666\,m \quad\Rightarrow\quad C_3 t_1 + C_4 = 666\,m \quad C_4 = -333{,}3\,m$

$\text{Treffen:}\quad x_G(t_2) = x_P(t_2) \quad\Rightarrow\quad v_G t_2 = C_3 t_2 + C_4 \quad t_2 = \frac{C_4}{v_G - C_3} = 66{,}67\,s$

$x_G(t_2) = v_G t_2 = 1000\,m \quad v_2 = \dot{x}_P(t_2) - \dot{x}_G(t_2) = 5\,\frac{m}{s}$

Lösung 1.10



$\text{AB:}\quad x(t=0) = 0 \quad \dot{x}(t=0) = v_0$

$a(t) = \ddot{x}(t) = a_0$

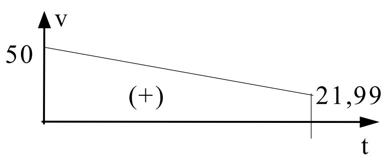
$v(t) = \dot{x}(t) = a_0 t + C_1 \qquad C_1 = v_0$

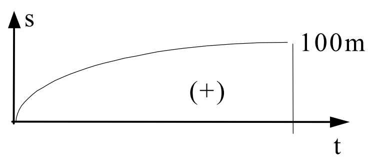
$s(t) = x(t) = \frac{1}{2}a_0 t^2 + C_1 t + C_2 \qquad C_2 = 0$

Endbedingung : x (t*) = *l*

$l = \frac{1}{2}a_0 t^{*2} + v_0 t^* \quad a_0 = \frac{2}{t^{*2}}\left(l - v_0 t^*\right) = -0{,}778\,\frac{m}{s^2}\,(\mathit{Verzögerung})$

$v(t^*) = a_0 t^* + v_0 = 6{,}11\,\frac{m}{s} = 21{,}99\,\frac{km}{h}$





Lösung 1.11

a.) $a(t) = a_0\left(1 - \dfrac{t}{2t_e}\right)$

$v(t) = a_0 t - \dfrac{a_0}{4t_e} t^2 + C_1 \qquad v(0) = 0: \quad C_1 = 0$

$s(t) = \dfrac{1}{2} a_0 t^2 - \dfrac{a_0}{12 t_e} t^3 + C_1 t + C_2 \qquad s(0) = 0: \quad C_2 = 0$

$s(t_e) = l: \qquad l = \dfrac{1}{2} a_0 t_e{}^2 - \dfrac{a_0}{12} t_e{}^2 = \dfrac{5}{12} a_0 t_e^2$

$t_e = \sqrt{\dfrac{12 \cdot l}{5 a_0}} = 2\sqrt{\dfrac{3}{5}\dfrac{l}{a_0}}$

$v(t_e) = v_e: \quad \boxed{v_e = \dfrac{3}{2}\sqrt{\dfrac{3}{5} a_0 l} = 1{,}16\sqrt{a_0 l}}$

b.) $a(x) = a_0\left(1 - \dfrac{x}{2l}\right)$

$a = \dfrac{dv}{dx} v \qquad v dv = a(x) dx$

$\int_0^{v_e} v dv = a_0 \int_0^l \left(1 - \dfrac{x}{2l}\right) dx \qquad \dfrac{1}{2} v_e^2 = \dfrac{3}{4} a_0 l \qquad \boxed{v_e = \sqrt{\dfrac{3}{2} a_0 l} = 1{,}225\sqrt{a_0 l}}$

c.) $a(v) = a_0\left(1 - \dfrac{v}{2v_e}\right)$

$a = \dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt} = \dfrac{dv}{dx} v \qquad dx = \dfrac{v}{a(v)} dv$

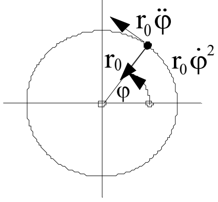
$\int_0^l dx = \int_0^{v_e} \dfrac{v dv}{a_0\left(1 - \dfrac{v}{2v_e}\right)} = \dfrac{2v_e}{a_0} \int_0^{v_e} \dfrac{v dv}{2v_e - v}$

Tafel: $\int \dfrac{x dx}{a + bx} = \dfrac{x}{b} - \dfrac{a}{b^2} \ln(a + bx)$

$l = \dfrac{2v_e}{a_0}\left[-v - 2v_e \ln(2\mathrm{v_e} - v)\right]_0^{v_e} = \dfrac{2v_e^2}{a_0}(2\ln 2 - 1)$

$\boxed{v_e = \sqrt{\dfrac{a_0 l}{2(2\ln 2 - 1)}} = 1{,}14\sqrt{a_0 l}}$

Lösung 1.12



1. $\omega_0 = 2\pi n_0 \quad \Rightarrow \quad n_0 = \dfrac{\omega_0}{2\pi} = 0{,}318\,\mathrm{s}^{-1} \approx 19\,\mathrm{min}^{-1}$

2. $\ddot{\varphi}(t) = \alpha \quad \dot{\varphi}(t) = \alpha t + C$

   $t = 0$: $\dot{\varphi}(0) = \omega_0 \quad \Rightarrow \quad C = \omega_0 \quad \dot{\varphi}(t) = \alpha t + \omega_0$

$$t = t_1: \quad \dot{\varphi} = \omega_1 = 2\pi n_1 = \frac{2\pi \cdot 1500}{60\text{s}} = 157\text{s}^{-1}$$

$$\omega_1 = \alpha t_1 + \omega_0 \quad \Rightarrow \quad t_1 = \frac{\omega_1 - \omega_0}{\alpha} = 15{,}5\text{s}$$

3. $$\vec{a} = \left(\ddot{r} - r\dot{\varphi}^2\right)\vec{e}_r + \left(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}\right)\vec{e}_\varphi \quad \text{für} \quad r = r_0 = \text{konst.}$$

$$\vec{a} = \left(-r_0\dot{\varphi}^2\right)\vec{e}_r + \left(r_0\ddot{\varphi}\right)\vec{e}_\varphi$$

$$t = 0: \quad \dot{\varphi} = \omega_0 \quad a_r = -r_0\omega_0^2 = -4\frac{m}{s^2} \quad a_\varphi = r_0\alpha = 10\frac{m}{s^2}$$

$$a = \sqrt{a_r^2 + a_\varphi^2} = 10{,}8\frac{m}{s^2}$$

Lösung 1.13

$$\ddot{\varphi}_1 = \alpha = \frac{a_t}{r} \qquad \ddot{\varphi}_2 = -\alpha = -\frac{a_t}{r}$$

$$\dot{\varphi}_1 = \alpha t + C_1 \qquad \dot{\varphi}_2 = -\alpha t + C_3$$

$$\varphi_1 = \frac{1}{2}\alpha t^2 + C_1 t + C_2 \qquad \varphi_2 = -\frac{1}{2}\alpha t^2 + C_3 t + C_4$$

$$\text{AB}: \quad t = 0 \quad \dot{\varphi}_1(0) = \dot{\varphi}_2(0) = \omega_0 = \frac{v_0}{r} \quad \Rightarrow \quad C_1 = C_3 = \omega_0$$

$$\varphi_1(0) = \varphi_2(0) = 0 \quad \Rightarrow \quad C_2 = C_4 = 0$$

$$\varphi_1(t) = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \varphi_2(t) = -\frac{1}{2}\alpha t^2 + \omega_0 t$$

1. Treffen: $\varphi_1(t_B) + \varphi_2(t_B) = 2\pi$

$$2\omega_0 t_B = 2\pi \quad \Rightarrow \quad t_B = \frac{\pi}{\omega_0} = \frac{\pi r}{v_0}$$

2. $$\varphi_2(t_B) = \varphi_B = -\frac{1}{2}\alpha\frac{\pi^2 r^2}{v_0^2} + \frac{v_0}{r}\frac{\pi r}{v_0} \qquad \varphi_B - \pi = -\frac{1}{2}\alpha\frac{\pi^2 r^2}{v_0^2}$$

$$\alpha = \frac{2v_0^2}{\pi^2 r^2}\left(\pi - \varphi_B\right) \quad \alpha = \frac{a_t}{r} \quad a_t = 2\frac{v_0^2}{\pi^2 r}\left(\pi - \varphi_B\right)$$

3. $$\dot{\varphi}_2(t_0) = 0 \quad \Rightarrow \quad -\alpha t_0 + \omega_0 = 0 \quad t_0 = \frac{\omega_0}{\alpha} = \frac{v_0}{a_t}$$

$$\varphi_2(t_0) = \varphi_0 = -\frac{1}{2}\alpha\frac{\omega_0^2}{\alpha^2} + \frac{\omega_0^2}{\alpha} = \frac{1}{2}\frac{\omega_0^2}{\alpha} = \frac{\pi^2}{4\left(\pi - \varphi_B\right)}$$

Lösung 1.14

Flugzeug I:

$\ddot{s}_I = 0 \quad \dot{s}_I = C_1 \quad s_I = C_1 t + C_2$

AB: $t = 0 \quad \dot{s}_I = v_I \quad s_I = 0 \quad \Rightarrow \quad C_1 = v_I \quad C_2 = 0$

$\dot{s}_I = v_I \quad s_I = v_I t$

Punkt E wird zur Zeit $t = t_1$ erreicht:

$t = t_1\textbf{:} \quad l_3 = v_I t_1 \quad \Rightarrow \quad t_1 = \frac{l_3}{v_I}$

Bewegung auf der Kreisbahn:

$\ddot{\varphi}(t) = 0 \quad \dot{\varphi}(t) = C_3 \quad \varphi(t) = C_3 t + C_4$

AB: $t = t_1 \quad \dot{\varphi}(t_1) = \frac{v_I}{R} \quad \Rightarrow \quad C_3 = \frac{v_I}{R}$

$\qquad \varphi(t_1) = 0 \quad \Rightarrow \quad C_4 = -\frac{v_I}{R} t_1$

$\dot{\varphi}(t) = \frac{v_I}{R} \quad \varphi(t) = \frac{v_I}{R}(t - t_1)$

Punkt D wird zur Zeit $t = t_2$ erreicht:

$t = t_2 \quad \varphi(t_2) = \pi \quad \Rightarrow \quad \pi = \frac{v_I}{R}(t_2 - t_1) \quad t_2 = \frac{R\pi}{v_I} + t_1 = \frac{1}{v_I}(\pi R + l_3)$

Flugzeug II:

$\ddot{s}_{II} = a_{II} \quad \dot{s}_{II} = a_{II} t + C_5 \quad s_{II} = \frac{1}{2} a_{II} t^2 + C_5 t + C_6$

AB: $t = 0 \quad \dot{s}_{II}(0) = v_{II} \quad \Rightarrow \quad C_5 = v_{II}$

$\qquad s_{II}(0) = 0 \quad \Rightarrow \quad C_6 = 0$

$\dot{s}_{II}(t) = a_{II} t + v_{II} \quad s_{II}(t) = \frac{1}{2} a_{II} t^2 + v_{II} t$

Das Flugzeug II soll den Punkt C erreicht haben, wenn sich das Flugzeug I im Punkt D befindet.

$$t = t_2 \quad s_{II}(t_2) = l_1 - l_3 + l_2 = \frac{1}{2} a_{II} t_2^2 + v_{II} t_2 = \frac{1}{2} a_{II}\left(\frac{\pi R}{v_I} + \frac{l_3}{v_I}\right)^2 + v_{II}\left(\frac{\pi R}{v_I} + \frac{l_3}{v_I}\right)$$

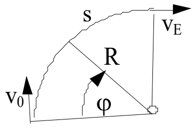
oder

$$l_3^2 + 2 l_3\left[\pi R + \frac{v_I}{a_{II}}(v_I + v_{II})\right] - 2\frac{v_I^2}{a_{II}}\left(l_1 + l_2 - \frac{v_{II}}{v_I}\pi R\right) + \pi^2 R^2 = 0$$

$$l_3 = \frac{v_I}{a_{II}}\left\{-\frac{\pi R a_{II}}{v_I} - v_I - v_{II} + \sqrt{2 a_{II}(l_1 + l_2 + \pi R) + (v_I + v_{II})^2}\right\}$$

$$t = t_2 = \frac{1}{v_I}(\pi R + l_3)$$

Lösung 1.15



$$R\ddot{\varphi} = -a_t \quad R\dot{\varphi} = -a_t t + C_1 \quad R\varphi = -\frac{1}{2} a_t t^2 + C_1 t + C_2$$

$$\text{AB:} \quad t = 0 \quad R\varphi = 0 \quad R\dot{\varphi} = v_0 \quad \Rightarrow C_2 = 0 \quad C_1 = v_0$$

$$t = t_E \quad R\dot{\varphi} = v_E \quad \Rightarrow \quad v_E = -a_t t_E + v_0 \quad t_E = \frac{v_0 - v_E}{a_t}$$

$$R\varphi = s \quad \Rightarrow \quad s = -\frac{a_t}{2}\left(\frac{v_0 - v_E}{a_t}\right)^2 + v_0 \frac{v_0 - v_E}{a_t} \qquad a_t = \frac{1}{2s}\left(v_0^2 - v_E^2\right)$$

$$t_E = \frac{v_0 - v_E}{a_t} = \frac{2s(v_0 - v_E)}{v_0^2 - v_E^2} = \frac{2s}{v_0 + v_E}$$

Zahlenwerte:

$$t_E = 80\mathrm{s} \quad a_t = 0{,}125\frac{\mathrm{m}}{\mathrm{s}^2} \quad a_0 = \sqrt{a_t^2 + \left(\frac{v_0^2}{R}\right)^2} = 0{,}308\frac{\mathrm{m}}{\mathrm{s}^2} \quad a_E = \sqrt{a_t^2 + \left(\frac{v_E^2}{R}\right)^2} = 0{,}1288\frac{\mathrm{m}}{\mathrm{s}^2}$$

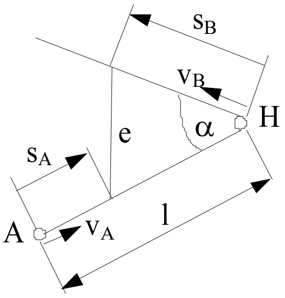
Lösung 1.16

$$e^2 + x^2 = d^2 \quad d + s = l \quad \Rightarrow \quad s(t) = l - \sqrt{e^2 + x^2(t)}$$

$$\dot{s}(t) = \frac{ds(t)}{dt} = \frac{ds}{dx}\cdot\frac{dx}{dt} = -\frac{2x}{2\sqrt{e^2 + x^2}}\cdot\dot{x} = -\frac{x\dot{x}}{\sqrt{e^2 + x^2}}$$

$$\ddot{s}(t) = \frac{d\dot{s}(t)}{dt} = -\frac{\sqrt{e^2 + x^2}\left(\dot{x}^2 + x\ddot{x}\right) - \dfrac{x\dot{x}2x\dot{x}}{2\sqrt{e^2 + x^2}}}{e^2 + x^2} = \frac{e^2\dot{x}^2 + x\ddot{x}\left(e^2 + x^2\right)}{\sqrt{\left(e^2 + x^2\right)^3}}$$

Lösung 1.17



$$s_A = v_A t s_B = v_B t$$

$$e^2 = s_B^2 + (l - s_A)^2 - 2 s_B (l - s_A)\cos\alpha \quad oder$$

$$e(t) = \sqrt{t^2\left(v_B^2 + v_A^2 + 2 v_A v_B \cos\alpha\right) - 2lt\left(v_A + v_B\cos\alpha\right) + l^2}$$

$$\frac{de(t)}{dt} = 0 = \frac{2t\left(v_B^2 + v_A^2 + 2 v_A v_B \cos\alpha\right) - 2l\left(v_A + v_B\cos\alpha\right)}{2\sqrt{\ldots}} \quad \Rightarrow$$

$$t_{\min} = \frac{l\left(v_A + v_B\cos\alpha\right)}{v_B^2 + v_A^2 + 2 v_A v_B \cos\alpha}$$

Hafenentfernungen:

$$s_{B\min} = v_B \cdot t_{\min} = \frac{v_B l\left(v_A + v_B\cos\alpha\right)}{v_B^2 + v_A^2 + 2 v_A v_B \cos\alpha}$$

$$(l - s_A)_{\min} = l - v_A \cdot t_{\min} = l - \frac{v_A l\left(v_A + v_B\cos\alpha\right)}{v_B^2 + v_A^2 + 2 v_A v_B \cos\alpha}$$

$$\left(\frac{s_B}{l - s_A}\right)_{\min} = \frac{v_A + v_B\cos\alpha}{v_B + v_A\cos\alpha}$$

Lösung 1.18

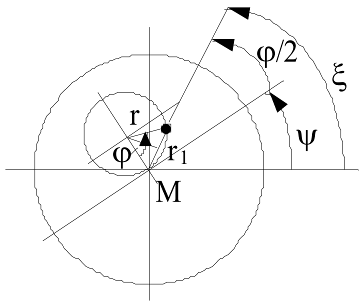
$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\varphi}\vec{e}_\varphi \qquad \vec{a} = \left(\ddot{r} - r\dot{\varphi}^2\right)\vec{e}_r + \left(2\dot{r}\dot{\varphi} + r\ddot{\varphi}\right)\vec{e}_\varphi$$

mit $\quad \dot{\varphi} = \omega_0 \quad \ddot{\varphi} = 0 \quad \dot{r} = v_0 \quad \ddot{r} = 0$

$$\vec{v} = v_0\vec{e}_r + r\omega_0\vec{e}_\varphi \qquad v = \sqrt{v_0^2 + \left(r\omega_0\right)^2}$$

$$\vec{a} = \left(-r\omega_0{}^2\right)\vec{e}_r + \left(2v_0\omega_0\right)\vec{e}_\varphi \qquad a = \sqrt{\left(r\omega_0{}^2\right)^2 + \left(2v_0\omega_0\right)^2}$$

Lösung 1.19



Geometrische Zusammenhänge:

$$r_1 = 2r\sin\frac{\varphi}{2} \qquad \xi = \psi + \frac{\varphi}{2}$$

Lösung in Polarkoordinaten:

$$\vec{v} = \dot{r}_1\vec{e}_{r_1} + r_1\dot{\xi}\vec{e}_\xi \qquad \vec{a} = \left(\ddot{r}_1 - r_1\dot{\xi}^2\right)\vec{e}_{r_1} + \left(2\dot{r}_1\dot{\xi} + r_1\ddot{\xi}\right)\vec{e}_\xi$$

$$\dot{\xi} = \dot{\psi} + \frac{\dot{\varphi}}{2} \qquad \dot{r}_1 = 2r\cdot\cos\frac{\varphi}{2}\cdot\frac{1}{2}\cdot\dot{\varphi} = r\dot{\varphi}\cos\frac{\varphi}{2}$$

$$\ddot{\xi} = \ddot{\psi} + \frac{\ddot{\varphi}}{2} \qquad \ddot{r}_1 = r\left[\ddot{\varphi}\cos\frac{\varphi}{2} - \frac{1}{2}\dot{\varphi}^2\sin\frac{\varphi}{2}\right]$$

$$\vec{v} = r\dot{\varphi}\cos\frac{\varphi}{2}\vec{e}_{r_1} + 2r\sin\frac{\varphi}{2}\dot{\xi}\vec{e}_\xi \qquad v = 2r\sqrt{\left(\frac{1}{2}\dot{\varphi}\cos\frac{\varphi}{2}\right)^2 + \left(\sin\frac{\varphi}{2}\cdot\dot{\xi}\right)^2}$$

$$\vec{a} = \left[r\left(\ddot{\varphi}\cos\frac{\varphi}{2} - \frac{1}{2}\dot{\varphi}^2\sin\frac{\varphi}{2}\right) - 2r\dot{\xi}^2\sin\frac{\varphi}{2}\right]\vec{e}_{r_1} + \left(2r\dot{\varphi}\cos\frac{\varphi}{2}\dot{\xi} + 2r\sin\frac{\varphi}{2}\ddot{\xi}\right)\vec{e}_\xi = a_{r_1}\vec{e}_{r_1} + a_\xi\vec{e}_\xi$$

$$a = 2r\sqrt{\left[\frac{\ddot{\varphi}}{2}\cos\frac{\varphi}{2} - \sin\frac{\varphi}{2}\cdot\left(\frac{\dot{\varphi}^2}{4} + \dot{\xi}^2\right)\right]^2 + \left[\dot{\varphi}\cos\frac{\varphi}{2}\dot{\xi} + \sin\frac{\varphi}{2}\ddot{\xi}\right]^2}$$

Lösung 1.20

$\ddot{\varphi} = \alpha_0 \qquad\qquad \ddot{r} = -a_0$

$\dot{\varphi} = \alpha_0 t + C_1 \qquad\qquad \dot{r} = -a_0 t + C_3$

$\varphi = \frac{1}{2}\alpha_0 t^2 + C_1 t + C_2 \qquad r = -\frac{1}{2}a_0 t^2 + C_3 t + C_4$

AB. liefern $\quad C_1 = 0 \quad C_3 = 0 \quad C_2 = 0 \quad C_4 = l$

$v_r = \dot{r} \qquad v_\varphi = r\dot{\varphi} \quad v = \sqrt{v_r^2 + v_\varphi^2}$

$a_r = \ddot{r} - r\dot{\varphi}^2 \quad a_\varphi = r\ddot{\varphi} + 2\dot{r}\dot{\varphi} \quad a = \sqrt{a_r^2 + a_\varphi^2}$

Endb.: $\; t = t_e: \;\; \varphi_e = \frac{\pi}{6} \qquad \frac{\pi}{6} = \frac{1}{2}\alpha_0 t_e^2 \quad t_e^2 = \frac{\pi}{3\alpha_0}$

$t_e^2 = 3{,}49 s^2 \quad t_e = 1{,}87 s \qquad r_e = -\frac{1}{2}a_0 t_e^2 + l = 56{,}375 cm$

$v_{re} = -a_0 t_e = -0{,}467 \frac{m}{s} \qquad v_{\varphi e} = \left(-\frac{1}{2}a_0 t_e^2 + l\right)\alpha_0 t_e = 0{,}316 \frac{m}{s} \qquad \boxed{v_e = 0{,}564 \frac{m}{s}}$

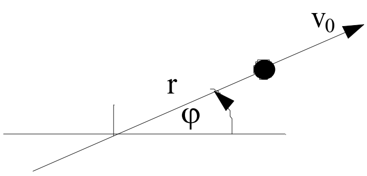
$a_{re} = -a_0 - r_e \alpha_0^2 t_e^2 = -0{,}427 \frac{m}{s^2} \quad a_{\varphi e} = r_e \alpha_0 - 2a_0 \alpha_0 t_e^2 = -0{,}354 \frac{m}{s^2}$

$\boxed{a_e = 0{,}555 \frac{m}{s^2}}$

Körper erreicht O für $\quad r(t^*) = 0 = -\frac{1}{2}a_0 t^{*2} + l$

$t^* = \sqrt{\frac{2l}{a_0}} = 2{,}83 s \qquad \varphi(t^*) = \frac{1}{2}\alpha_0 t^{*2} = 1{,}2 \quad (68{,}75°)$

Lösung 1.21



$\vec{v} = \dot{r}\vec{e}_r + r\dot{\varphi}\vec{e}_\varphi \quad \vec{a} = \left(\ddot{r} - r\dot{\varphi}^2\right)\vec{e}_r + \left(2\dot{r}\dot{\varphi} + r\ddot{\varphi}\right)\vec{e}_\varphi$

$\dot{r} = v_0 \quad \ddot{r} = 0$

$\varphi = \hat{\varphi}\sin\Omega t \quad \dot{\varphi} = \hat{\varphi}\Omega\cos\Omega t \quad \ddot{\varphi} = -\hat{\varphi}\Omega^2\sin\Omega t$

$\vec{v} = v_0\vec{e}_r + r\hat{\varphi}\Omega\cos\Omega t\vec{e}_\varphi$

$\vec{a} = -r\left(\hat{\varphi}\Omega\cos\Omega t\right)^2\vec{e}_r + \left(2v_0\hat{\varphi}\Omega\cos\Omega t - r\hat{\varphi}\Omega^2\sin\Omega t\right)\vec{e}_\varphi$

$v = \sqrt{v_0^2 + \left(r\hat{\varphi}\Omega\cos\Omega t\right)^2}$

$a = \sqrt{\left(-r\hat{\varphi}^2\Omega^2\cos^2\Omega t\right)^2 + \left(2v_0\hat{\varphi}\Omega\cos\Omega t - r\hat{\varphi}\Omega^2\sin\Omega t\right)^2}$

Lösung 1.22

$$v_r = \dot{r} = \frac{dr}{d\varphi}\frac{d\varphi}{dt} = \frac{dr}{d\varphi}\cdot\dot{\varphi} \qquad v_\varphi = r\cdot\dot{\varphi} \qquad r(\varphi) = R\frac{\varphi}{\pi} \quad \frac{dr}{d\varphi} = \frac{R}{\pi}$$

$$v_r = \frac{R}{\pi}\cdot\dot{\varphi} \qquad v_\varphi = R\frac{\varphi}{\pi}\cdot\dot{\varphi} \qquad v_K = \sqrt{v_r^2 + v_\varphi^2} = \frac{R}{\pi}\cdot\frac{d\varphi}{dt}\sqrt{1+\varphi^2} = \text{konst.}$$

$$v_K\int_0^t d\bar{t} = \frac{R}{\pi}\int_0^\varphi \sqrt{1+\bar{\varphi}^2}\,d\bar{\varphi} \quad v_K t = \frac{R}{2\pi}\left[\varphi\sqrt{1+\varphi^2} + \operatorname{arsinh}\varphi\right]$$

Maus: $v = v_M \qquad s = v_M t$

$\varphi = \pi$ und $t = T$: $\quad s = v_M T = \pi R \quad T = \dfrac{\pi R}{v_M}$

$$v_K = \frac{R}{2\pi T}\left[\pi\sqrt{1+\pi^2} + \operatorname{arsinh}\pi\right] = v_M\cdot\frac{1}{2\pi^2}\left[\pi\sqrt{1+\pi^2} + \operatorname{arsinh}\pi\right] = 0{,}62\,v_M$$

Kinematik der ebenen Bewegung des starren Körpers

Lösung 2.1

$$\omega_W \cdot \frac{D}{2} = v \quad \Rightarrow \quad \omega_W = \frac{2v}{D}$$

$$\omega_W \cdot r_G = \omega_{ZW} \cdot r_K \quad \Rightarrow \quad \omega_{ZW} = \frac{r_G}{r_K} \cdot \omega_W = \frac{r_G}{r_K} \cdot \frac{2v}{D}$$

$$\omega_{ZW} \cdot r_G = \omega_M \cdot r_K \quad \Rightarrow \quad \frac{r_G}{r_K} \cdot \frac{2v}{D} = \frac{r_K}{r_G} \cdot 2\pi n_M \qquad \left(\frac{r_K}{r_G}\right)^2 = \frac{v}{\pi n_M D} = i^2$$

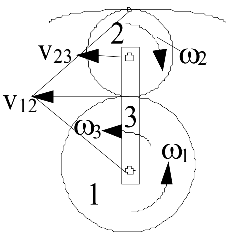
$$i = \frac{r_K}{r_G} = \sqrt{\frac{v}{\pi n_M D}} = 0{,}18 = \frac{z_K}{z_G}$$

Lösung 2.2

$$\omega_{Mot} \cdot \frac{d_1}{2} = \omega_2 \cdot \frac{d_2}{2} \quad \text{und} \quad \omega_2 \cdot \frac{d_3}{2} = v \quad \Rightarrow \quad \omega_{Mot} = \frac{2vd_2}{d_1 d_3} = 142{,}7\,s^{-1}$$

$$\omega = 2\pi n \quad \Rightarrow \quad n_{Mot} = \frac{\omega_{Mot}}{2\pi} = 22{,}72\,s^{-1} = 1363\,\text{min}^{-1}$$

Lösung 2.3

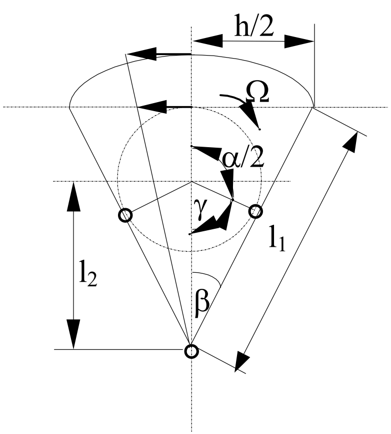


$$v_{12} = r_1\omega_1 = 2r_2\omega_2 \qquad v_{23} = r_2\omega_2 = (r_1 + r_2)\omega_3$$

$$\omega_2 = \left(\frac{r_1}{r_2} + 1\right)\omega_3 \qquad \omega_1 = 2\frac{r_2}{r_1}\omega_2 = 2\left(\frac{r_2}{r_1} + 1\right)\omega_3$$

$$\varphi_2 = \left(\frac{r_1}{r_2} + 1\right)\varphi_3 \qquad \varphi_1 = 2\left(\frac{r_2}{r_1} + 1\right)\varphi_3$$

Lösung 2.4



$$\sin\beta = \frac{r}{l_2} = \frac{\frac{h}{2}}{l_1} \quad \Rightarrow \quad \beta = \arcsin\frac{r}{l_2} \qquad h = 2l_1\frac{r}{l_2}$$

$$\frac{\alpha}{2} + \gamma = 180° \qquad \beta + \gamma = 90° \quad \Rightarrow \quad \alpha = 180° + 2\beta$$

$$\Omega = 2\pi n$$

$$\frac{v_{Amax}}{r\Omega} = \frac{l_1}{l_2 + r} \quad \Rightarrow \quad v_{Amax} = \frac{l_1}{l_2 + r} \cdot r\,2\pi n$$

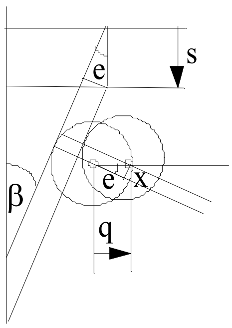
$$\frac{v_{Rmax}}{r\Omega} = \frac{l_1}{l_2 - r} \quad \Rightarrow \quad v_{Rmax} = \frac{l_1}{l_2 - r} \cdot r\,2\pi n$$

Zahlenwerte:

1. $\alpha = 209°$ 2. $\beta = 14{,}5°$ 3. $h = 450\,\text{mm}$

4. $v_{Amax} = 0{,}4712\,\frac{m}{s}$ 5. $v_{Rmax} = 0{,}7854\,\frac{m}{s}$

Lösung 2.5



$$\sin\beta = \frac{e}{s} \quad e = s\sin\beta \quad \beta = \frac{\alpha}{2} \quad \cos\beta = \frac{e}{q} \quad q = \frac{e}{\cos\beta}$$

$$q(t) = s(t)\cdot\tan\frac{\alpha}{2} \quad \dot{q}(t) = \dot{s}(t)\cdot\tan\frac{\alpha}{2} \quad \dot{q}_m = \dot{s}_m\cdot\tan\frac{\alpha}{2}$$

$$\tan\frac{\alpha}{2} = \frac{\dot{q}_m}{\dot{s}_m} = \frac{v_{mst}}{v_{mk}} = 0{,}05714 \quad \Rightarrow \quad \alpha = 6{,}54°$$

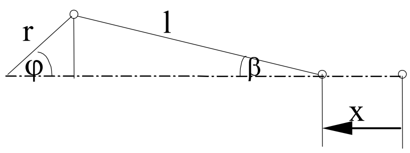
$$\tan\beta = \frac{x}{e} \quad x = e\cdot\tan\beta = s\cdot\tan\beta\sin\beta$$

$$x(t) = s(t)\cdot\tan\beta\sin\beta \quad \dot{x}(t) = \dot{s}(t)\cdot\tan\beta\sin\beta$$

$$\dot{x}_m = \dot{s}_m\cdot\tan\beta\sin\beta = v_{mk}\cdot\tan\beta\sin\beta = 0{,}00114\,\frac{m}{s}$$

$$\omega_R\cdot\frac{d}{2} = \dot{x} \quad \omega_R = \frac{2\dot{x}}{d} \quad \omega_{Rm} = \frac{2\dot{x}_m}{d} = 0{,}0456\,s^{-1}$$

Lösung 2.6



$$x = l + r - r\cos\varphi - l\cos\beta = r(1-\cos\varphi) + l(1-\cos\beta)$$

$$\frac{x}{r} = 1-\cos\varphi + \frac{l}{r}(1-\cos\beta) = 1-\cos\varphi + \frac{1}{\lambda}(1-\cos\beta)$$

$$r\sin\varphi = l\sin\beta \quad \sin\beta = \lambda\sin\varphi \quad \cos\beta = \sqrt{1-\sin^2\beta}$$

$$\varphi = \omega t$$

$$\frac{x}{r} = 1-\cos\omega t + \frac{1}{\lambda}\left(1-\sqrt{1-\lambda^2\sin^2\omega t}\right)$$

$$\frac{\dot{x}}{\omega r} = \sin\omega t\left\{1 + \frac{\lambda\cos\omega t}{\sqrt{1-\lambda^2\sin^2\omega t}}\right\}$$

$$\frac{\ddot{x}}{\omega^2 r} = \cos\omega t + \lambda\frac{\cos^2\omega t - \sin^2\omega t\left(1-\lambda^2\sin^2\omega t\right)}{\left(\sqrt{1-\lambda^2\sin^2\omega t}\right)^3}$$

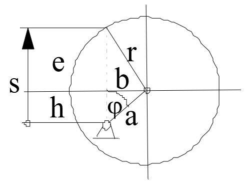
Näherung für λ << 1 durch eine Reihenentwicklung der Form $\sqrt{1-y} = 1-\frac{1}{2}y\ldots$

$$\frac{x}{r} \approx 1-\cos\omega t + \frac{1}{\lambda}\left[1-\left(1-\frac{1}{2}\lambda^2\sin^2\omega t\right)\right] = 1-\cos\omega t + \frac{\lambda}{2}\sin^2\omega t$$

$$\frac{\dot{x}}{\omega r} \approx \sin\omega t + \frac{\lambda}{2}2\sin\omega t\cos\omega t = \sin\omega t + \lambda\sin\omega t\cos\omega t = \sin\omega t + \frac{\lambda}{2}\sin 2\omega t$$

$$\frac{\ddot{x}}{\omega^2 r} \approx \cos\omega t + \frac{1}{2}\lambda 2\cos 2\omega t = \cos\omega t + \lambda\cos 2\omega t$$

Lösung 2.7



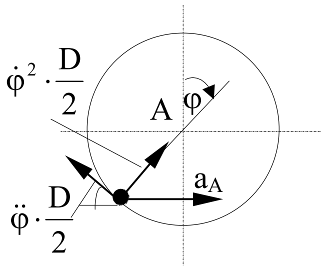
$$s = h + e \quad \dot{\varphi} = C \quad \ddot{\varphi} = 0 \quad h = a\cos\varphi \quad b = a\sin\varphi \quad e = \sqrt{r^2 - b^2}$$

$$s(t) = a\cos\varphi(t) + \sqrt{r^2 - a^2\sin^2\varphi} \qquad \frac{s}{r} = \lambda\cos\varphi + \sqrt{1 - \lambda^2\sin^2\varphi}$$

$$\frac{\dot{s}}{r} = -\lambda\dot{\varphi}\left\{\sin\varphi + \frac{\lambda\sin\varphi\cos\varphi}{\sqrt{1-\lambda^2\sin^2\varphi}}\right\} \qquad \frac{\ddot{s}}{r} = -\lambda\dot{\varphi}^2\left\{\cos\varphi + \lambda\frac{\cos^2\varphi + \sin^2\varphi\left(\lambda^2\sin^2\varphi - 1\right)}{\left(1-\lambda^2\sin^2\varphi\right)^{\frac{3}{2}}}\right\}$$

Lösung 2.8

Lösung durch Überlagerung von Translation und Drehung um den Schwerpunkt:



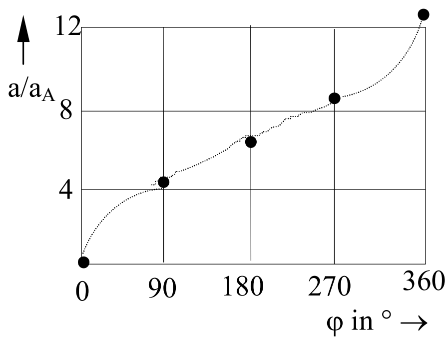
$$\ddot{x} = a_A + \dot{\varphi}^2\frac{D}{2}\sin\varphi - \ddot{\varphi}\frac{D}{2}\cos\varphi \quad x_A = \frac{D}{2}\varphi \quad \ddot{x}_A = a_A = \frac{D}{2}\ddot{\varphi}$$

$$\ddot{y} = \ddot{\varphi}\frac{D}{2}\sin\varphi + \dot{\varphi}^2\frac{D}{2}\cos\varphi \quad \ddot{\varphi} = \frac{d\dot{\varphi}}{d\varphi}\dot{\varphi} = \frac{2a_A}{D}\dot{\varphi}^2 = \frac{4a_A}{D}\varphi$$

$$\ddot{x} = a_A\left(1 + 2\varphi\sin\varphi - \cos\varphi\right) \quad \ddot{y} = a_A\left(\sin\varphi + 2\varphi\cos\varphi\right)$$

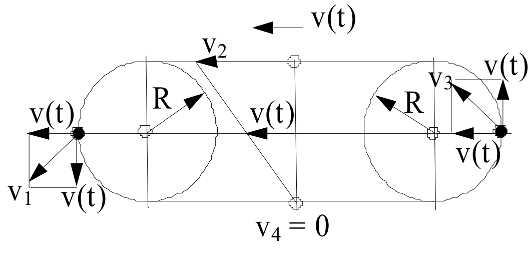
$$a = a_A\sqrt{2\left(1-\cos\varphi\right) + 4\varphi\left(\varphi + \sin\varphi\right)} oder$$

$$a = a_A\sqrt{\left(1-\cos\varphi\right)^2 + \left(2\varphi + \sin\varphi\right)^2}$$



Lösung 2.9

Geschwindigkeitsverhältnisse:



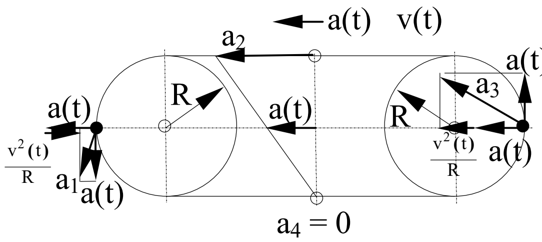
$$v_1 = \sqrt{2}\,v(t)$$

$$v_2 = 2v(t)$$

$$v_3 = \sqrt{2}\,v(t)$$

$$v_4 = 0$$

Beschleunigungsverhältnisse:



$$a_1=\sqrt{\left[a(t)-\frac{v^2(t)}{R}\right]^2+a^2(t)}$$

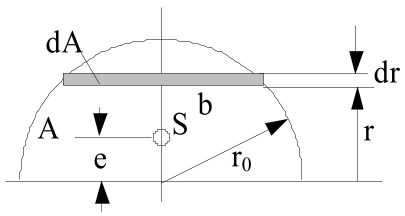
$$a_2=2a(t)$$

$$a_3=\sqrt{\left[a(t)+\frac{v^2(t)}{R}\right]^2+a^2(t)}$$

$$a_4=0$$

Lösung 2.10

1. Schwerpunkt der Halbscheibe:

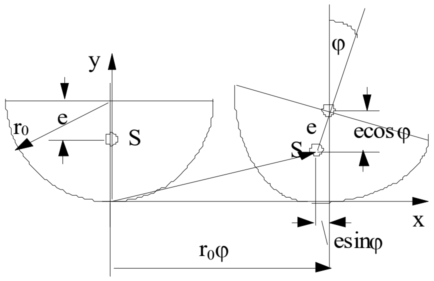


$$A\cdot e=\int_0^{r_0} r\,dA \qquad A=\frac{\pi r_0^2}{2} \qquad dA=2b\,dr \qquad b=\sqrt{r_0^2-r^2}$$

$$e=\frac{4}{\pi r_0^2}\int_0^{r_0} r\sqrt{r_0^2-r^2}\,dr \qquad z=\sqrt{r_0^2-r^2} \qquad r=\sqrt{r_0^2-z^2}$$

$$dr=-\frac{z}{\sqrt{r_0^2-z^2}}dz \qquad \int_0^{r_0} r\sqrt{r_0^2-r^2}\,dr=-\int_{r_0}^{0} z^2dz=\frac{r_0^3}{3} \qquad \boxed{e=\frac{4r_0}{3\pi}}$$

2. Beschreibung der Schwerpunktsbewegung in kart. Koordinaten:



$$x_S=r_0\varphi-e\sin\varphi$$

$$\dot{x}_S=r_0\dot{\varphi}-e\dot{\varphi}\cos\varphi$$

$$\ddot{x}_S=r_0\ddot{\varphi}-e\left(-\dot{\varphi}^2\sin\varphi+\ddot{\varphi}\cos\varphi\right)=\ddot{\varphi}\left(r_0-e\cos\varphi\right)+e\dot{\varphi}^2\sin\varphi$$

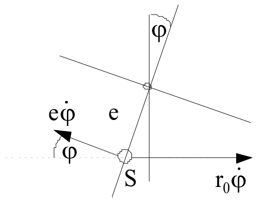
$$y_S=r_0-e\cos\varphi$$

$$\dot{y}_S=e\dot{\varphi}\sin\varphi$$

$$\ddot{y}_S=e\left(\dot{\varphi}^2\cos\varphi+\ddot{\varphi}\sin\varphi\right)$$

$$v_S=\sqrt{\dot{x}_S^2+\dot{y}_S^2}=\dot{\varphi}\sqrt{\left(r_0-e\cos\varphi\right)^2+\left(e\sin\varphi\right)^2}$$

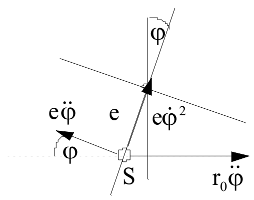
$$a_S=\sqrt{\ddot{x}_S^2+\ddot{y}_S^2}=\sqrt{r_0^2\ddot{\varphi}^2+e^2\left(\dot{\varphi}^4+\ddot{\varphi}^2\right)+2r_0e\ddot{\varphi}\left(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi\right)}$$



$$\dot{x}_S=r_0\dot{\varphi}-e\dot{\varphi}\cos\varphi$$

$$=\dot{\varphi}\left(r_0-e\cos\varphi\right)$$

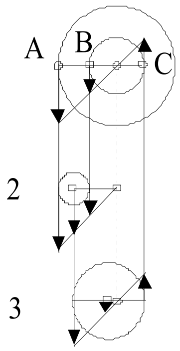
$$\dot{y}_S=e\dot{\varphi}\sin\varphi$$



$$\ddot{x}_S=r_0\ddot{\varphi}+e\dot{\varphi}^2\sin\varphi-e\ddot{\varphi}\cos\varphi$$

$$\ddot{y}_S=e\dot{\varphi}^2\cos\varphi+e\ddot{\varphi}\sin\varphi$$

Lösung 2.11



(1) $v_A = R\omega_1 \quad v_B = r\omega_1 \quad v_C = -r\omega_1$, $r_2 = \dfrac{R-r}{2} \quad r_3 = \dfrac{2r + r_2}{2} = \dfrac{R+3r}{4}$

(2) $v_A = v_2 + r_2\omega_2 \quad v_B = v_2 - r_2\omega_2 \quad v_A + v_B = 2v_2$

$$v_2 = \frac{1}{2}(v_A + v_B) = \frac{1}{2}(R+r)\omega_1 \qquad \omega_2 = \frac{v_A - v_2}{r_2} = \omega_1 \quad \omega_2 = \omega_1$$

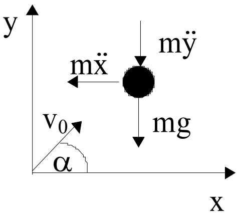
$$v_2 = v_3 + r_3\omega_3 \quad v_C = v_3 - r_3\omega_3 \qquad v_2 + v_C = 2v_3$$

$$v_3 = \frac{1}{2}(v_2 + v_C) = \frac{1}{4}(R-r)\omega_1 \quad \text{und} \quad \omega_3 = \frac{v_2 - v_3}{r_3} = \omega_1 \quad \omega_3 = \omega_1$$

Die Momentanpole liegen auf einer Geraden.

Kinetik der ebenen Bewegung von Punktmassen und starren Körpern

Lösung 3.1



$$\leftarrow: \quad \ddot{x} = 0 \quad \dot{x} = v_0 \cos\alpha \qquad x = v_0 \cos\alpha \cdot t$$

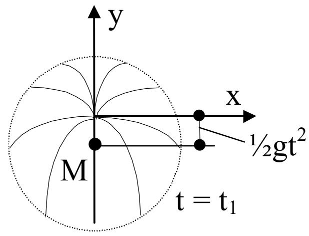
$$\downarrow: \quad \ddot{y} = -g \quad \dot{y} = -gt + v_0 \sin\alpha \quad y = -\frac{1}{2} g t^2 + v_0 \sin\alpha \cdot t$$

$$t = t_1: \quad x_1 = v_0 \cos\alpha \cdot t_1 \quad y_1 = -\frac{1}{2} g t_1{}^2 + v_0 \sin\alpha \cdot t_1$$

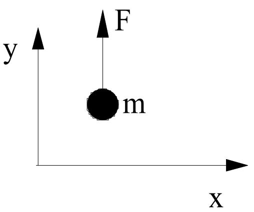
Elimination von α durch Quadrieren und Addieren liefert:

$x_1^2 + \left(y_1 + \frac{1}{2} g t_1^2\right)^2 = \left(v_0 t_1\right)^2$ Kreisgleichung mit dem Radius $R = v_0\, t_1$, Mittelpunkt versetzt um

$$y = -\frac{1}{2} g t_1^2$$



Lösung 3.2



Dynamisches Grundgesetz:

$$\vec{F} = m\vec{a} \qquad \vec{F} = \{0; F\} \qquad \vec{a} = \{\ddot{x}; \ddot{y}\}$$

$$0 = m\ddot{x} \quad \Rightarrow \quad \ddot{x} = 0 \qquad F = m\ddot{y} \quad \Rightarrow \quad \ddot{y} = \frac{1}{m} \hat{F} \cos\Omega t$$

$$\ddot{x} = 0 \quad \dot{x} = C_1 \quad x = C_1 t + C_2$$

$$\ddot{y} = \frac{1}{m} \hat{F} \cos\Omega t \quad \dot{y} = \frac{\hat{F}}{m\Omega} \sin\Omega t + C_3 \quad y = -\frac{\hat{F}}{m\Omega^2} \cos\Omega t + C_3 t + C_4$$

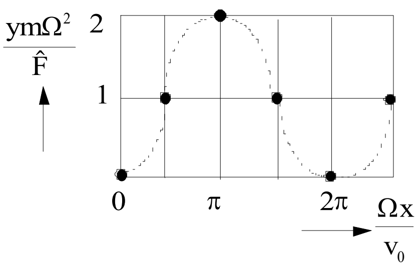
AB: $t = 0 \quad x = 0 \quad \Rightarrow \quad C_2 = 0$

$$y = 0 \quad \Rightarrow \quad 0 = -\frac{\hat{F}}{m\Omega^2} + C_4 \quad C_4 = \frac{\hat{F}}{m\Omega^2}$$

$$\dot{x} = v_0 \quad \Rightarrow \quad C_1 = v_0$$

$$\dot{y} = 0 \quad \Rightarrow \quad C_3 = 0$$

$$x = v_0 t \quad \Rightarrow \quad t = \frac{x}{v_0} \qquad y = \frac{\hat{F}}{m\Omega^2}(1 - \cos\Omega t) = \frac{\hat{F}}{m\Omega^2}\left(1 - \cos\frac{\Omega x}{v_0}\right)$$

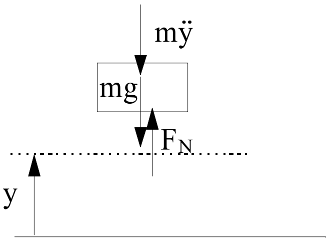


$\frac{\Omega x}{v_0}$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$\cos\frac{\Omega x}{v_0}$	1	0	−1	0	1
$\frac{ym\Omega^2}{\hat{F}}$	0	1	2	1	0

Extremwerte:

$$\frac{d\left(\frac{ym\Omega^2}{\hat{F}}\right)}{d\left(\frac{\Omega x}{v_0}\right)} = 0 \quad \Rightarrow \quad \left(\frac{\Omega x}{v_0}\right)_{Ex} = n\pi \qquad n = 0,1,2,3,...$$

Lösung 3.3



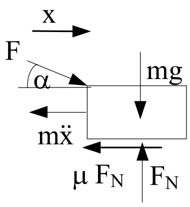
$$F_N - mg - m\ddot{y} = 0 \qquad y = \hat{y}\sin\Omega t \quad \ddot{y} = -\hat{y}\Omega^2\sin\Omega t$$

$$F_N = mg - m\hat{y}\Omega^2\sin\Omega t$$

$$\text{Abheben:} \quad F_N = 0 \qquad mg = m\hat{y}\Omega^2\sin\Omega t \quad g = \hat{y}\Omega^2\sin\Omega t$$

$$\Omega = \sqrt{\frac{g}{\hat{y}\sin\Omega t}} \qquad \Omega_{min} \quad \text{für} \quad \sin\Omega t = 1 \quad \Omega_{min} = \sqrt{\frac{g}{\hat{y}}} = 99\,s^{-1}$$

Lösung 3.4

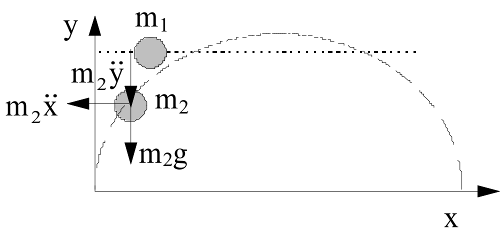


$$\uparrow: \quad F_N = mg + F\sin\alpha$$

$$\leftarrow: \; m\ddot{x} + \mu\left(mg + F\sin\alpha\right) = F\cos\alpha$$

$$F = mg\,\frac{\frac{\ddot{x}}{g} + \mu}{\cos\alpha - \mu\sin\alpha} = 371{,}45\,N$$

Lösung 3.5



$$y_1 = H_0 \quad \dot{x}_1 = v_1 \quad x_1(t) = v_1 t + C_1 \quad C_1 = 0$$

$$\ddot{y}_2 = -g \quad \dot{y}_2 = -gt + C_2 \quad y_2(t) = -\frac{1}{2}gt^2 + C_2 t + C_3$$

$$\ddot{x}_2 = 0 \quad \dot{x}_2 = C_4 \quad x_2(t) = C_4 t + C_5$$

$$\begin{aligned} AB: \quad t = 0 \quad & \dot{x}_2 = v_2\cos\alpha && \Rightarrow \quad C_4 = v_2\cos\alpha \\ & \dot{y}_2 = v_2 sin\alpha && \Rightarrow \quad C_2 = v_2 sin\alpha \\ & x_2 = 0 && \Rightarrow \quad C_5 = 0 \\ & y_2 = 0 && \Rightarrow \quad C_3 = 0 \end{aligned}$$

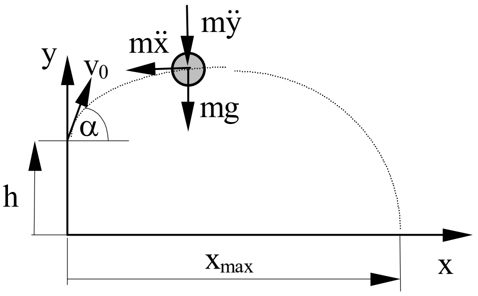
$$EB: \quad t = t_E \quad x_1(t_E) = x_2(t_E) \quad \Rightarrow \quad v_2 = \frac{v_1}{\cos\alpha}$$

$$y_1(t_E) = y_2(t_E) \quad \Rightarrow \quad H_0 = -\frac{1}{2}gt_E^2 + v_1\tan\alpha \cdot t_E$$

$$t_E^2 - 2\frac{v_1\tan\alpha}{g}t_E + 2\frac{H_0}{g} = 0 \quad \Rightarrow \quad t_{E1,2} = \frac{v_1\tan\alpha}{g} \pm \sqrt{\left(\frac{v_1\tan\alpha}{g}\right)^2 - 2\frac{H_0}{g}}$$

Zahlenwerte: $v_2 = 1200\,\frac{km}{h}$, $\quad t_{E1} = 3{,}67\,s \quad (t_{E2} = 55{,}1\,s)$

Lösung 3.6



$$\leftarrow: \quad \ddot{x} = 0 \qquad \downarrow: \quad \ddot{y} = -g$$

$$\dot{x} = C_1 \qquad \dot{y} = -gt + C_3$$

$$x = C_1 t + C_2 \qquad y = -\frac{1}{2} g t^2 + C_3 t + C_4$$

AB: $x(0) = 0 \quad y(0) = h \quad \dot{x}(0) = v_0 \cos\alpha \quad \dot{y}(0) = v_0 \sin\alpha$

$C_1 = v_0 \cos\alpha \quad C_2 = 0 \quad C_3 = v_0 \sin\alpha \quad C_4 = h$

$$x(t) = v_0 \cos\alpha \cdot t \qquad y(t) = -\frac{1}{2} g t^2 + v_0 \sin\alpha \cdot t + h$$

Endbedingung: $y(x_{max}) = 0$

Elimination von t:

$$t = \frac{x}{v_0 \cos\alpha} \quad \Rightarrow \quad y = -\frac{1}{2}\frac{g x^2}{v_0^2 \cos^2\alpha} + x \tan\alpha + h$$

$$y(x_{max}, \alpha) = 0$$

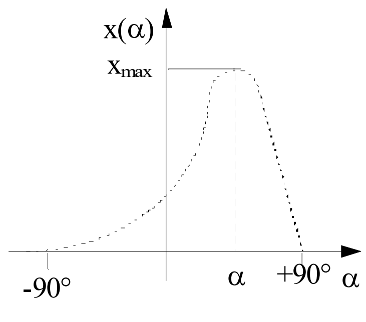
$$-\frac{1}{2}\frac{g x_{max}^2}{v_0^2 \cos^2\alpha} + x_{max} \tan\alpha + h = 0 \qquad (*)$$

$$\frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha \qquad \frac{d\tan\alpha}{d\alpha} = 1 + \tan^2\alpha$$

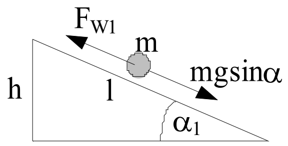
$$-\frac{1}{2}\frac{g x^2}{v_0^2}\left(1 + \tan^2\alpha\right) + x \tan\alpha + h = y(x, \alpha)$$

$$\frac{dy(x_{max}, \alpha)}{d\alpha} = 0 \quad \Rightarrow \quad -\frac{1}{2}\frac{g x_{max}^2}{v_0^2} \cdot 2\tan\alpha\left(1 + \tan^2\alpha\right) + x_{max}\left(1 + \tan^2\alpha\right) = 0$$

$$\tan\alpha = \frac{v_0^2}{g \cdot x_{max}} \quad \textit{in} \quad (*) \quad \textit{eingesetzt} \quad x_{max} = \frac{v_0^2}{g}\sqrt{1 + \frac{2gh}{v_0^2}}$$



Lösung 3.7

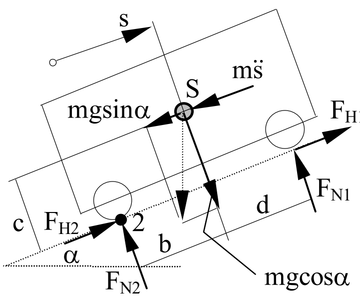


$$F_{W1} = mg\sin\alpha_1 = kv_0^2 \quad F_{W2} = mg\sin\alpha_2 = kv_2^2$$

$$\frac{v_0^2}{v_2^2} = \frac{\sin\alpha_1}{\sin\alpha_2} \quad v_2 = v_0\sqrt{\frac{\sin\alpha_2}{\sin\alpha_1}} \quad \sin\alpha_1 = \frac{h}{l} = 0{,}14 \quad \sin\alpha_2 = 0{,}08$$

$$v_2 = 60\sqrt{\frac{0{,}08}{0{,}14}}\,\frac{km}{h} = 45{,}3\,\frac{km}{h}$$

Lösung 3.8



$$\nwarrow: \quad F_{N1} + F_{N2} - mg\cos\alpha = 0$$

$$\nearrow: \quad F_{H1} + F_{H2} - mg\sin\alpha - m\ddot{s} = 0$$

$$\curvearrowleft 2: \quad F_{N1}(b+d) + m\ddot{s}c + mg\sin\alpha \cdot c - mg\cos\alpha \cdot b = 0$$

$$F_H \le \mu F_N$$

$$F_{N1} = mg\frac{c}{(b+d)}\left(\frac{b}{c}\cos\alpha - \frac{\ddot{s}}{g} - \sin\alpha\right)$$

$$F_{N2} = mg\frac{c}{(b+d)}\left(\frac{d}{c}\cos\alpha + \frac{\ddot{s}}{g} + \sin\alpha\right)$$

3. Allrad: $F_{H1} + F_{H2} = mg\left(\frac{\ddot{s}}{g} + \sin\alpha\right) \qquad F_{H1} + F_{H2} \le \mu(F_{N1} + F_{N2}) = \mu mg\cos\alpha$

$$mg\left(\frac{\ddot{s}}{g} + \sin\alpha\right) \le \mu mg\cos\alpha \qquad \frac{\ddot{s}}{g} \le \mu\cos\alpha - \sin\alpha$$

1. Frontantrieb: $F_{H1} \le \mu F_{N1} \qquad F_{H2} \approx 0 \qquad F_{H1} = mg\left(\frac{\ddot{s}}{g} + \sin\alpha\right)$

$$mg\left(\frac{\ddot{s}}{g} + \sin\alpha\right) \le \mu mg\frac{c}{(b+d)}\left(\frac{b}{c}\cos\alpha - \frac{\ddot{s}}{g} - \sin\alpha\right) \qquad \frac{\ddot{s}}{g} \le \frac{b}{b+d} \cdot \frac{\mu\cos\alpha}{1 + \mu\frac{c}{b+d}} - \sin\alpha$$

2. Heckantrieb: $F_{H1} \approx 0 \qquad F_{H2} \le \mu F_{N2}$

$$mg\left(\frac{\ddot{s}}{g} + \sin\alpha\right) \le \mu mg\frac{c}{(b+d)}\left(\frac{d}{c}\cos\alpha + \frac{\ddot{s}}{g} + \sin\alpha\right) \qquad \frac{\ddot{s}}{g} \le \frac{d}{b+d} \cdot \frac{\mu\cos\alpha}{1 - \mu\frac{c}{b+d}} - \sin\alpha$$

$$\left(\frac{\ddot{s}}{g}\right)_1 = \frac{2}{9}\cos\alpha - \sin\alpha \qquad \left(\frac{\ddot{s}}{g}\right)_2 = \frac{2}{7}\cos\alpha - \sin\alpha \qquad \left(\frac{\ddot{s}}{g}\right)_3 = \frac{1}{2}\cos\alpha - \sin\alpha \qquad \left(\frac{\ddot{s}}{g}\right)_G = 2\cos\alpha - \sin\alpha$$

$$\tan\alpha = 0 \quad \Rightarrow \quad \cos\alpha = 1 \quad \sin\alpha = 0$$

3. $\frac{\ddot{s}}{g} \le 0{,}5$ 1. $\frac{\ddot{s}}{g} \le 0{,}2222$ 2. $\frac{\ddot{s}}{g} \le 0{,}286$

Umkippen, wenn $F_{N1} < 0$ wird.

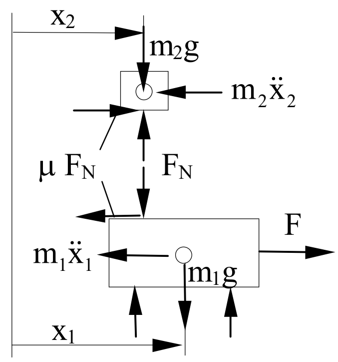
Grenzbedingung: $\frac{\ddot{s}}{g} \le \frac{b}{c}\cos\alpha - \sin\alpha \qquad \tan\alpha = 0 \qquad \left(\frac{\ddot{s}}{g}\right)_G = 2$

$$\tan\alpha = 0{,}2 \quad \Rightarrow \quad \cos\alpha = 0{,}9806 \quad \sin\alpha = 0{,}196$$

$\left(\frac{\ddot{s}}{g}\right)_G = 1{,}765 \qquad$ 3. $\frac{\ddot{s}}{g} \le 0{,}2943 \qquad$ 1. $\frac{\ddot{s}}{g} \le 0{,}02169 \qquad$ 2. $\frac{\ddot{s}}{g} \le 0{,}0844$

Lösung 3.9

1. Voraussetzung: m_1 gleitet auf m_2



$$m_2: \quad F_N = m_2 g \qquad m_2\ddot{x}_2 - \mu F_N = 0 \quad \Rightarrow \quad \ddot{x}_2 = \mu g$$

$$m_1: \quad m_1\ddot{x}_1 + \mu F_N - F = 0 \quad \ddot{x}_1 = \frac{F}{m_1} - \mu\frac{m_2}{m_1}g$$

$$\text{Zahlenwerte:} \quad \ddot{x}_1 = 3{,}088\,\frac{m}{s^2} \quad \ddot{x}_2 = 1{,}472\,\frac{m}{s^2}$$

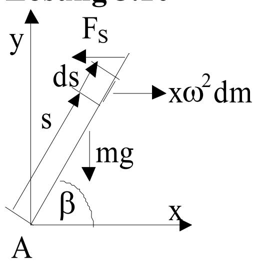
2. Kein Gleiten zwischen m_1 und m_2

$\ddot{x}_1 = \ddot{x}_2$ und $\mu = \mu_0$ $F = F^*$

$$\frac{F^*}{m_1} - \mu_0\frac{m_2}{m_1}g = \mu_0 g \quad \Rightarrow \qquad F^* = \mu_0 g(m_1 + m_2)$$

$$\text{Zahlenwert:} \quad F^* = 1717\,N$$

Lösung 3.10



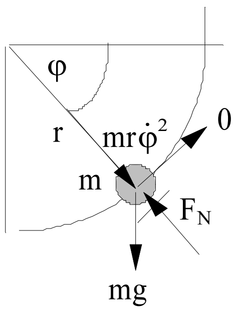
$$\curvearrowleft A: \quad F_S l\sin\beta - \frac{1}{2}mgl\cos\beta - \int_0^l x\omega^2 y\,dm = 0$$

$$x = s\cos\beta \quad y = s\sin\beta \quad dm = \frac{m}{l}ds$$

$$F_S l\sin\beta - \frac{1}{2}mgl\cos\beta - \omega^2\sin\beta\cos\beta\,\frac{m}{l}\int_0^l s^2 ds$$

$$F_S = \frac{1}{2}mg\cot\beta + \frac{1}{3}ml\omega^2\cos\beta = 1931{,}64\,N$$

Lösung 3.11



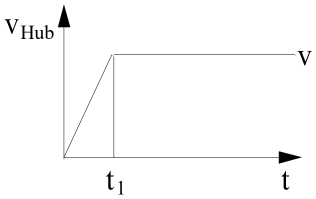
$$F_N - mg\sin\varphi - mr\dot{\varphi}^2 = 0$$

$$F_N = mg\sin\varphi + mr\dot{\varphi}^2 \quad \dot{\varphi} = \frac{v}{r}$$

$$F_{N_{max}} = mg + \frac{mv^2}{r} = m\left(g + \frac{v^2}{r}\right) = 11{,}073\,kN$$

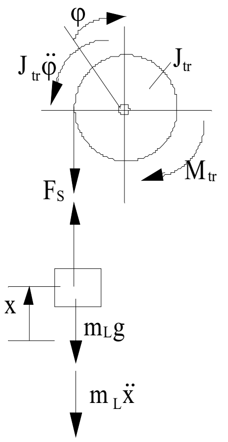
Lösung 3.12

Aus Aufgabe 2.1 : $M_{mot} = \left(\frac{r_k}{r_g}\right)^2 M_{tr} \qquad \left(\frac{r_k}{r_g}\right) = 0{,}18$



1. Bereich: $0 \le t \le t_1 \qquad \ddot{x} = \text{konst.} = a = \frac{v}{t}$

2. Bereich: $t \ge t_1 \qquad \ddot{x} = 0$



freie Koordinaten: x, φ

Zwangsbedingungen: $x = \frac{D}{2}\varphi \quad \Rightarrow \quad \varphi = \frac{2x}{D}$

Masse m_L: $F_S - m_L\ddot{x} - m_L g = 0 \quad F_S = m_L\ddot{x} + m_L g$

Trommel: $J_{tr}\ddot{\varphi} + F_S \cdot \frac{D}{2} - M_{tr} = 0$

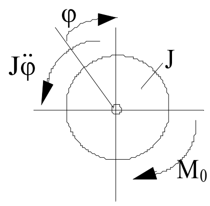
$$M_{tr} = J_{tr} \cdot \frac{2\ddot{x}}{D} + \frac{D}{2} m_L(\ddot{x} + g) = \frac{D}{2} m_L g + \ddot{x}\left(\frac{D}{2} m_L + \frac{2J_{tr}}{D}\right)$$

$$J_{tr} = 0{,}9 J_{Ring} = 0{,}9\left(\frac{D}{2}\right)^2 m_{tr}$$

$$M_{mot} = \left(\frac{r_k}{r_g}\right)^2 \frac{D}{2}\left[m_L g + \ddot{x}(m_L + 0{,}9 m_{tr})\right]$$

1. Bereich: $M_{mot} = 224$ Nm 2. Bereich: $M_{mot} = 215$ Nm

Lösung 3.13



$J\ddot{\varphi} = M_0 \quad J(\omega - \omega_0) = M_0(t - t_0)$

$t_0 = 0 \quad \omega_0 = 0$

$t = t_1\text{:} \quad J\omega_1 = M_0 t_1 \quad \Rightarrow \quad M_0 = \frac{J\omega_1}{t_1}$

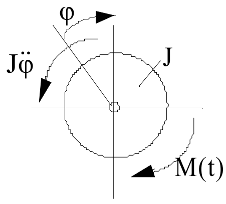
$$J = \frac{1}{2} m_1\left(\frac{D}{2}\right)^2 - 6\left\{\frac{1}{2} m_2\left(\frac{d}{2}\right)^2 + m_2 e^2\right\}$$

$m_1 = \rho a \frac{\pi D^2}{4} \quad m_2 = \rho b \frac{\pi d^2}{4} \quad \Rightarrow \quad J = \frac{\rho\pi}{32}\left[aD^4 - 6bd^2(d^2 + 8e^2)\right]$

$M_0 = \frac{\omega_1 \rho\pi}{32 t_1}\left[aD^4 - 6bd^2(d^2 + 8e^2)\right] \qquad n_1 = \frac{\omega_1}{2\pi}$

Zahlenwerte: $n_1 = 4{,}77\,s^{-1} \equiv 286{,}5\,\text{min}^{-1} \quad M_0 = 0{,}3933\,\text{Nm}$

Lösung 3.14



$$J\ddot{\varphi} = M(t) \quad \ddot{\varphi} = \frac{M(t)}{J}$$

$$0 \le t \le t_1: \quad M(t) = M_1\frac{t}{t_1} \quad \ddot{\varphi} = \frac{M_1}{Jt_1}\cdot t \quad \dot{\varphi} = \frac{M_1}{Jt_1}\cdot\frac{t^2}{2} + C_1$$

$$\varphi = \frac{M_1}{Jt_1}\cdot\frac{t^3}{6} + C_1 t + C_2 \quad t = 0: \quad \varphi = 0 \quad \dot{\varphi} = 0$$

$$\ddot{\varphi} = \frac{M_1}{Jt_1}\cdot t \qquad \dot{\varphi} = \frac{M_1}{Jt_1}\cdot\frac{t^2}{2} \qquad \varphi = \frac{M_1}{Jt_1}\cdot\frac{t^3}{6}$$

$$t = t_1: \quad \ddot{\varphi} = \frac{M_1}{J} \qquad \dot{\varphi} = \frac{M_1}{J}\cdot\frac{t_1}{2} \qquad \varphi = \frac{M_1}{J}\cdot\frac{t_1^2}{6}$$

$$t_1 \le t: \quad M(t) = M_1 \quad \ddot{\varphi} = \frac{M_1}{J} \quad \dot{\varphi} = \frac{M_1}{J}t + C_3 \quad \varphi = \frac{M_1}{J}\frac{t^2}{2} + C_3 t + C_4$$

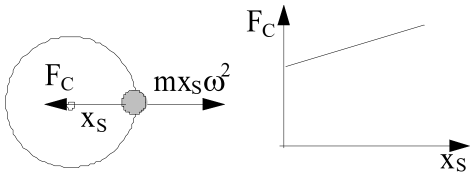
$$t = t_1: \quad \dot{\varphi}(t_1) = \frac{M_1}{J}\cdot\frac{t_1}{2} = \frac{M_1}{J}t_1 + C_3 \quad \Rightarrow \quad C_3 = -\frac{M_1}{J}\cdot\frac{t_1}{2}$$

$$\varphi(t_1) = \frac{M_1}{J}\cdot\frac{t_1^2}{6} = \frac{M_1}{J}\frac{t_1^2}{2} - \frac{M_1}{J}\cdot\frac{t_1^2}{2} + C_4 \quad \Rightarrow \quad C_4 = \frac{M_1}{J}\cdot\frac{t_1^2}{6}$$

$$\dot{\varphi} = \frac{M_1}{J}t - \frac{M_1}{J}\cdot\frac{t_1}{2} \qquad \varphi = \frac{M_1}{J}\frac{t^2}{2} - \frac{M_1}{J}\cdot\frac{t_1}{2}t + \frac{M_1}{J}\cdot\frac{t_1^2}{6}$$

$$t = 2t_1: \quad \dot{\varphi} = \frac{3M_1}{2J}t_1 \quad \omega = 2\pi n \quad n = \frac{\dot{\varphi}}{2\pi}$$

Lösung 3.15



$$F_C = F_a + cx_S \quad x_S = x_{S0} \quad \Rightarrow \quad F_C = F_0$$

$$F_0 = F_a + cx_{S0} \quad \Rightarrow \quad F_a = F_0 - cx_{S0}$$

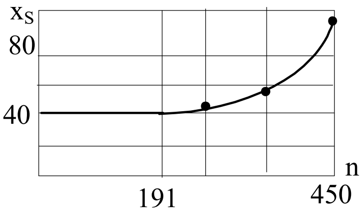
$$F_C = F_0 + c(x_S - x_{S0})$$

$$F_C = mx_S\omega^2 \quad \Rightarrow \quad mx_S\omega^2 = F_0 + c(x_S - x_{S0})$$

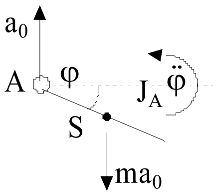
$$\omega = \omega_a: \quad x_S = x_{S0} \quad \Rightarrow \quad F_0 = mx_{S0}\omega_a^2 \quad \omega_a = \sqrt{\frac{F_0}{mx_{S0}}} \quad n_a = \frac{\omega_a}{2\pi} = 3{,}183\,s^{-1} \equiv 191\,min^{-1}$$

$$x_S = \frac{F_0 - cx_{S0}}{m\omega^2 - c}: \quad n_1 = 250\,min^{-1} \quad \Rightarrow \quad x_{S1} = 43{,}9\,mm \quad n_2 = 355\,min^{-1} \quad \Rightarrow \quad x_{S2} = 57{,}7\,mm$$

$$n_3 = 450\,min^{-1} \quad \Rightarrow \quad x_{S3} = 92{,}8\,mm$$



Lösung 3.16



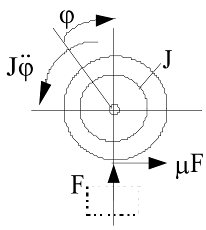
$$\curvearrowleft A:\quad J_A\ddot{\varphi} - m a_0 b\cos\varphi = 0 \qquad J_A = J_S + mb^2$$

$$\ddot{\varphi} = \frac{m a_0 b}{J_S + mb^2}\cos\varphi = \frac{d\dot{\varphi}}{d\varphi}\dot{\varphi} \quad \text{Für } \varphi = 0 \text{ ist } \dot{\varphi} = 0$$

$$\dot{\varphi}d\dot{\varphi} = \frac{m a_0 b}{J_S + mb^2}\cos\varphi\, d\varphi \Big| \text{ int.}$$

$$\frac{1}{2}\dot{\varphi}^2 = \frac{m a_0 b}{J_S + mb^2}\sin\varphi \qquad \dot{\varphi}\left(\frac{\pi}{2}\right) = \sqrt{\frac{2 m a_0 b}{J_S + mb^2}}$$

Lösung 3.17



$$J\ddot{\varphi} + \mu F \frac{d_2}{2} = 0 \qquad J(\omega_1 - \omega_0) = -\mu F \frac{d_2}{2}(t_1 - t_0)$$

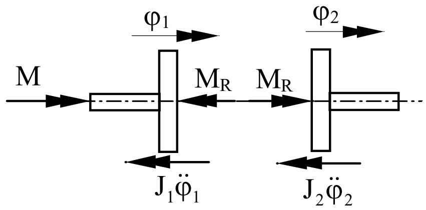
$$t_0 = 0 \quad \omega = \omega_0 \quad t = t_1 \quad \omega = \omega_1 = 0$$

$$J\omega_0 = \mu F \frac{d_2}{2} t_1 \quad \Rightarrow \quad F = \frac{2J\omega_0}{\mu d_2 t_1}$$

$$J = \frac{1}{2} m_1 \left(\frac{d_1}{2}\right)^2 + \frac{1}{2} m_2 \left(\frac{d_2}{2}\right)^2 \qquad m_i = \rho l \frac{\pi}{4} d_i^2$$

$$F = \frac{\omega_0 \rho l \pi}{16 \mu d_2 t_1}\left(d_1^4 + d_2^4\right)$$

Lösung 3.18



Scheibe 1: $J_1\ddot{\varphi}_1 + M_R - M = 0$

Scheibe 2: $J_2\ddot{\varphi}_2 - M_R = 0$

$$\ddot{\varphi}_1 = \frac{M - M_R}{J_1} \qquad \dot{\varphi}_1 = \frac{M - M_R}{J_1} t + C_1$$

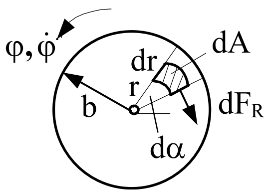
$$\varphi_1 = \frac{M - M_R}{J_1}\frac{t^2}{2} + C_1 t + C_2$$

$$\ddot{\varphi}_2 = \frac{M_R}{J_2} \qquad \dot{\varphi}_2 = \frac{M_R}{J_2} t + C_3 \qquad \varphi_2 = \frac{M_R}{J_2}\frac{t^2}{2} + C_3 t + C_4$$

AB: $t = 0 \quad \dot{\varphi}_1 = \dot{\varphi}_{10} \quad \varphi_1 = 0 \quad \dot{\varphi}_2 = 0 \quad \varphi_2 = 0 \quad \Rightarrow \quad C_1 = \dot{\varphi}_{10} \quad C_2 = 0 \quad C_3 = 0 \quad C_4 = 0$

$$\dot{\varphi}_1 = \frac{M - M_R}{J_1} t + \dot{\varphi}_{10} \qquad \varphi_1 = \frac{M - M_R}{J_1}\frac{t^2}{2} + \dot{\varphi}_{10} t \qquad \dot{\varphi}_2 = \frac{M_R}{J_2} t \qquad \varphi_2 = \frac{M_R}{J_2}\frac{t^2}{2}$$

Reibmoment M_R:



$$dM_R = dF_R \cdot r = \mu \cdot p \cdot dr \cdot r d\alpha \cdot r$$

$$M_R = \int_{\alpha=0}^{2\pi} \int_{r=0}^{b} \mu p r^2 d\alpha dr = \frac{2}{3}\pi \mu p b^3 = 125{,}7\,Nm$$

Kuppelzeit t_K:

$$\dot{\varphi}_1(t_K) = \dot{\varphi}_2(t_K) \quad \Rightarrow \quad \frac{M - M_R}{J_1} t_K + \dot{\varphi}_{10} = \frac{M_R}{J_2} t_K$$

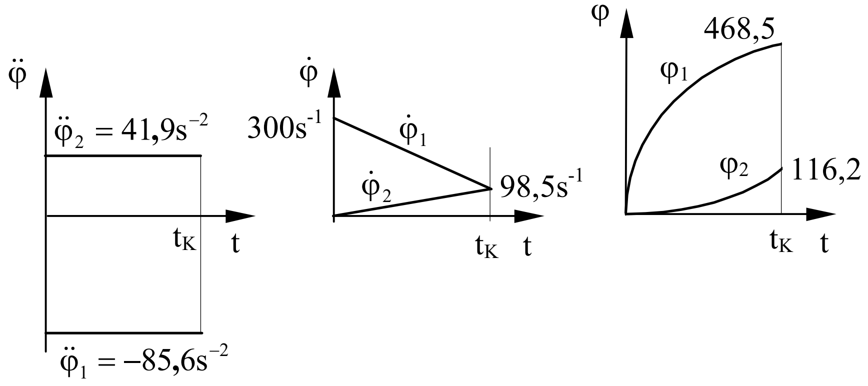
$$t_K = \frac{\dot{\varphi}_{10}}{\left(\frac{M_R}{J_2} - \frac{M - M_R}{J_1}\right)} = 2{,}35\text{s}$$

Kuppelzeit t_K muß endlich sein und $t_K > 0$:

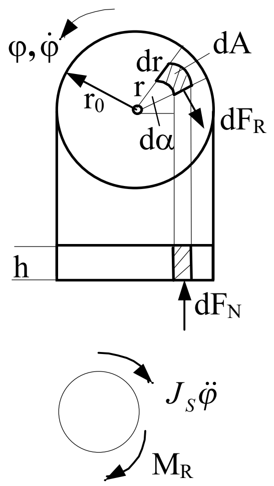
$$\left(\frac{M_R}{J_2} - \frac{M - M_R}{J_1}\right) > 0 \quad \Rightarrow \quad p > \frac{3M}{2\pi \mu b^3 \left(1 + \frac{J_1}{J_2}\right)} = 14{,}47 \frac{N}{cm^2}$$

Energieverlust:

$$W_V = M_R \cdot \left[\varphi_1(t_K) - \varphi_2(t_K)\right] = \frac{M_R}{2}\dot{\varphi}_{10} t_K = 4{,}43 \cdot 10^4\, Nm$$



Lösung 3.19



$$J_S\ddot{\varphi} = -M_R \qquad dA = r\,d\alpha\,dr \qquad m = \rho\pi r_0^2 h$$

$$dF_N = \rho g h\, dA \quad dF_R = \mu\, dF_N \qquad J_S = \frac{1}{2} m r_0^2$$

$$M_R = \int r\,dF_R = \mu\rho g h \int_0^{r_0}\int_0^{2\pi} r^2\, d\alpha\, dr = \frac{2}{3}\mu g r_0 \rho\pi r_0^2 h = \frac{2}{3}\mu g r_0 m$$

$$\frac{1}{2} m r_0^2 \ddot{\varphi} = -\frac{2}{3}\mu r_0 m g \qquad \ddot{\varphi} = -\frac{4}{3}\frac{\mu g}{r_0}$$

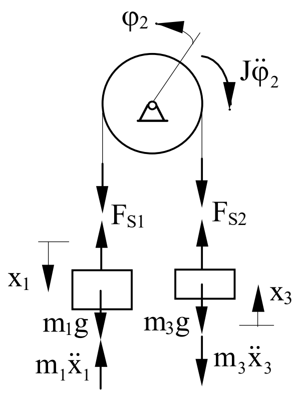
AB: $t = 0 \quad \varphi(0) = 0 \quad \dot{\varphi}(0) = \omega_0$

$$\dot{\varphi} = -\frac{4}{3}\frac{\mu g}{r_0} t + \omega_0 \qquad \varphi = -\frac{2}{3}\frac{\mu g}{r_0} t^2 + \omega_0 t$$

Endb.: $\dot{\varphi}(T) = 0 \quad 0 = -\frac{4}{3}\frac{\mu g}{r_0} T + \omega_0 \quad \boxed{T = \frac{3\omega_0 r_0}{4\mu g}}$

$$\varphi(T) = \frac{3}{8}\frac{\omega_0^2 r_0}{\mu g} \quad \text{Umdrehungen} \quad \boxed{U = \frac{\varphi(T)}{2\pi} = \frac{3}{16\pi}\frac{\omega_0^2 r_0}{\mu g}}$$

Lösung 3.20



ZB: $\quad x_3 = x_1 = x \qquad \varphi_2 = \varphi = \dfrac{x}{r}$

$$J = \frac{1}{2} m_2 r^2$$

$$F_{S1} = m_1 g - m_1 \ddot{x}_1 = m_1 g - m_1 \ddot{x}$$

$$F_{S2} = m_3 g + m_3 \ddot{x}_3 = m_3 g + m_3 \ddot{x}$$

$$J\ddot{\varphi}_2 + F_{S2} r - F_{S1} r = 0 = J\frac{\ddot{x}}{r} + m_3 g r + m_3 \ddot{x} r - m_1 g r + m_1 \ddot{x} r$$

$$\ddot{x} = g\,\frac{m_1 - m_3}{m_1 + \frac{1}{2} m_2 + m_3}$$

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{2} J \dot{\varphi}_2^2 = \frac{1}{2}\dot{x}^2\left(m_1 + \frac{1}{2} m_2 + m_3\right)$$

$$U = -m_1 g x_1 + m_3 g x_3 = -gx\left(m_1 - m_3\right)$$

$$L = T - U = \frac{1}{2}\dot{x}^2\left(m_1 + \frac{1}{2} m_2 + m_3\right) + gx\left(m_1 - m_3\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

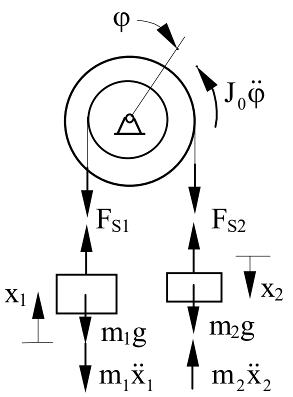
$$\frac{\partial L}{\partial x} = g\left(m_1 - m_3\right) \qquad \frac{\partial L}{\partial \dot{x}} = \dot{x}\left(m_1 + \frac{1}{2} m_2 + m_3\right) \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \ddot{x}\left(m_1 + \frac{1}{2} m_2 + m_3\right)$$

$$T + U = \text{konst.} \qquad \Rightarrow \qquad \frac{d}{dt}\left(T + U\right) = 0$$

$$T + U = \frac{1}{2}\dot{x}^2\left(m_1 + \frac{1}{2} m_2 + m_3\right) - gx\left(m_1 - m_3\right)$$

$$\ddot{x}\dot{x}\left(m_1 + \frac{1}{2} m_2 + m_3\right) - g\dot{x}\left(m_1 - m_3\right) = 0$$

Lösung 3.21



$$ZB: \quad r_1\varphi = x_1 \quad r_2\varphi = x_2 \quad \Rightarrow \quad \varphi = \frac{x_1}{r_1} \quad x_2 = \frac{r_2}{r_1}x_1$$

$$F_{S1} = m_1 g + m_1\ddot{x}_1$$

$$F_{S2} = m_2 g - m_2\ddot{x}_2 = m_2 g - m_2\frac{r_2}{r_1}\ddot{x}_1$$

$$J_0\ddot{\varphi} - F_{S2}r_2 + F_{S1}r_1 = 0 = J_0\frac{\ddot{x}_1}{r_1} - m_2 g r_2 + m_2\frac{r_2}{r_1}\ddot{x}_1 r_2 + m_1 g r_1 + m_1\ddot{x}_1 r_1$$

$$\ddot{x}_1 = g\frac{\frac{r_2}{r_1}m_2 - m_1}{m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2} \qquad \ddot{x}_2 = g\frac{r_2}{r_1}\frac{\frac{r_2}{r_1}m_2 - m_1}{m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2}$$

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}J_0\dot{\varphi}^2 = \frac{1}{2}\dot{x}_1^2\left(m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2\right)$$

$$U = m_1 g x_1 - m_2 g x_2 = g x_1\left(m_1 - \frac{r_2}{r_1}m_2\right)$$

$$L = T - U = \frac{1}{2}\dot{x}_1^2\left(m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2\right) - g x_1\left(m_1 - \frac{r_2}{r_1}m_2\right)$$

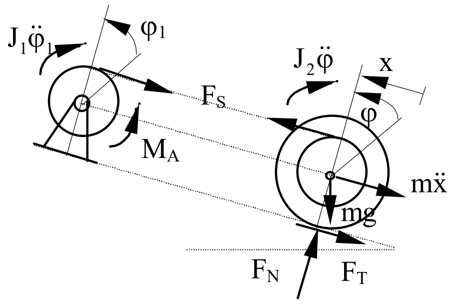
$$\frac{\partial L}{\partial x_1} = -g\left(m_1 - \frac{r_2}{r_1}m_3\right) \quad \frac{\partial L}{\partial \dot{x}_1} = \dot{x}_1\left(m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2\right) \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \ddot{x}_1\left(m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2\right)$$

$$T + U = konst. \quad \Rightarrow \quad \frac{d}{dt}(T + U) = 0$$

$$T + U = \frac{1}{2}\dot{x}_1^2\left(m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2\right) + g x_1\left(m_1 - \frac{r_2}{r_1}m_2\right)$$

$$\ddot{x}_1\dot{x}_1\left(m_1 + \frac{J_0}{r_1^2} + \left(\frac{r_2}{r_1}\right)^2 m_2\right) + g\dot{x}_1\left(m_1 - \frac{r_2}{r_1}m_2\right) = 0$$

Lösung 3.22



$$r_1\varphi_1 = x_S \qquad \frac{x_S}{x} = \frac{r_2 + r_3}{r_3} \quad \Rightarrow \quad \varphi_1 = \left(\frac{r_2}{r_3} + 1\right)\frac{x}{r_1}$$

$$r_3\varphi = x \quad \Rightarrow \quad \varphi = \frac{x}{r_3} \qquad r_1 = r_2$$

$$J_1\ddot{\varphi}_1 + F_S r_1 - M_A = 0 \qquad F_S = \frac{M_A}{r_1} - J_1\frac{\ddot{\varphi}_1}{r_1}$$

$$J_2\ddot{\varphi} - F_S(r_2 + r_3) + m\ddot{x}\cdot r_3 + mg\sin\alpha\cdot r_3 = 0$$

$$J_2\ddot{\varphi} - \frac{M_A}{r_1}(r_2 + r_3) + J_1\frac{\ddot{\varphi}_1}{r_1}(r_2 + r_3) + m\ddot{x}\cdot r_3 + mg\sin\alpha\cdot r_3 = 0$$

$$M_{A\min} \quad für \quad \ddot{\varphi} = \ddot{\varphi}_1 = 0, \quad \ddot{x} = 0 \quad \Rightarrow \quad M_{A\min} = \frac{mgr_1r_3\sin\alpha}{(r_2 + r_3)}$$

$$J_2\frac{\ddot{x}}{r_3^2} - \frac{M_A}{r_1}\left(\frac{r_2 + r_3}{r_3}\right) + J_1\frac{\ddot{x}}{r_3^2}\frac{(r_2 + r_3)^2}{r_1^2} + m\ddot{x} + mg\sin\alpha = 0$$

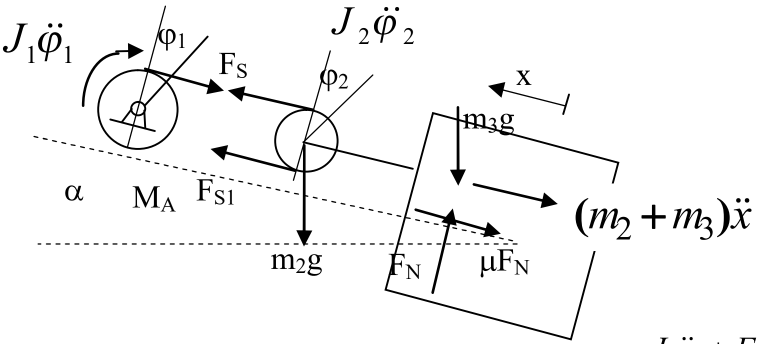
$$M_A = 2M_{A\min} = \frac{2mgr_1r_3\sin\alpha}{(r_2 + r_3)}$$

$$\ddot{x}\left\{m + \frac{J_2}{r_3^2} + \frac{J_1}{r_1^2}\left(1 + \frac{r_1}{r_3}\right)^2\right\} = mg\sin\alpha$$

$$\ddot{x}(t) = \ddot{s}(t) = \frac{mg\sin\alpha}{\left\{m + \frac{J_2}{r_3^2} + \frac{J_1}{r_1^2}\left(1 + \frac{r_1}{r_3}\right)^2\right\}} = K \qquad \dot{s}(t) = Kt + 0 \qquad s(t) = \frac{1}{2}Kt^2$$

$$K = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v \quad \Rightarrow \quad K\int_0^s d\bar{s} = \int_0^v \bar{v}\,d\bar{v}\,v = \sqrt{2Ks}$$

Lösung 3.23



$$J_1\ddot{\varphi}_1 + F_S r_1 - M_A = 0(1)$$
$$J_2\ddot{\varphi}_2 - F_S r_2 + F_{S1} r_2 = 0(2)$$
$$F_S + F_{S1} - (m_2+m_3)\ddot{x} - (m_2+m_3)g\sin\alpha$$
$$-\mu F_N = 0\ (3)$$
$$F_N - (m_2+m_3)g\cos\alpha = 0(4)$$
$$ZB:\varphi_2 = \frac{2x}{r_2}(5);\varphi_1 = 2\frac{x}{r_1}(6)$$

Ab hier falsch da, ZB1 falsch eingesetzt-> Faktor 2

$$F_S = \frac{M_A}{r_1} - 2J_1\frac{\ddot{x}}{r_1^2} \qquad (1')$$

$$F_{S1} = \frac{M_A}{r_1} - 2J_1\frac{\ddot{x}}{r_1^2} - J_2\frac{\ddot{x}}{r_2^2} \qquad (2')$$

in (3) eingesetzt:

$$\frac{M_A}{r_1} - 2J_1\frac{\ddot{x}}{r_1^2} + \frac{M_A}{r_1} - 2J_1\frac{\ddot{x}}{r_1^2} - J_2\frac{\ddot{x}}{r_2^2} - (m_2+m_3)\ddot{x} - (m_2+m_3)g\sin\alpha - \mu(m_2+m_3)g\cos\alpha = 0$$

$$-\ddot{x}\left(m_2+m_3+4J_1\frac{1}{r_1^2}+J_2\frac{1}{r_2^2}\right) = -2\frac{M_A}{r_1} + (m_2+m_3)g\sin\alpha + \mu(m_2+m_3)g\cos\alpha$$

$$\ddot{x} = \frac{2\frac{M_A}{r_1} - (m_2+m_3)g(\sin\alpha+\mu\cos\alpha)}{m_2+m_3+4J_1\frac{1}{r_1^2}+J_2\frac{1}{r_2^2}} = \text{konst.} = a$$

$$v = at \quad s = \frac{1}{2}at^2$$

Energiebilanz:

$$M_A\varphi_1 - \mu(m_2 + m_3)g\cos\alpha \cdot x = \frac{1}{2}J_1\dot{\varphi}_1^2 + \frac{1}{2}J_2\dot{\varphi}_2^2 + \frac{1}{2}(m_2 + m_3)\dot{x}^2 + (m_2 + m_3)g\sin\alpha \cdot x$$

$$\frac{1}{2}J_1\dot{\varphi}_1^2 + \frac{1}{2}J_2\dot{\varphi}_2^2 + \frac{1}{2}(m_2 + m_3)\dot{x}^2 + (m_2 + m_3)g\sin\alpha \cdot x - M_A\varphi_1 + \mu(m_2 + m_3)g\cos\alpha \cdot x = 0$$

ZB einsetzen:

$$\frac{1}{2}\dot{x}^2\left(4J_1\frac{1}{r_1^2} + \frac{1}{2}m_2 + m_2 + m_3\right) + x\left[(m_2 + m_3)g(\sin\alpha + \mu\cos\alpha) - 2\frac{M_A}{r_1}\right] = 0 \,\Big|\, \frac{d}{dt}$$

$$\dot{x}\ddot{x}\left(4J_1\frac{1}{r_1^2} + \frac{3}{2}m_2 + m_3\right) + \dot{x}\left[(m_2 + m_3)g(\sin\alpha + \mu\cos\alpha) - 2\frac{M_A}{r_1}\right] = 0 \quad \dot{x} \neq 0 \quad \Rightarrow$$

$$\ddot{x} = \frac{2\frac{M_A}{r_1} - (m_2 + m_3)g(\sin\alpha + \mu\cos\alpha)}{4J_1\frac{1}{r_1^2} + \frac{3}{2}m_2 + m_3}$$

Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=Q^* \quad Q^*=\frac{\delta W^*}{\delta x}$$

$$L=T-U=\frac{1}{2}\left[J_1\dot{\varphi}_1^2+J_2\dot{\varphi}_2^2+\left(m_2+m_3\right)\dot{x}^2\right]-\left(m_2+m_3\right)g\sin\alpha\cdot x$$

$$=\frac{1}{2}\dot{x}^2\left(m_2+m_3+4\frac{J_1}{r_1^2}+\frac{J_2}{r_2^2}\right)-\left(m_2+m_3\right)g\sin\alpha\cdot x$$

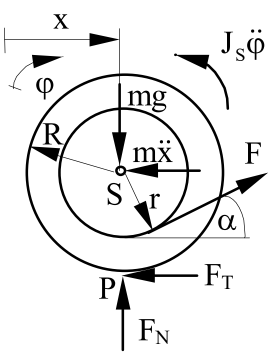
$$W^*=M_A\varphi_1-\left(m_2+m_3\right)\mu g\cos\alpha\cdot x=x\left(2\frac{M_A}{r_1}-\left(m_2+m_3\right)\mu g\cos\alpha\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=\left(m_2+m_3+4\frac{J_1}{r_1^2}+\frac{J_2}{r_2^2}\right)\ddot{x} \quad \frac{\partial L}{\partial x}=-\left(m_2+m_3\right)g\sin\alpha$$

$$Q^*=\left(2\frac{M_A}{r_1}-\left(m_2+m_3\right)\mu g\cos\alpha\right)$$

$$\left(m_2+m_3+4\frac{J_1}{r_1^2}+\frac{J_2}{r_2^2}\right)\ddot{x}+\left(m_2+m_3\right)g\sin\alpha=\left(2\frac{M_A}{r_1}-\left(m_2+m_3\right)\mu g\cos\alpha\right)$$

Lösung 3.24



$$R\varphi=x \quad\Rightarrow\quad \ddot{\varphi}=\frac{\ddot{x}}{R} \qquad (1)$$

$$\leftarrow:\quad F_T+m\ddot{x}-F\cos\alpha=0 \qquad (2)$$

$$S:\quad J_S\ddot{\varphi}+Fr-F_TR=0 \qquad (3)$$

aus (2): $F_T=-m\ddot{x}+F\cos\alpha$

in (3): $J_S\dfrac{\ddot{x}}{R}+Fr+m\ddot{x}R-FR\cos\alpha=0$

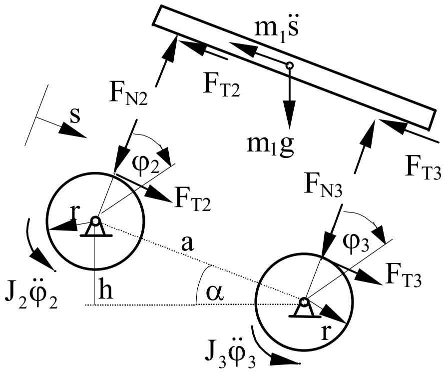
$$\ddot{x}\left(m+\frac{J_S}{R^2}\right)=F\left(\cos\alpha-\frac{r}{R}\right) \qquad \ddot{x}=\frac{F\left(\cos\alpha-\frac{r}{R}\right)}{m+\frac{J_S}{R^2}}$$

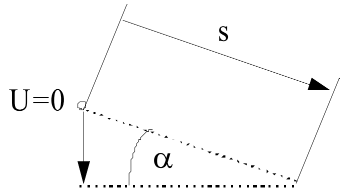
für $\cos\alpha>\frac{r}{R} \quad\Rightarrow\quad \ddot{x}>0$ **(**Beschl. in x – Richtung**)**

$\cos\alpha=\frac{r}{R} \quad\Rightarrow\quad \ddot{x}=0$

$\cos\alpha<\frac{r}{R} \quad\Rightarrow\quad \ddot{x}<0$ **(**Beschl. in neg. x – Richtung**)**

Lösung 3.25





$$\varphi_2 = \frac{s}{r} \quad \varphi_3 = \frac{s}{r}$$

$$\text{Rolle 2:} \quad J_2\ddot{\varphi}_2 - F_{T2}r = 0$$

$$\text{Rolle 3:} \quad J_3\ddot{\varphi}_3 - F_{T3}r = 0$$

$$\text{Masse 1:} \quad m_1\ddot{s} + F_{T2} + F_{T3} - m_1 g\sin\alpha = 0$$

$$m_1\ddot{s} + \frac{J_2}{r^2}\ddot{s} + \frac{J_3}{r^2}\ddot{s} = m_1 g\sin\alpha \quad J_2 = \frac{1}{2}m_2r^2 \quad J_3 = \frac{1}{2}m_3r^2$$

$$\ddot{s} = \frac{m_1 g\sin\alpha}{m_1 + \frac{m_2}{2} + \frac{m_3}{2}}$$

$$L = T - U = \frac{1}{2}\left(m_1\dot{s}^2 + J_2\dot{\varphi}_2^2 + J_3\dot{\varphi}_3^2\right) + m_1 gs\cdot\sin\alpha$$

$$= \frac{1}{2}\dot{s}^2\left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) + m_1 gs\cdot\sin\alpha$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) = \ddot{s}\left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right) \qquad \frac{\partial L}{\partial s} = m_1 g\sin\alpha$$

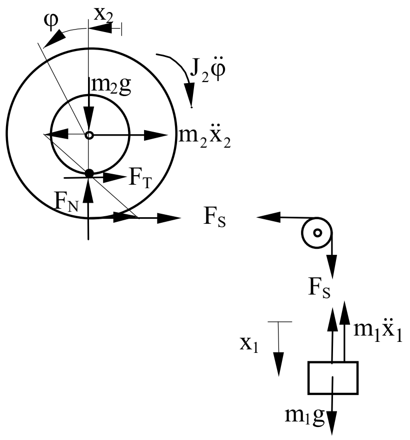
$$\ddot{s} = \frac{m_1 g\sin\alpha}{\left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right)} = K \qquad \dot{s} = Kt + C_1 \qquad s = \frac{1}{2}Kt^2 + C_1 t + C_2$$

$$AB: \quad t = 0 \quad \dot{s} = 0, \quad s = 0 \quad \Rightarrow \quad C_1 = 0, \quad C_2 = 0 \quad \dot{s} = Kt \quad s = \frac{1}{2}Kt^2$$

$$EB: \quad t = t_E \quad s = l - a \quad \Rightarrow \quad l - a = \frac{1}{2}Kt_E^2 \quad \Rightarrow \quad t_E = \sqrt{\frac{2(l-a)}{K}} = \sqrt{\frac{2(l-a)\left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right)}{m_1 g\sin\alpha}}$$

$$v_E = Kt_E = \sqrt{2(l-a)K} = \sqrt{\frac{2m_1 g\sin\alpha\cdot(l-a)}{\left(m_1 + \frac{m_2}{2} + \frac{m_3}{2}\right)}}$$

Lösung 3.26



$$\varphi = \frac{x_2}{r} \qquad x_1 = (R-r)\varphi = \left(\frac{R}{r}-1\right)x_2$$

$$F_S = m_1 g - m_1\left(\frac{R}{r}-1\right)\ddot{x}_2$$

$$F_S(R-r) - J_2\frac{\ddot{x}_2}{r} - m_2\ddot{x}_2 r = 0 \;\Big|\cdot\frac{1}{r}$$

$$\ddot{x}_2\left\{m_2 + \frac{J_2}{r^2} + m_1\left(\frac{R}{r}-1\right)^2\right\} = m_1 g\left(\frac{R}{r}-1\right)$$

$$J_2 = 2J_A + J_B \quad J_A = \frac{1}{2}m_A r^2 \quad J_B = \frac{1}{2}m_B R^2$$

$$m_2 = 2m_A + m_B \quad m_A = \pi r^2 \frac{d}{2} \cdot \rho \quad m_B = \pi R^2 b \cdot \rho$$

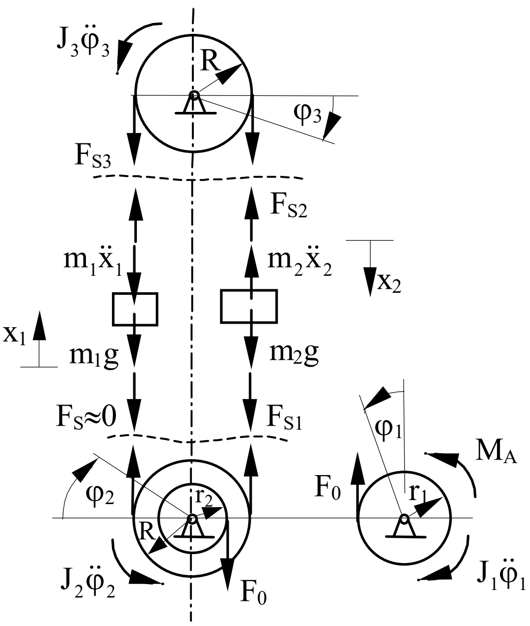
$$J_2 = \frac{1}{2}\pi\rho\left\{dr^4 + bR^4\right\} \qquad m_2 = \pi\rho\left\{dr^2 + bR^2\right\} \quad \Rightarrow \quad \rho = \frac{m_2}{\pi\left\{dr^2 + bR^2\right\}}$$

$$J_2 = \frac{1}{2} m_2 \frac{\left\{dr^4 + bR^4\right\}}{\left\{dr^2 + bR^2\right\}}$$

$$\ddot{x}_1 = \left(\frac{R}{r} - 1\right)\ddot{x}_2 = \frac{m_1 g}{m_1 + \dfrac{m_2}{\left(\dfrac{R}{r} - 1\right)^2} + \dfrac{J_2}{r^2\left(\dfrac{R}{r} - 1\right)^2}}$$

$$\mathrm{T} = \frac{1}{2}\left(\mathrm{m}_2\dot{\mathrm{x}}_2^2 + \mathrm{m}_1\dot{\mathrm{x}}_1^2 + \mathrm{J}_2\dot{\varphi}^2\right) \qquad \mathrm{U} = -\mathrm{m}_1 \mathrm{g} \mathrm{x}_1 \quad \text{z.B.} \quad \frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{T} + \mathrm{U}) = 0$$

Lösung 3.27



$$\mathrm{r}_1\varphi_1 = \mathrm{r}_2\varphi_2 \quad \varphi_2 = \varphi_3 \quad \mathrm{x}_2 = \mathrm{R}\varphi_3 \quad \mathrm{x}_1 = \mathrm{x}_2$$

$$\text{ZB:} \quad \mathrm{x}_2 = \mathrm{x}_1 = \mathrm{x} \quad \varphi_3 = \varphi_2 = \frac{\mathrm{x}}{\mathrm{R}} \quad \varphi_1 = \frac{\mathrm{r}_2}{\mathrm{r}_1}\frac{\mathrm{x}}{\mathrm{R}}$$

$$\text{Antrieb:} \quad \mathrm{M}_A - \mathrm{F}_0\mathrm{r}_1 - \mathrm{J}_1\frac{\mathrm{r}_2}{\mathrm{r}_1}\frac{\ddot{\mathrm{x}}}{\mathrm{R}} = 0$$

$$\text{Rad 2:} \quad \mathrm{J}_2\frac{\ddot{\mathrm{x}}}{\mathrm{R}} + \mathrm{F}_{S1}\mathrm{R} - \mathrm{F}_0\mathrm{r}_2 = 0$$

$$\text{Rad 3:} \quad \mathrm{J}_3\frac{\ddot{\mathrm{x}}}{\mathrm{R}} - \mathrm{F}_{S2}\mathrm{R} + \mathrm{F}_{S3}\mathrm{R} = 0$$

$$\text{Masse 1:} \quad \mathrm{m}_1\mathrm{g} + \mathrm{m}_1\ddot{\mathrm{x}} - \mathrm{F}_{S3} = 0$$

$$\text{Masse 2:} \quad -\mathrm{m}_2\mathrm{g} + \mathrm{m}_2\ddot{\mathrm{x}} - \mathrm{F}_{S1} + \mathrm{F}_{S2} = 0$$

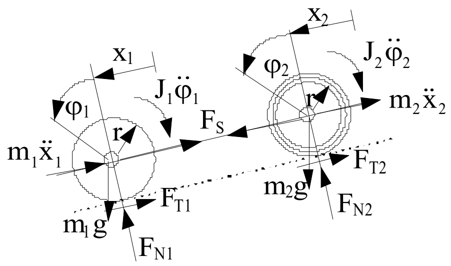
$$F_0 = \frac{1}{r_1}\left[M_A - J_1\frac{r_2}{r_1}\frac{\ddot{x}}{R}\right] \qquad F_{S1} = \frac{1}{R}\left[\frac{r_2}{r_1}\left(M_A - J_1\frac{r_2}{r_1}\frac{\ddot{x}}{R}\right) - J_2\frac{\ddot{x}}{R}\right]$$

$$F_{S3} = m_1 g + m_1\ddot{x} \qquad F_{S2} = \frac{1}{R}\left[\frac{r_2}{r_1}\left(M_A - J_1\frac{r_2}{r_1}\frac{\ddot{x}}{R}\right) - J_2\frac{\ddot{x}}{R}\right] + m_2 g - m_2\ddot{x}$$

$$J_3\frac{\ddot{x}}{R^2} - \frac{1}{R}\left[\frac{r_2}{r_1}\left(M_A - J_1\frac{r_2}{r_1}\frac{\ddot{x}}{R}\right) - J_2\frac{\ddot{x}}{R}\right] - m_2 g + m_2\ddot{x} + m_1 g + m_1\ddot{x} = 0$$

$$\ddot{x} = \frac{\dfrac{r_2}{r_1}\dfrac{M_A}{R} + m_2 g - m_1 g}{m_1 + m_2 + \left(\dfrac{r_2}{r_1}\right)^2\dfrac{J_1}{R^2} + \dfrac{J_2}{R^2} + \dfrac{J_3}{R^2}}$$

Lösung 3.28



$$x_1 = x_2 = x \qquad \varphi_1 = \varphi_2 = \varphi = \frac{x}{r}$$

$$m_1: \quad J_1\ddot{\varphi} + F_S r + m_1\ddot{x}r - m_1 g\sin\alpha \cdot r = 0$$

$$m_2: \quad J_2\ddot{\varphi} - F_S r + m_2\ddot{x}r - m_2 g\sin\alpha \cdot r = 0$$

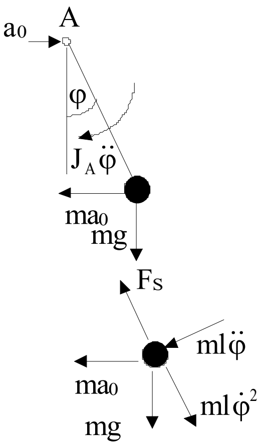
$$J_2\frac{\ddot{x}}{r^2} + m_2\ddot{x} - m_2 g\sin\alpha + J_1\frac{\ddot{x}}{r^2} + m_1\ddot{x} - m_1 g\sin\alpha = 0$$

$$m_1 = m_2 = m \qquad J_1 = \frac{1}{2}mr^2 \qquad J_2 = mr^2$$

$$\ddot{x}\left(m + m + \frac{1}{2}m + m\right) = 2mg\sin\alpha \qquad \ddot{x} = \frac{4}{7}g\sin\alpha$$

$$F_S = m(\ddot{x} + \ddot{x} - g\sin\alpha) = \frac{1}{7}mg\sin\alpha$$

Lösung 3.29



$$A: \quad J_A\ddot{\varphi} + mgl\sin\varphi + ma_0 l\cos\varphi = 0 \qquad J_A = ml^2$$

$$ml^2\ddot{\varphi} + mgl\sin\varphi + ma_0 l\cos\varphi = 0$$

$$l\ddot{\varphi} + a_0\cos\varphi + g\sin\varphi = 0$$

$$\nwarrow: \quad F_S - ml\dot{\varphi}^2 - mg\cos\varphi + ma_0\sin\varphi = 0$$

$$F_S = m\left(l\dot{\varphi}^2 + g\cos\varphi - a_0\sin\varphi\right)$$

$$\ddot{\varphi} = -\frac{1}{l}\left(a_0\cos\varphi + g\sin\varphi\right) = \frac{d\dot{\varphi}}{d\varphi}\dot{\varphi} \qquad -\frac{1}{l}\int_0^{\varphi}\left(a_0\cos\varphi^* + g\sin\varphi^*\right)d\varphi^* = \frac{1}{2}\dot{\varphi}^2$$

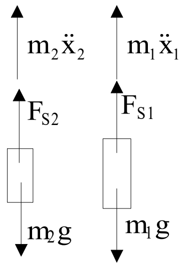
$$l\dot{\varphi}^2 = -2\int_0^{\varphi}\left(a_0\cos\varphi^* + g\sin\varphi^*\right)d\varphi^*$$

Lösung 3.30

$$L = T - U = \frac{1}{2}J_A\dot{\varphi}^2 + \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 - \left(-m_1 g x_1 - m_2 g x_2\right) \qquad x_1 = r_1\varphi \quad x_2 = r_2\varphi$$

$$L = \frac{1}{2}\dot{\varphi}^2\left(J_A + m_1 r_1^2 + m_2 r_2^2\right) + g\varphi\left(m_1 r_1 + m_2 r_2\right) \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \frac{\partial L}{\partial\varphi} = 0$$

$$\ddot{\varphi}\left(J_A + m_1 r_1^2 + m_2 r_2^2\right) - g\left(m_1 r_1 + m_2 r_2\right) = 0 \qquad \boxed{\ddot{\varphi} = \frac{g\left(m_1 r_1 + m_2 r_2\right)}{\left(J_A + m_1 r_1^2 + m_2 r_2^2\right)}}$$

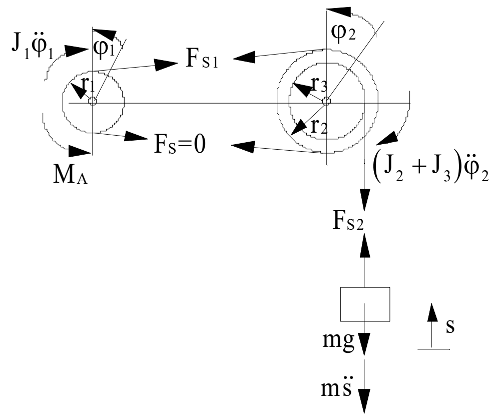


$$F_{S1} = m_1 g - m_1 r_1 \ddot{\varphi} = m_1 g\left(1 - \frac{r_1\ddot{\varphi}}{g}\right) = m_1 g\,\frac{J_A - m_2 r_2 (r_1 - r_2)}{J_A + m_1 r_1^2 + m_2 r_2^2}$$

$$F_{S2} = m_2 g - m_2 r_2 \ddot{\varphi} = m_2 g\left(1 - \frac{r_2\ddot{\varphi}}{g}\right) = m_2 g\,\frac{J_A + m_1 r_1 (r_1 - r_2)}{J_A + m_1 r_1^2 + m_2 r_2^2}$$

$F_{S2} > 0$, Forderung: $F_{S1} > 0$ liefert $\boxed{J_A > m_2 r_2 (r_1 - r_2)}$

Lösung 3.31



ZB: $\varphi_2 = \frac{s}{r_3} \quad r_2\varphi_2 = r_1\varphi_1 \quad \Rightarrow \quad \varphi_1 = \frac{r_2}{r_1}\frac{s}{r_3}$

Masse m: $m\ddot{s} + mg - F_{S2} = 0$

Scheibe 1: $J_1\ddot{\varphi}_1 - M_A + F_{S1} r_1 = 0$

Scheibe 2 / 3: $(J_2 + J_3)\ddot{\varphi}_2 - F_{S1} r_2 + F_{S2} r_3 = 0$

$F_{S2} = m\ddot{s} + mg \quad F_{S1} = \frac{M_A}{r_1} - J_1 \frac{r_2}{r_1^2}\frac{\ddot{s}}{r_3}$

$$(J_2 + J_3)\frac{\ddot{s}}{r_3} - \frac{r_2}{r_1} M_A + J_1 \frac{r_2^2}{r_1^2}\frac{\ddot{s}}{r_3} + m\ddot{s} r_3 + mg r_3 = 0$$

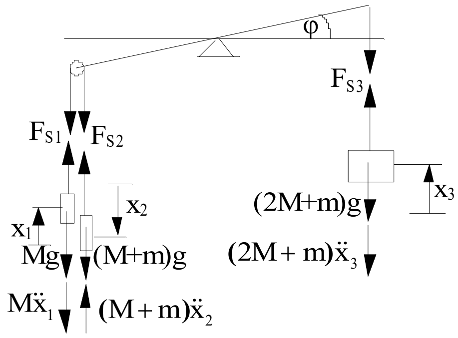
($F_S = 0$, da Riemenabtriebsseite ohne Last)

$$\ddot{s} = \frac{\frac{r_2}{r_1 r_3} M_A - mg}{\left(\frac{r_2}{r_3}\right)^2 \frac{J_1}{r_1^2} + \frac{(J_2 + J_3)}{r_3^2} + m} \qquad 1.\ M_{A\min}\ \ f\ddot{u}r\ \ \ddot{s} = 0 \quad \Rightarrow \quad M_A = M_{A\min} = \frac{r_1 r_3}{r_2} mg$$

$$2.\ M_A = 2M_{A\min} \quad \Rightarrow \quad \ddot{s} = \frac{mg}{\left(\frac{r_2}{r_3}\right)^2 \frac{J_1}{r_1^2} + \frac{(J_2 + J_3)}{r_3^2} + m} = K \qquad \dot{s} = Kt \qquad s = \frac{1}{2}Kt^2$$

$$3.\ t = \frac{\dot{s}}{K} \quad \Rightarrow \quad s = \frac{1}{2}\frac{\dot{s}^2}{K} \quad oder \quad \ddot{s} = K = \frac{d\dot{s}}{ds}\dot{s} \qquad \dot{s}d\dot{s} = Kds \qquad \dot{s} = \sqrt{2Ks}$$

Lösung 3.32



$$ZB: f = 2 \qquad x_3 = \varphi \cdot l$$

$$x_1 = -x_3 + \bar{x}_1 \quad x_2 = x_3 + \bar{x}_1 \quad \Rightarrow \quad x_2 = x_1 + 2\varphi \cdot l$$

$$F_{S1} = M(g + \ddot{x}_1) \qquad F_{S2} = (M+m)(g - \ddot{x}_2)$$

$$F_{S3} = (2M+m)(g + \ddot{x}_3) \qquad F_{S1} = F_{S2}$$

$$M(g + \ddot{x}_1) = (M+m)(g - \ddot{x}_2)$$

$$(2M+m)(g + \ddot{x}_3) \cdot l = 2M(g + \ddot{x}_1) \cdot l$$

$$(2M+m)\ddot{x}_1 + 2(M+m)l\ddot{\varphi} = mg \quad (1)$$

$$-2M\ddot{x}_1 + (2M+m)l\ddot{\varphi} = -mg \qquad (2)$$

$$\ddot{x}_3 = l\ddot{\varphi} = -\frac{g}{1 + 8\frac{M}{m}\left(1 + \frac{M}{m}\right)} \quad \ddot{x}_1 = \frac{g\left[4\left(\frac{M}{m} + 1\right) - 1\right]}{1 + 8\frac{M}{m}\left(1 + \frac{M}{m}\right)} \quad \ddot{x}_2 = \frac{g\left[4\left(\frac{M}{m} + 1\right) - 3\right]}{1 + 8\frac{M}{m}\left(1 + \frac{M}{m}\right)}$$

$$F_{S2} = F_{S1} = 4Mg\frac{\left(\frac{M}{m} + 1\right)\left(2\frac{M}{m} + 1\right)}{1 + 8\frac{M}{m}\left(1 + \frac{M}{m}\right)} \qquad \ddot{\varphi} = \frac{\ddot{x}_3}{l} = -\frac{g}{l} \cdot \frac{1}{1 + 8\frac{M}{m}\left(1 + \frac{M}{m}\right)}$$

Lösung mit Lagrange:

$$L = \frac{1}{2}\left\{M\dot{x}_1^2 + (M+m)(\dot{x}_1 + 2l\dot{\varphi})^2 + (2M+m)l^2\dot{\varphi}^2\right\} - g\left\{Mx_1 - (M+m)(x_1 + 2l\varphi) + (2M+m)l\varphi\right\}$$

$$\frac{\partial L}{\partial \dot{x}_1} = M\dot{x}_1 + (M+m)(\dot{x}_1 + 2l\dot{\varphi}) \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = M\ddot{x}_1 + (M+m)(\ddot{x}_1 + 2l\ddot{\varphi}) = (2M+m)\ddot{x}_1 + 2(M+m)l\ddot{\varphi}$$

$$\frac{\partial L}{\partial x_1} = -g\{M - M - m\} = mg$$

$$(2M+m)\ddot{x}_1 + 2(M+m)l\ddot{\varphi} = mg \quad (I)$$

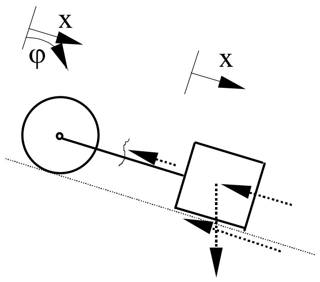
$$\frac{\partial L}{\partial \dot{\varphi}} = (M+m)(\dot{x}_1 + 2l\dot{\varphi}) \cdot 2l + (2M+m)l^2\dot{\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = (M+m)(\ddot{x}_1 + 2l\ddot{\varphi}) \cdot 2l + (2M+m)l^2\ddot{\varphi} = 2(M+m)l\ddot{x}_1 + (6M+5m)l^2\ddot{\varphi}$$

$$\frac{\partial L}{\partial \varphi} = -g\{-2l(M+m) + l(2M+m)\} = mgl$$

$$(6M+5m)l\ddot{\varphi} + 2(M+m)\ddot{x}_1 = mg \quad (II^*) \Big| II^* - 2I \quad \Rightarrow \quad (2M+m)l\ddot{\varphi} - 2M\ddot{x}_1 = -mg \quad (II)$$

Lösung 3.33



$$L = T - U = \frac{1}{2} J_1 \dot{\varphi}^2 + \frac{1}{2}\left(m_1 + m_2\right)\dot{x}^2 - \left\{-\left(m_1 + m_2\right) g \sin\alpha \cdot x\right\}$$

$$W^* = -\mu_2 m_2 g \cos\alpha \cdot x - \mu_1 m_1 g \cos\alpha\left(x - r\varphi\right)$$

1. ZB: $x_1 = x_2 = x \quad \varphi = \dfrac{x}{r}$

$$L = \frac{1}{2}\dot{x}^2\left(m_1 + m_2 + \frac{J_1}{r^2}\right) + xg\left(m_1 + m_2\right)\sin\alpha$$

$$W^* = -\mu_2 m_2 g \cos\alpha \cdot x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = Q^* \qquad J_1 = \frac{1}{2} m_1 r^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \ddot{x}\left(\frac{3}{2} m_1 + m_2\right) \qquad \frac{\partial L}{\partial x} = \left(m_1 + m_2\right) g \sin\alpha \qquad Q^* = -\mu_2 m_2 g \cos\alpha$$

$$\ddot{x} = g\,\frac{\left(m_1 + m_2\right)\sin\alpha - \mu_2 m_2 \cos\alpha}{\left(\frac{3}{2} m_1 + m_2\right)}$$

Kräftegleichgewicht in Richtung der Schiefen Ebene für die Masse m_2:

$$F_S = m_2 g \sin\alpha - \mu_2 m_2 g \cos\alpha - m_2 \ddot{x} = m_2 g \frac{m_1}{3m_1 + 2m_2}\left(\sin\alpha - 3\mu_2 \cos\alpha\right)$$

2. ZB: $x_1 = x_2 = x \quad r\varphi \neq x$

$$L = \frac{1}{2} J_1 \dot{\varphi}^2 + \frac{1}{2}\left(m_1 + m_2\right)\dot{x}^2 + xg\left(m_1 + m_2\right)\sin\alpha$$

$$W^* = -\left(\mu_1 m_1 + \mu_2 m_2\right) g \cos\alpha \cdot x + \mu_1 m_1 g \cos\alpha \cdot r\varphi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = Q_1^* \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = Q_2^* \quad J_1 = \frac{1}{2} m_1 r^2$$

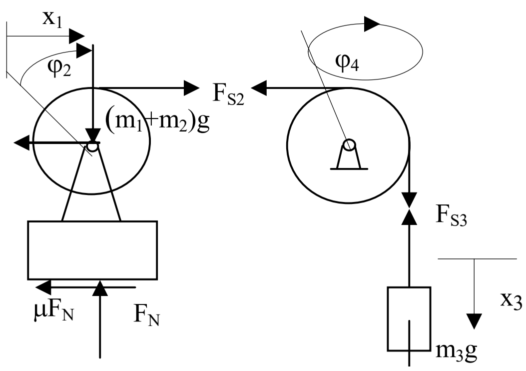
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \ddot{x}\left(m_1 + m_2\right) \qquad \frac{\partial L}{\partial x} = \left(m_1 + m_2\right) g \sin\alpha \qquad Q_1^* = -\left(\mu_1 m_1 + \mu_2 m_2\right) g \cos\alpha$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = J_1 \ddot{\varphi} = \frac{1}{2} m_1 r^2 \ddot{\varphi} \quad \frac{\partial L}{\partial \varphi} = 0 \quad Q_2^* = \mu_1 m_1 g r \cos\alpha$$

(1) $\ddot{x}\left(m_1 + m_2\right) - \left(m_1 + m_2\right) g \sin\alpha = -\left(\mu_1 m_1 + \mu_2 m_2\right) g \cos\alpha$

(2) $\dfrac{1}{2} m_1 r^2 \ddot{\varphi} = \mu_1 m_1 g r \cos\alpha \quad \Rightarrow \quad \ddot{\varphi} = \dfrac{2\mu_1 g \cos\alpha}{r}$

$$\ddot{x} = g\left\{\sin\alpha - \cos\alpha \cdot \frac{\left(\mu_1 m_1 + \mu_2 m_2\right)}{\left(m_1 + m_2\right)}\right\} \quad F_S = \left(\mu_1 - \mu_2\right) \cdot \frac{m_1 m_2 g \cos\alpha}{m_1 + m_2}$$



Lösung 3.34

Freie Koordinaten: x_1, φ_2, x_3, φ_4

Zwangsbedingungen:

$$x_3 = r\varphi_4, \quad x_3 = x_1 + r\varphi_2 \quad \Rightarrow$$
$$\varphi_4 = \frac{x_3}{r}, \quad \varphi_2 = \frac{1}{r}(x_3 - x_1)$$

D´Alembert:

$$Masse\,3: \qquad F_{S3} = m_3 g - m_3 \ddot{x}_3 \qquad (1)$$
$$Masse\,4: \qquad J_4\ddot{\varphi}_4 + F_{S2}r - F_{S3}r = 0 \qquad (2)$$
$$Masse\,2: \qquad J_2\ddot{\varphi}_2 - F_{S2}r = 0 \qquad (3)$$
$$Massen\,1\,u\,2: \quad F_{S2} = (m_1 + m_2)\ddot{x}_1 + \mu F_N \quad (4) \qquad F_N = (m_1 + m_2)g \quad (5)$$
$$(3),(5)\ in\ (4)\ und\ ZB:$$
$$\ddot{x}_1\left(m_1 + m_2 + \frac{J_2}{r^2}\right) - \frac{J_2}{r^2}\ddot{x}_3 + \mu(m_1 + m_2)g = 0 \quad (I)$$
$$(2)\ mit\ (1),(3)\ und\ ZB:$$
$$-\frac{J_2}{r^2}\ddot{x}_1 + \ddot{x}_3\left(m_3 + \frac{J_2}{r^2} + \frac{J_4}{r^2}\right) - m_3 g = 0 \qquad (II)$$

Lagrange:

$$L = T - U = \frac{1}{2}\left\{(m_1 + m_2)\dot{x}_1^2 + J_2 \cdot \frac{1}{r^2}(\dot{x}_3 - \dot{x}_1)^2 + J_4 \cdot \frac{1}{r^2}\dot{x}_3^2 + m_3\dot{x}_3^2\right\} - \{-m_3 g x_3\}$$
$$W^* = -\mu(m_1 + m_2)g x_1$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = Q_1^* \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) - \frac{\partial L}{\partial x_3} = Q_3^*$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \left(m_1 + m_2 + \frac{J_2}{r^2}\right)\ddot{x}_1 - \frac{J_2}{r^2}\ddot{x}_3 \qquad \frac{\partial L}{\partial x_1} = 0 \qquad Q_1^* = -\mu(m_1 + m_2)g$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = \left(m_3 + \frac{J_4}{r^2} + \frac{J_2}{r^2}\right)\ddot{x}_3 - \frac{J_2}{r^2}\ddot{x}_1 \qquad \frac{\partial L}{\partial x_3} = m_3 g \qquad Q_3^* = 0$$
$$\left(m_1 + m_2 + \frac{J_2}{r^2}\right)\ddot{x}_1 - \frac{J_2}{r^2}\ddot{x}_3 = -\mu(m_1 + m_2)g \quad (I)$$
$$-\frac{J_2}{r^2}\ddot{x}_1 + \left(m_3 + \frac{J_4}{r^2} + \frac{J_2}{r^2}\right)\ddot{x}_3 - m_3 g = 0 \qquad (II)$$

Lösung:

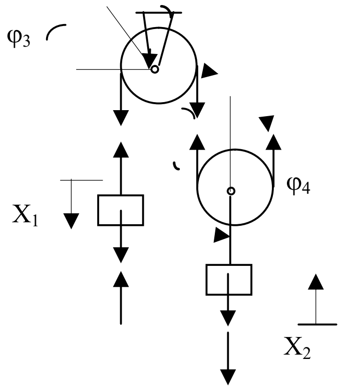
$$\frac{5}{2}m\ddot{x}_1 - \frac{1}{2}m\ddot{x}_3 = -2\mu mg \left| \cdot \frac{8}{m} \right. \qquad \Rightarrow \quad 20\ddot{x}_1 - 4\ddot{x}_3 = -16\mu g$$

$$-\frac{1}{2}m\ddot{x}_1 + 2m\ddot{x}_3 = mg \left| \cdot \frac{2}{m} \right. \qquad \Rightarrow \quad -\ddot{x}_1 + 4\ddot{x}_3 = 2g$$

$$add.: \quad 19\ddot{x}_1 = 2g(1-8\mu) \quad \ddot{x}_1 = \frac{2}{19}g(1-8\mu) = \frac{6}{95}g$$

$$\ddot{x}_3 = 5\ddot{x}_1 + 4\mu g = \frac{2}{19}g(5-2\mu) = \frac{49}{95}g$$

Lösung 3.35



Zwangsbedingungen: $\varphi_3 = \dfrac{x_1}{r} \quad \varphi_4 = \dfrac{x_1}{2r} \quad x_2 = \dfrac{x_1}{2}$

$$T = \frac{1}{2}\left\{m_1\dot{x}_1^2 + \left(m_2 + m_4\right)\dot{x}_2^2 + J_3\dot{\varphi}_3^2 + J_4\dot{\varphi}_4^2\right\}$$

$$U = -m_1 g x_1 + \left(m_2 + m_4\right) g x_2$$

$$L = T - U = \frac{1}{2}\dot{x}_1^2\left\{m_1 + \frac{1}{4}\left(m_2 + m_4\right) + \frac{J_3}{r^2} + \frac{J_4}{4r^2}\right\} + g x_1\left(m_1 - \frac{1}{2}\left(m_2 + m_4\right)\right)$$

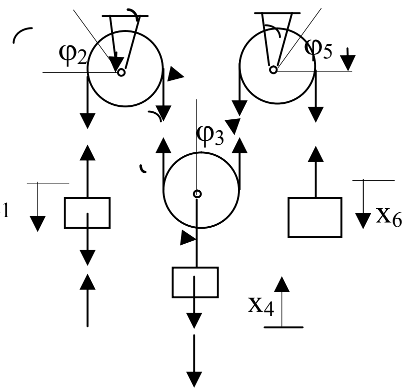
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = 0 \quad \Rightarrow \quad \ddot{x}_1\left\{m_1 + \frac{1}{4}\left(m_2 + m_4\right) + \frac{J_3}{r^2} + \frac{J_4}{4r^2}\right\} - g\left(m_1 - \frac{1}{2}\left(m_2 + m_4\right)\right) = 0$$

$$\ddot{x}_1 = \frac{g\left(m_1 - \frac{1}{2}\left(m_2 + m_4\right)\right)}{\left\{m_1 + \frac{1}{4}\left(m_2 + m_4\right) + \frac{J_3}{r^2} + \frac{J_4}{4r^2}\right\}}$$

$$F_{S1} = m_1 g - m_1\ddot{x}_1 = m_1\left(g - \ddot{x}_1\right) \quad F_{S2} = F_{S1} - J_3\frac{\ddot{\varphi}_3}{r} = m_1 g - \ddot{x}_1\left(m_1 + \frac{J_3}{r^2}\right)$$

$$F_{S3} = F_{S2} - J_4\frac{\ddot{\varphi}_4}{r} = m_1 g - \ddot{x}_1\left(m_1 + \frac{J_3}{r^2} + \frac{J_4}{2r^2}\right)$$

Lösung 3.36



ZB:

$$\varphi_2 = \frac{x_1}{r} \quad \varphi_5 = \frac{x_6}{r} \quad x_4 = \frac{1}{2}\left(x_1 + x_6\right)$$

$$\varphi_3 = \frac{1}{2r}\left(x_1 - x_6\right)$$

$$L = T - U$$

$$T = \frac{1}{2}\left\{m_1\dot{x}_1^2 + J_2\dot{\varphi}_2^2 + \left(m_3 + m_4\right)\dot{x}_4^2 + J_3\dot{\varphi}_3^2 + J_5\dot{\varphi}_5^2 + m_6\dot{x}_6^2\right\}$$

$$U = -m_1 g x_1 + \left(m_3 + m_4\right) g x_4 - m_6 g x_6$$

$$L=\frac{1}{2}\left\{2m\dot{x}_1^2+\frac{1}{2}m\dot{x}_1^2+2m\cdot\frac{1}{4}(\dot{x}_1+\dot{x}_6)^2+\frac{1}{2}m\cdot\frac{1}{4}(\dot{x}_1-\dot{x}_6)^2+\frac{1}{2}m\dot{x}_6^2+m\dot{x}_6^2\right\}+$$

$$2mgx_1+mgx_6-2mg\cdot\frac{1}{2}(x_1+x_6)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}_1}\right)-\frac{\partial L}{\partial x_1}=0\qquad\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}_6}\right)-\frac{\partial L}{\partial x_6}=0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}_1}\right)=\ddot{x}_1\left(2m+\frac{1}{2}m\right)+\frac{1}{2}m(\ddot{x}_1+\ddot{x}_6)+\frac{1}{8}m(\ddot{x}_1-\ddot{x}_6)=\frac{25}{8}m\ddot{x}_1+\frac{3}{8}m\ddot{x}_6\qquad\frac{\partial L}{\partial x_1}=2mg-mg$$

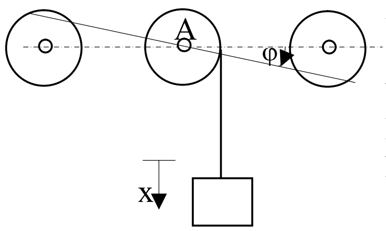
$$\frac{25}{8}m\ddot{x}_1+\frac{3}{8}m\ddot{x}_6-mg=0\qquad 25\ddot{x}_1+3\ddot{x}_6-8g=0\qquad(I)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}_6}\right)=\ddot{x}_1\left(\frac{1}{2}m-\frac{1}{8}m\right)+\ddot{x}_6\left(\frac{1}{2}m+\frac{1}{8}m+\frac{1}{2}m+m\right)=\frac{3}{8}m\ddot{x}_1+\frac{17}{8}m\ddot{x}_6\qquad\frac{\partial L}{\partial x_6}=mg-mg=0$$

$$\frac{3}{8}m\ddot{x}_1+\frac{17}{8}m\ddot{x}_6=0\qquad 3\ddot{x}_1+17\ddot{x}_6=0\qquad(II)\quad\Rightarrow\quad\ddot{x}_6=-\frac{3}{17}\ddot{x}_1\quad in\ (I)$$

$$25\ddot{x}_1-\frac{9}{17}\ddot{x}_1=8g\quad\Rightarrow\quad\ddot{x}_1=\frac{17}{52}g\qquad\Rightarrow\quad\ddot{x}_6=-\frac{3}{52}g$$

Lösung 3.37



Freie Koordinaten: x, φ

Zwangsbedingung: $\varphi=\frac{x}{a}$

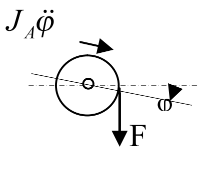
Fall 1) und 2) unterscheiden sich nur durch die Massenträgheitsmomente.

Massenträgheitsmomente Bezüglich A:

1) $J_A=\frac{m_0}{12}(6a)^2+\frac{m_1}{2}a^2+2m_2(3a)^2=a^2\left(3m_0+\frac{m_1}{2}+18m_2\right)$ $(m_2\ als\ Punktmasse)$

2) $J_A=\frac{m_0}{12}(6a)^2+\frac{m_1}{2}a^2+2\left[\frac{m_2}{2}a^2+m_2(3a)^2\right]=a^2\left(3m_0+\frac{m_1}{2}+19m_2\right)$

1. D'Alembert



$$J_A\ddot{\varphi}-F_Sa=0\qquad F_S+m_3\ddot{x}-m_3g=0$$

$$\ddot{x}=\frac{m_3g}{\left(m_3+\frac{J_A}{a^2}\right)}$$

2.Lagrange II: nur Potentialkräfte

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0 \qquad L=T-U \quad T=\frac{1}{2}\left(J_A\dot{\varphi}^2+m_3\dot{x}^2\right)=\frac{\dot{x}^2}{2}\left(\frac{J_A}{a^2}+m_3\right) \quad U=-m_3gx$$

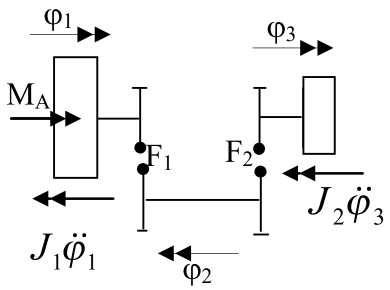
$$L=\frac{\dot{x}^2}{2}\left(\frac{J_A}{a^2}+m_3\right)+m_3gx \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=\ddot{x}\left(\frac{J_A}{a^2}+m_3\right) \qquad \frac{\partial L}{\partial x}=m_3g$$

$$\ddot{x}\left(\frac{J_A}{a^2}+m_3\right)-m_3g=0 \qquad \ddot{x}=\frac{m_3g}{\left(\frac{J_A}{a^2}+m_3\right)}$$

3. Zahlenwerte:

1) $\ddot{x} = \frac{2}{45} g = 0{,}436 \frac{m}{s^2}$ 2) $\ddot{x} = \frac{2}{47} g = 0{,}417 \frac{m}{s^2}$

Lösung 3.38



Freie Koordinaten: φ_1, φ_2, φ_3

ZB: $r_1\varphi_1 = r_2\varphi_2$ $r_3\varphi_2 = r_4\varphi_3$

gen. Koordinate: φ_3

$$\ddot{\varphi}_2 = \frac{r_4}{r_3}\ddot{\varphi}_3 \quad \ddot{\varphi}_1 = \frac{r_2}{r_1}\frac{r_4}{r_3}\ddot{\varphi}_3$$

$$F_1 r_1 + J_1\ddot{\varphi}_1 - M_A = 0 \qquad F_1 r_2 - F_2 r_3 = 0 \qquad J_2\ddot{\varphi}_3 - F_2 r_4 = 0 \quad \Rightarrow \quad F_2 = \frac{J_2\ddot{\varphi}_3}{r_4}$$

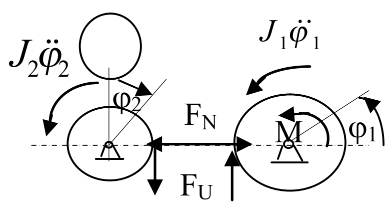
$$F_1 = \frac{r_3}{r_2}F_2 = \frac{r_3}{r_2}\frac{J_2\ddot{\varphi}_3}{r_4} \qquad J_1\frac{r_2}{r_1}\frac{r_4}{r_3}\ddot{\varphi}_3 - M_A + \frac{r_3}{r_2}\frac{r_1}{r_4}J_2\ddot{\varphi}_3 = 0 \quad \Rightarrow \quad \ddot{\varphi}_3 = \frac{M_A}{J_1\frac{r_2}{r_1}\frac{r_4}{r_3} + \frac{r_3}{r_2}\frac{r_1}{r_4}J_2} = K$$

$$\ddot{\varphi}_3 = K \qquad \dot{\varphi}_3 = Kt + C \qquad AB: \; t = 0 \;\; \dot{\varphi}_3 = 0 \;\; \Rightarrow \;\; C = 0, \;\; d.h. \;\; \dot{\varphi}_3 = Kt$$

$$t = t_0: \;\; \dot{\varphi}_3 = \omega_3 \;\; \Rightarrow \;\; \omega_3 = K \cdot t_0 \quad \omega_3 = \frac{M_A \cdot t_0}{J_1\frac{r_2}{r_1}\frac{r_4}{r_3} + \frac{r_3}{r_2}\frac{r_1}{r_4}J_2} \qquad \omega_3 = 2\pi n_3$$

$$M_A = \frac{2\pi n_3}{t_0}\left(J_1\frac{r_2}{r_1}\frac{r_4}{r_3} + \frac{r_3}{r_2}\frac{r_1}{r_4}J_2\right) = 663{,}2\,Nm \qquad P_{\max} = M_A \cdot \omega_1 = M_A\frac{r_2}{r_1}\frac{r_4}{r_3}\omega_3 = 41{,}67\;kW$$

Lösung 3.39



$$r_1\dot{\varphi}_1 = r_2\dot{\varphi}_2 \quad \Rightarrow \quad \ddot{\varphi}_1 = \frac{r_2}{r_1}\ddot{\varphi}_2$$

$$J_1\ddot{\varphi}_1 + F_U r_1 - M = 0$$

$$J_2\ddot{\varphi}_2 - F_U r_2 \qquad = 0 \quad \Rightarrow \quad F_U = \frac{J_2}{r_2}\ddot{\varphi}_2$$

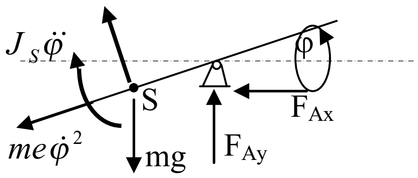
$$\ddot{\varphi}_2\left(J_1\frac{r_2}{r_1} + J_2\frac{r_1}{r_2}\right) - M = 0 \quad \ddot{\varphi}_2 = \frac{M}{\left(J_1\frac{r_2}{r_1} + J_2\frac{r_1}{r_2}\right)} = K$$

$$\ddot{\varphi}_2 = K \qquad \dot{\varphi}_2 = Kt + C \quad \dot{\varphi}_2(0) = 0 \quad \Rightarrow \quad C = 0 \qquad \dot{\varphi}_2 = Kt$$

$$t = t_1: \qquad \dot{\varphi}_2 = \omega_2 \qquad \omega_2 = Kt_1 \quad \Rightarrow \quad M = \frac{\omega_2}{t_1}\left(J_1\frac{r_2}{r_1} + J_2\frac{r_1}{r_2}\right) \qquad \omega_2 = 2\pi n_2$$

$$J_1 = \frac{m_1}{2}r_1^2 = \frac{r_1^4}{2}\pi\rho b_1 \qquad J_2 = \frac{m_2}{2}r_2^2 = \frac{r_2^4}{2}\pi\rho b_2 \qquad M = \frac{\pi^2 n_2\rho r_1 r_2}{t_1}\left(r_1^2 b_1 + r_2^2 b_2\right) = 4{,}6\,Nm$$

Lösung 3.40



$$m = m_1 + m_2$$

$$e = \frac{-m_2\frac{l_3}{2} + m_1\frac{l_2}{2}}{m} = \frac{m_1 l_2 - m_2 l_3}{2(m_1 + m_2)}$$

$$A: \quad J_S\ddot{\varphi} - mge\cos\varphi + me^2\ddot{\varphi} = 0 \qquad J_A = J_S + me^2$$

$$\ddot{\varphi} = \frac{mge\cos\varphi}{J_A} = \frac{d\dot{\varphi}}{d\varphi}\dot{\varphi}$$

$$\leftarrow: \quad me\ddot{\varphi}\sin\varphi + F_{Ax} + me\dot{\varphi}^2\cos\varphi = 0 \quad \Rightarrow \quad F_{Ax} = -me\left[\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi\right]$$

$$\frac{1}{2}\dot{\varphi}^2 = \frac{mge}{J_A}\int_0^{\varphi}\cos\varphi^* d\varphi^* = \frac{mge}{J_A}\sin\varphi \qquad mit \quad \dot{\varphi} = 0 \quad für \quad \varphi = 0 \quad \dot{\varphi}^2 = 2\frac{mge}{J_A}\sin\varphi$$

$$F_{Ax} = -3mg\frac{me^2}{J_A}\sin\varphi\cos\varphi$$

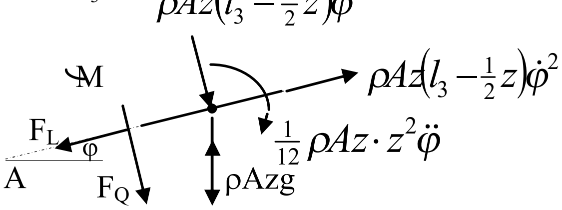
$$\uparrow: \quad F_{Ay} - mg + me\ddot{\varphi}\cos\varphi - me\dot{\varphi}^2\sin\varphi = 0 \quad \Rightarrow \quad F_{Ay} = mg + me\left[\dot{\varphi}^2\sin\varphi - \ddot{\varphi}\cos\varphi\right]$$

$$F_{Ay} = mg\left\{1 + \frac{me^2}{J_A}\left(2\sin^2\varphi - \cos^2\varphi\right)\right\}$$

$$\varphi = 0: \qquad F_{Ax} = 0 \qquad F_{Ay} = mg\left[1 - \frac{me^2}{J_A}\right] \qquad J_A = \frac{1}{12}m_2 l_3^2 + m_2\frac{l_3^2}{4} + \frac{1}{12}m_1\left(l_1^2 + l_2^2\right) + m_1\frac{l_2^2}{4}$$

Ergänzung: Schnittgrößen im dünnen Stab

$0 \le z \le l_3$



$$F_L = \rho Az\left\{\left(l_3 - \frac{1}{2}z\right)\dot{\varphi}^2 - g\sin\varphi\right\}$$

$$F_L = \rho Azg\left\{2\frac{me\left(l_3 - \frac{1}{2}z\right)}{J_A} - 1\right\}\sin\varphi$$

$$F_Q = -\rho Az\left\{\left(l_3 - \frac{1}{2}z\right)\ddot{\varphi} + g\cos\varphi\right\}$$

$$\varphi = 0 \quad \Rightarrow \quad F_L = 0$$

$$\varphi = 0 \quad \Rightarrow \quad M = -\frac{1}{2} m_2 g \frac{z^2}{l_3} \left\{ 1 + \frac{me}{J_A} \left(l_3 - \frac{1}{3} z \right) \right\} = -\frac{1}{2} m_2 g \frac{z^2}{l_3} \left\{ 1 + \frac{3\left(l_3 - \frac{1}{3} z \right) (m_1 l_2 - m_2 l_3)}{2\left[m_1 \left(l_2^2 + \frac{l_1^2}{4} \right) + m_2 l_3^2 \right]} \right\}$$

$$M(z = l_3) = -\frac{1}{2} m_2 g l_3 \left\{ 1 + \frac{l_3 (m_1 l_2 - m_2 l_3)}{m_1 \left(l_2^2 + \frac{l_1^2}{4} \right) + m_2 l_3^2} \right\}$$

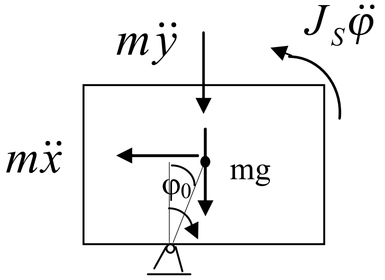
$$F_Q = -\rho g A z \left\{ 1 + \frac{me\left(l_3 - \frac{z}{2} \right)}{J_A} \right\} \cos\varphi \qquad \varphi = 0 \quad \Rightarrow \quad F_Q = -\rho g A z \left\{ 1 + \frac{me\left(l_3 - \frac{z}{2} \right)}{J_A} \right\}$$

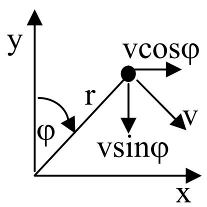
$$mit \quad m_2 = \rho A l_3 \quad und \quad \rho A = \frac{m_2}{l_3} \quad \Rightarrow \quad F_Q = -m_2 g \frac{z}{l_3} \left\{ 1 + \frac{me\left(l_3 - \frac{z}{2} \right)}{J_A} \right\}$$

$$M = -\frac{1}{2} \rho A z^2 \left\{ \frac{1}{6} z \ddot{\varphi} + \left(l_3 - \frac{1}{2} z \right) \ddot{\varphi} + g \cos\varphi \right\} = -\frac{1}{2} m_2 g \frac{z^2}{l_3} \cos\varphi \left\{ 1 + \frac{me}{J_A} \left(l_3 - \frac{1}{3} z \right) \right\}$$

$$M(z) = -\frac{1}{2} q z^2 = -\frac{1}{2} m_2 g \frac{z^2}{l_3}$$

Lösung 3.41





$$r = \sqrt{a^2 + \frac{a^2}{4}} = \frac{a}{2}\sqrt{5}$$

$$v = r\dot{\varphi} \quad \Rightarrow \quad \dot{x} = r\dot{\varphi}\cos\varphi \quad \dot{y} = -r\dot{\varphi}\sin\varphi$$

$$\tan\varphi_0 = \frac{a}{2a} \quad \sin\varphi_0 = \frac{a}{2r} \quad \cos\varphi_0 = \frac{a}{r}$$

$$F_0 = \sqrt{F_{0x}^2 + F_{0x}^2} = 0{,}918\ mg$$

$$\dot{\varphi}\big|_{t=0} = 0$$

$$\ddot{x} = r\ddot{\varphi}\cos\varphi - r\dot{\varphi}^2\sin\varphi \qquad \ddot{y} = -r\ddot{\varphi}\sin\varphi - r\dot{\varphi}^2\cos\varphi \qquad J_S = \frac{m}{12}\left(9a^2 + 4a^2\right) = \frac{13}{12}ma^2$$

$\varphi = \varphi_0$:

$$GGW: \quad \uparrow \quad F_{0y} = mg + m\ddot{y} \qquad \rightarrow \quad F_{0x} = m\ddot{x} \qquad 0: \quad J_S\ddot{\varphi} - mg\frac{a}{2} + m\ddot{x}a - m\ddot{y}\frac{a}{2} = 0$$

$$\frac{13}{12}ma^2\ddot{\varphi} - mg\frac{a}{2} + mar\ddot{\varphi}\cos\varphi_0 + m\frac{a}{2}r\ddot{\varphi}\sin\varphi_0 = 0$$

$$\ddot{\varphi} = \frac{g}{r\sin\varphi_0 + 2r\cos\varphi_0 + \frac{13}{6}a} = \frac{6g}{28a}$$

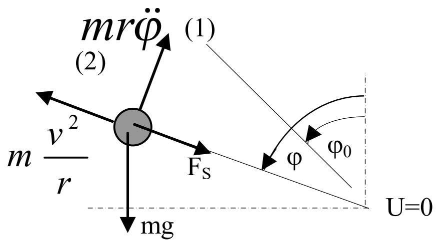
$$F_{0x} = mr\ddot{\varphi}\cos\varphi_0 = mr\cdot\frac{6g}{28a}\cdot\frac{a}{r} = \frac{3}{14}mg \qquad F_{0y} = mg - mr\ddot{\varphi}\sin\varphi_0 = mg - \frac{3}{28}mg = \frac{25}{28}mg$$

Lösung 4.1

$$W = \int \vec{F}d\vec{r} = \int F\cos\alpha|d\vec{r}| = F_S\cos\alpha\cdot s = 10\cdot\frac{1}{2}\sqrt{3}\cdot 3\cdot 10^3\,\mathrm{kNm} = 25980{,}76\,\mathrm{kJ}$$

$$P = \vec{F}\vec{v} = F_S\cos\alpha\cdot v = 10\cdot\frac{1}{2}\sqrt{3}\cdot\frac{9\cdot 10^3}{3{,}6\cdot 10^3}\,\mathrm{kN}\frac{\mathrm{m}}{\mathrm{s}} = 21{,}65\,\mathrm{kW}$$

Lösung 4.2



$$U_1 + T_1 = U_2 + T_2$$

$$U_1 = mgr\cos\varphi_0 \qquad T_1 = 0$$

$$U_2 = mgr\cos\varphi \qquad T_2 = \frac{1}{2}mv^2(\varphi)$$

$$mgr\cos\varphi_0 = \frac{1}{2}mv^2(\varphi) + mgr\cos\varphi$$

$$v(\varphi) = \sqrt{2gr(\cos\varphi_0 - \cos\varphi)}$$

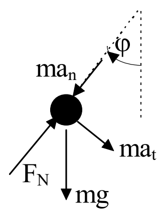
$$v(\pi) = \sqrt{2gr(\cos\varphi_0 + 1)} = 6{,}05\frac{m}{s}$$

$$F_S(\varphi) = \frac{m}{r}v^2(\varphi) - mg\cos\varphi = mg(2\cos\varphi_0 - 3\cos\varphi) \qquad F_S(\pi) = mg(\sqrt{3} +$$

Lösung 4.3

$a.)\quad T_1 + U_1 = T_2 + U_2$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_2^2 + mgl(1+\cos\alpha) \qquad v_0^2 = v_2^2 + 2gl(1+\cos 60°) = v_2^2 + 3gl$$



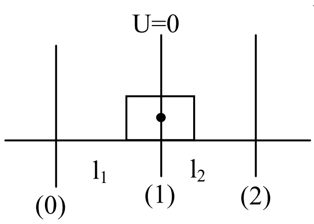
$$:\quad F_N - ma_n - mg\cos\varphi = 0 \qquad a_n = \frac{v^2}{l}$$

$$\text{für } \varphi = \pi \text{ ist} \quad F_N = 0, \quad d.h. \quad \frac{v_2^2}{l} = g \qquad v_2^2 = gl$$

$$\text{und damit} \quad v_0^2 = gl + 3gl = 4gl \qquad \boxed{v_0 = 2\sqrt{gl}}$$

$$b.)\quad v_2 = 0 \qquad \text{Energiesatz:} \quad v_0^2 = 3gl \qquad \boxed{v_0 = \sqrt{3gl} \quad \text{mit} \quad F_N = -mg}$$

Lösung 4.4



Energiebilanzen:

1. $\frac{1}{2}c_1l_1^2 - \mu mgl_1 = \frac{1}{2}mv_1^2 \quad\Rightarrow\quad v_1 = \sqrt{\frac{c_1}{m}l_1^2 - 2\mu gl_1} = 89{,}7\frac{cm}{s}$

2. $\frac{1}{2}c_1l_1^2 - \mu mg(l_1 + l_2) = \frac{1}{2}c_2l_2^2$

$$l_2^2 + 2mg\mu\frac{l_2}{c_2} + 2mg\mu\frac{l_1}{c_2} - \frac{c_1}{c_2}l_1^2 = 0$$

$$l_2 = \frac{\mu m g}{c_2}\left[\pm\sqrt{1-\frac{2c_2 l_1}{\mu m g}+c_1 c_2\left(\frac{l_1}{\mu m g}\right)^2}-1\right] \qquad l_2 = 5{,}29\,cm$$

3. $\frac{1}{2}c_1 l_1^2 = \frac{1}{2}c_2 l_2^{*2} \quad \Rightarrow \quad l_2^* = l_1\sqrt{\frac{c_1}{c_2}} = 6{,}33\,cm$

Lösung 4.5

Impulserhaltung: $m_1 v_1 = m_2 v_2 \quad \Rightarrow \quad v_2 = \frac{m_1}{m_2}v_1$

Energieerhaltung: $\frac{1}{2}cw^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad \Rightarrow \quad w = v_1\sqrt{\frac{m_1}{c}\left(1+\frac{m_1}{m_2}\right)}$

Lösung 4.6

1. Energiebilanz (Ausgangslage und horizontale Lage (mit U=0):

$$3ag\left(M+\frac{m}{2}\right)\sin\varphi_0 = \frac{1}{2}J\dot{\varphi}_1^2 \qquad J = M(3a)^2 + \frac{1}{12}m(3a)^2 + m\left(\frac{3}{2}a\right)^2 = (3a)^2\left(M+\frac{m}{3}\right)$$

$$\dot{\varphi}_1 = \sqrt{\frac{2}{3}\frac{g}{a}\frac{\left(M+\frac{m}{2}\right)}{\left(M+\frac{m}{3}\right)}\sin\varphi_0}$$

2. Energiebilanz (Ausgangslage und Endlage):
für kleine Winkel φ_2 gilt für die Federwege $x_1 = a\varphi_2$ und $x_2 = 2a\varphi_2$ und $\sin\varphi_2 \approx \varphi_2 << \sin\varphi_0$

$$3ag\left(M+\frac{m}{2}\right)\sin\varphi_0 = \frac{1}{2}c_1 x_1^2 + \frac{1}{2}c_2 x_2^2 - 3ag\left(M+\frac{m}{2}\right)\sin\varphi_2$$

$$3ag\left(M+\frac{m}{2}\right)(\sin\varphi_0+\varphi_2) = \frac{1}{2}a^2\varphi_2^2(c_1+4c_2) \quad \Rightarrow \quad \varphi_2 \approx \sqrt{\frac{6g}{a}\frac{\left(M+\frac{m}{2}\right)}{(c_1+4c_2)}\cdot\sin\varphi_0}$$

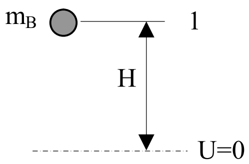
Lösung 4.7

Energiebilanz: Ausgangslage (U=0) – vertikale Lage – Lage mit maximaler Federzusammendrückung

$$0 = \frac{1}{2}J_A\dot{\varphi}_1^2 - mgl_1 = \frac{1}{2}c(l_2\varphi_{\max})^2 - mgl_1\cos\varphi_{\max}$$

$$0 = \frac{1}{2}ml_1^2\omega_1^2 - mgl_1 \quad \Rightarrow \quad \omega_1 = \sqrt{\frac{2g}{l_1}} \qquad \cos\varphi_{\max} \approx 1 \qquad \varphi_{\max} = \sqrt{\frac{2mgl_1}{cl_2^2}}$$

Lösung 4.8
Fallhöhe aus Vergleich der Energien ermittelt:
Die kinetische Energie des Beiles beim Auftreffen auf den Boden wird mit der kinetischen Energie des Rasenmähers gleichgesetzt.



$$T_{0B} = T_{0RM} \qquad T_{0RM} = \frac{1}{2} J_{RM} \omega^2 \qquad \omega = 2\pi n \qquad J_{RM} = J_{Anker} + J_{Messer}$$

$$J_{Messer} = \frac{1}{12} m_M l^2 \qquad J_{Anker} = \frac{1}{2} m_A \left(\frac{D}{2}\right)^2 \qquad m_M = \rho_{Fe} blh$$

$$m_A = \rho_{Fe} \frac{\pi D^2}{4} \cdot L$$

Für das Beil gilt:

$$T_1 + U_1 = T_0 + U_0 \qquad T_1 = 0 \quad U_1 = m_B g H \quad T_0 = T_{0B} = \frac{1}{2} m_B v_0^2 \quad U_0 = 0 \quad \frac{1}{2} m_B v_0^2 = m_B g H$$

$$m_B g H = \frac{1}{2} J_{RM} (2\pi n)^2 \quad \Rightarrow \quad H = \frac{2\pi^2 n^2}{m_B g} \cdot J_{RM}$$

Zahlenwerte: $m_M = 283\,g \quad m_A = 4{,}74 \cdot 10^3\,g \quad J_{RM} = 6{,}85 \cdot 10^4\,gcm^2 \quad \Rightarrow \quad H = 0{,}689\,m$

Lösung 4.9

$$T_I + U_I = T_{II} + U_{II} \quad T_I = 0 \quad U_I = \frac{1}{2} c (b - l_0)^2 \quad T_{II} = \frac{1}{2} m v_{II}^2 + \frac{1}{2} J \dot{\varphi}_{II}^2 \quad U_{II} = mgh + \frac{1}{2} c (h - l_0)^2$$

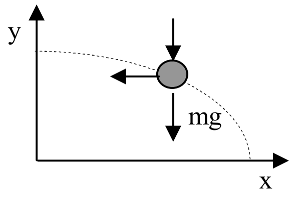
$$h = b \tan\alpha \qquad \varphi_{II} = \frac{v_{II}}{r}$$

$$\frac{1}{2} c (b - l_0)^2 = mgb \tan\alpha + \frac{1}{2} c (b \tan\alpha - l_0)^2 + \frac{1}{2} m v_{II}^2 + \frac{1}{2} J \frac{v_{II}^2}{r^2}$$

$$v_{II} = \sqrt{\frac{c(b - l_0)^2 - 2mgb \tan\alpha - c(b \tan\alpha - l_0)^2}{m + \frac{J}{r^2}}}$$

$$v_{II} = 0 \quad \Rightarrow \quad c^*(b - l_0)^2 - 2mgb \tan\alpha - c^*(b \tan\alpha - l_0)^2 = 0 \quad c^* = \frac{2mgb \tan\alpha}{(b - l_0)^2 - (b \tan\alpha - l_0)^2}$$

Lösung 4.10



Energiebilanz im Rohr: $\frac{1}{2} c l^2 = \frac{1}{2} m v_0^2 \quad \Rightarrow \quad v_0 = l \sqrt{\frac{c}{m}}$

$$\ddot{x} = 0 \qquad \dot{x} = C_1 \qquad x = C_1 t + C_2$$

$$\ddot{y} = -g \qquad \dot{y} = -gt + C_3 \qquad y = -\frac{1}{2} g t^2 + C_3 t + C_4$$

$$AB: t = 0 \quad x = 0 \quad \Rightarrow \quad C_2 = 0 \quad \dot{x} = v_0 \quad \Rightarrow \quad C_1 = v_0$$

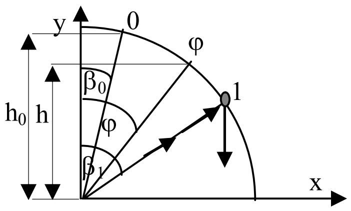
$$y = h \quad \Rightarrow \quad C_4 = h \quad \dot{y} = 0 \quad \Rightarrow \quad C_3 = 0$$

EB:

$$t = t_E \quad x = w \quad y = 0 \quad \Rightarrow \quad w = v_0 t_E \quad t_E = \frac{w}{v_0} \quad \frac{1}{2} g \left(\frac{w}{v_0} \right)^2 - h = 0 \quad l = \sqrt{\frac{w^2 mg}{2ch}}$$

Zahlenwerte: l = 9,9 cm; v_0 = 8,09 m/s

Lösung 4.11



Energiesatz:

$$U_0 + T_0 = U_\varphi + T_\varphi$$

$$U_0 = mgh_0 = mgr\cos\beta_0 \quad T_0 = \frac{1}{2}mv_0^2$$

$$U_\varphi = mgh = mgr\cos\varphi \quad T_\varphi = \frac{1}{2}mv_\varphi^2$$

$$mgr\cos\beta_0 + \frac{1}{2}mv_0^2 = mgr\cos\varphi + \frac{1}{2}mv_\varphi^2$$

$$v(\varphi) = \sqrt{2gr(\cos\beta_0 - \cos\varphi) + v_0^2}$$

Kräftegleichgewicht im Punkt 1 in Radiusrichtung:

$F_N = 0 \quad \textit{für} \quad \varphi = \beta_1 \quad \Rightarrow$

$$0 = mg\cos\beta_1 - m\frac{v^2(\beta_1)}{r} = mg\cos\beta_1 - \frac{m}{r}\left[2gr(\cos\beta_0 - \cos\beta_1) + v_0^2\right]$$

$$\cos\beta_1 = \frac{2}{3}\cos\beta_0 + \frac{v_0^2}{3gr}$$

Zahlenwerte:

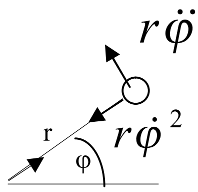
$v_0 = 0\frac{m}{s}: \quad \beta_0 = 0° \quad \Rightarrow \quad \beta_1 = 48{,}19°; \qquad \beta_0 = 15° \quad \Rightarrow \quad \beta_1 = 49{,}91°;$

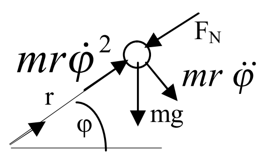
$v_0 = 2\frac{m}{s}: \quad \beta_0 = 0° \quad \Rightarrow \quad \beta_1 = 36{,}62°; \qquad \beta_0 = 15° \quad \Rightarrow \quad \beta_1 = 38{,}75°;$

Lösung 4.12

Bewegung auf der Kreisbahn: r = konst.

Ablösen von der Bahn



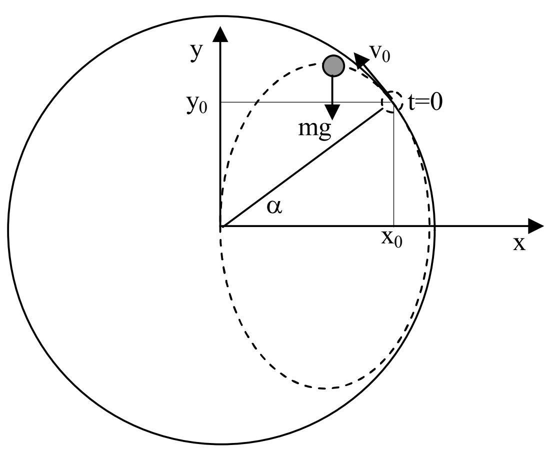


$mr\dot{\varphi}^2 - mg\sin\varphi - F_N = 0$

für $\varphi = \alpha \quad \dot{\varphi} = \omega \quad F_N = 0$

$$mr\omega^2 = mg\sin\alpha \qquad \omega^2 = \frac{v_0^2}{r^2} \quad v_0^2 = rg\sin\alpha$$

Schiefer Wurf:



$$\ddot{x} = 0 \qquad \dot{x} = C_1 \qquad x = C_1 t + C_2$$

$$\ddot{y} = -g \quad \dot{y} = -gt + C_3 \quad y = -\frac{1}{2}gt^2 + C_3 t + C_4$$

$$\begin{aligned} AB: \quad x(0) &= r\cos\alpha &\Rightarrow\quad C_2 &= r\cos\alpha \\ y(0) &= r\sin\alpha &\Rightarrow\quad C_4 &= r\sin\alpha \\ \dot{x}(0) &= -v_0\sin\alpha &\Rightarrow\quad C_1 &= -v_0\sin\alpha \\ \dot{y}(0) &= v_0\cos\alpha &\Rightarrow\quad C_3 &= v_0\cos\alpha \end{aligned}$$

$$x(t) = -v_0 \sin\alpha \cdot t + r\cos\alpha \qquad y(t) = -\frac{1}{2}gt^2 + v_0\cos\alpha \cdot t + r\sin\alpha$$

Bahnkurve (Zeit eliminieren):

$$t = \frac{r\cos\alpha - x}{v_0 \sin\alpha} \quad \Rightarrow \quad y(x,\alpha) = -\frac{1}{2}g\left(\frac{r\cos\alpha - x}{v_0\sin\alpha}\right)^2 + v_0\cos\alpha \cdot \frac{r\cos\alpha - x}{v_0\sin\alpha} + r\sin\alpha$$

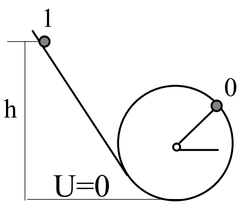
$$y(x,\alpha) = r\sin\alpha + \frac{\cos\alpha}{\sin\alpha}(r\cos\alpha - x) - \frac{1}{2}\frac{(r\cos\alpha - x)^2}{r\sin^3\alpha}$$

$$\mathit{Bedingung}: \quad y(0,\alpha) = 0 \quad \Rightarrow \quad 0 = r\sin\alpha + r\frac{\cos^2\alpha}{\sin\alpha} - \frac{1}{2}r\frac{\cos^2\alpha}{\sin^3\alpha}\bigg| \cdot \frac{\sin^3\alpha}{r}$$

$$\sin^2\alpha(\sin^2\alpha + \cos^2\alpha) - \frac{1}{2}\cos^2\alpha = \sin^2\alpha - \frac{1}{2}(1 - \sin^2\alpha) = \frac{3}{2}\sin^2\alpha - \frac{1}{2} = 0$$

$$\sin\alpha = \sqrt{\frac{1}{3}} \quad \Rightarrow \quad \alpha = 35{,}3° \quad \mathit{und} \quad v_0^2 = rg \cdot \sqrt{\frac{1}{3}}$$

Energiesatz:



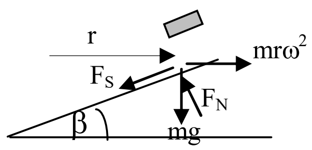
$$T_1 + U_1 = T_0 + U_0$$

$$0 + mgh = \frac{1}{2}mv_0^2 + mgr(1 + \sin\alpha)$$

$$h = r(1 + \sin\alpha) + \frac{1}{2}\frac{v_0^2}{g} = r\left(1 + \sqrt{\frac{1}{3}}\right) + \frac{1}{2}r\sqrt{\frac{1}{3}} = r\left(1 + \frac{1}{2}\sqrt{3}\right)$$

Lösung 4.13

Bewegung auf der schiefen Ebene: $mgl\sin\alpha = \frac{1}{2}mv_1^2 \quad \Rightarrow \quad v_1 = \sqrt{2gl\sin\alpha}$



$$\dot{\varphi} = \omega = \frac{v_1}{r}$$

$$\rightarrow: \quad mr\omega^2 - F_S\cos\beta - F_N\sin\beta = 0$$

$$\downarrow: \quad mg + F_S\sin\beta - F_N\cos\beta = 0 \qquad F_N = \frac{mg + F_S\sin\beta}{\cos\beta}$$

$$mr\omega^2 - F_S\cos\beta - \frac{mg + F_S\sin\beta}{\cos\beta} \cdot \sin\beta = 0$$

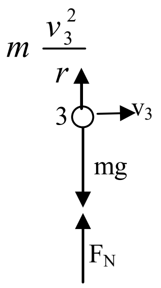
$$F_S = 0 \quad \mathit{für} \quad \beta = \beta^* \qquad \tan\beta^* = \frac{r\omega^2}{g} \quad \mathit{Überhöhung}: h = w\sin\beta^* = w\frac{\tan\beta^*}{\sqrt{1 + \tan^2\beta^*}}$$

$$h = \frac{w}{\sqrt{1 + \left(\frac{r}{2l\sin\alpha}\right)^2}} = 20{,}6\ cm$$

Lösung 4.14

$$T_1 + U_1 = T_2 + U_2 \quad \Rightarrow \quad mgh_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 \qquad v_2 = \sqrt{2gh_1 + v_1^2}$$

$$T_1 + U_1 = T_3 + U_3 \quad \Rightarrow \quad mgh_1 + \frac{1}{2}mv_1^2 = mgh_3 + \frac{1}{2}mv_3^2 \qquad v_3 = \sqrt{2g(h_1 - h_3) + v_1^2}$$

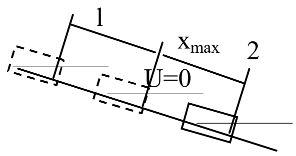


$$\uparrow: \quad F_N - mg + m\frac{v_3^2}{r} = 0 \qquad F_N = mg - m\frac{v_3^2}{r}$$

$$F_N = 0 \quad \textit{für} \quad r = r^* \quad \Rightarrow \quad r^* = \frac{v_3^2}{g}$$

$$r > r^* = 2(h_1 - h_3) + \frac{v_1^2}{g} \qquad v_2 = 6{,}95\frac{m}{s} \qquad r > 2{,}917m$$

Lösung 4.15



$$\frac{1}{2}mv_1^2 + mgl\sin\alpha = \frac{1}{2}cx_{\max}^2 - mgx_{\max}\sin\alpha$$

$$x_{\max}^2 - 2\frac{mg}{c}\sin\alpha \cdot x_{\max} - 2\frac{mgl}{c}\sin\alpha - \frac{m}{c}v_1^2 = 0$$

$$x_{\max}^2 - 1cm \cdot x_{\max} - 190cm^2 = 0 \qquad x_{\max} = 14{,}3cm$$

Lösung 4.16

0 – Ausgangszustand (U=0) 1 – Endzustand $m_S = ql$ Schwerpunkt des Seiles bei $\frac{l}{2}$.

$$T_0 + U_0 = T_1 + U_1$$

$$T_0 = 0 \quad U_0 = 0 \quad T_1 = \frac{1}{2}J_{ges}\dot{\varphi}_1^2 \quad \textit{mit} \quad J_{ges} = J + m_S\left(\frac{d}{2}\right)^2 \qquad U_1 = -m_S g\frac{l}{2}$$

$$0 = \frac{1}{2}\left(J + m_S\left(\frac{d}{2}\right)^2\right)\dot{\varphi}_1^2 - m_S g\frac{l}{2} \quad \Rightarrow \quad \dot{\varphi}_1 = \sqrt{\frac{m_S gl}{J + m_S\left(\frac{d}{2}\right)^2}} \qquad n = \frac{\dot{\varphi}_1}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{m_S gl}{J + m_S\left(\frac{d}{2}\right)^2}}$$

Lösung 4.17

0 – horizontale Lage mit U=0 1 – vertikale Lage

$$T_0 + U_0 = T_1 + U_1$$

$$T_0 = 0 \quad U_0 = 0 \quad T_1 = \frac{1}{2}J_A\dot{\varphi}_1^2 \quad \textit{mit} \quad J_A = m_1l_1^2 + m_2l_2^2 \quad U_1 = m_1gl_1 - m_2gl_2$$

$$\frac{1}{2}J_A\dot{\varphi}_1^2 + g(m_1l_1 - m_2l_2) = 0 \quad \Rightarrow \quad \dot{\varphi}_1^2 = \frac{v_2^2}{l_2^2} = \frac{2g(m_2l_2 - m_1l_1)}{J_A}$$

$$v_2 = \sqrt{\frac{2l_2^2 g(m_2l_2 - m_1l_1)}{m_1l_1^2 + m_2l_2^2}} \quad \textit{für} \quad m_2l_2 > m_1l_1 \qquad v_2 = \sqrt{\frac{2l_2^2 g(m_1l_1 - m_2l_2)}{m_1l_1^2 + m_2l_2^2}} \quad \textit{für} \quad m_1l_1 > m_2l_2$$

Lösung 4.18

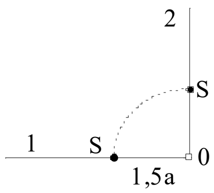
$$T_1 + U_1 = T_2 + U_2$$

$$0 + \frac{1}{2}c(l_1 - l_F)^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}c(l_2 - l_F)^2 - mgl \qquad l_2 = \sqrt{l_1^2 + l^2} = 25\mathrm{cm}$$

$$v_2^2 = \frac{c}{m}\left[(l_1 - l_F)^2 - (l_2 - l_F)^2\right] + 2gl$$

$$v_2 = \sqrt{2gl - \frac{c}{m}\left[(l_2 - l_F)^2 - (l_1 - l_F)^2\right]} = 148\frac{\mathrm{cm}}{\mathrm{s}} = 1{,}48\frac{\mathrm{m}}{\mathrm{s}}$$

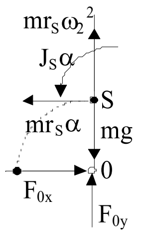
Lösung 4.19



$$T_1 + U_1 = T_2 + U_2 \qquad J_S = \frac{1}{12}m(5a)^2$$

$$0 + \frac{1}{2}cb^2 = \frac{1}{2}J_0\omega_2^2 + 1{,}5mga \quad J_0 = J_S + m(1{,}5a)^2 = \frac{13}{3}ma^2$$

$$\frac{1}{2}cb^2 = \frac{13}{6}ma^2\omega_2^2 + 1{,}5mga \quad \omega_2 = \sqrt{\frac{3}{13}\frac{c}{m}\left(\frac{b}{a}\right)^2 - \frac{9}{13}\frac{g}{a}} = 5{,}72\frac{1}{s}$$



$$0: \quad J_S\alpha + mr_S^2\alpha = 0 \qquad \alpha\left(J_S + mr_S^2\right) = 0 \quad \alpha = 0$$

$$\rightarrow: \quad F_{0x} - mr_S\alpha = 0 \qquad F_{0x} = 0$$

$$\uparrow: \quad F_{0y} - mg + mr_S\omega_2^2 = 0 \quad F_{0y} = m\left(g - r_S\omega_2^2\right) = 73{,}88\mathrm{N}$$

Lösung 4.20

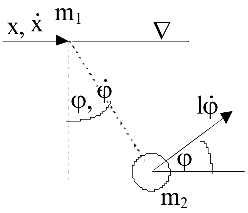
$$L = T - U = \frac{1}{2}J_R\dot{\varphi}^2 + \frac{1}{2}m\dot{x}^2 + 6\left(\frac{1}{2}mv_Z^2 + \frac{1}{2}J_Z\dot{\varphi}_Z^2\right) + mgx \qquad J_R = mr_R^2\,, \quad J_Z = \frac{1}{2}mr_Z^2$$

$$x = r_R\varphi \quad x_Z = r_Z\varphi_Z \quad x = 2r_Z\varphi_Z = 2x_Z \quad \dot{\varphi} = \frac{\dot{x}}{r_R} \quad v_Z = \frac{1}{2}\dot{x} \quad \dot{\varphi}_Z = \frac{\dot{x}}{2r_Z}$$

$$L = \frac{1}{2}\dot{x}^2\left(m + m + \frac{3}{2}m + \frac{3}{4}m\right) + mgx$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \qquad \frac{17}{4}\ddot{x} - g = 0 \qquad \boxed{\ddot{x} = \frac{4}{17}g}$$

Lösung 4.21



$$T=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\left[\left(\dot{x}+l\dot{\varphi}\cos\varphi\right)^2+\left(l\dot{\varphi}\sin\varphi\right)^2\right]\qquad U=-m_2 gl\cos\varphi$$

$$L=\frac{1}{2}\left(m_1+m_2\right)\dot{x}^2+m_2 l\dot{x}\dot{\varphi}\cos\varphi+\frac{1}{2}m_2 l^2\dot{\varphi}^2+m_2 gl\cos\varphi$$

$$\frac{\partial L}{\partial\dot{\varphi}}=m_2 l\dot{x}\cos\varphi+m_2 l^2\dot{\varphi}\qquad\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\bullet}=m_2 l\ddot{x}\cos\varphi-m_2 l\dot{x}\dot{\varphi}\sin\varphi+m_2 l^2\ddot{\varphi}$$

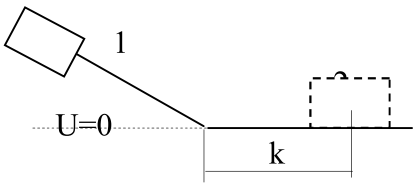
$$\frac{\partial L}{\partial\varphi}=-m_2 l\dot{x}\dot{\varphi}\sin\varphi-m_2 gl\sin\varphi$$

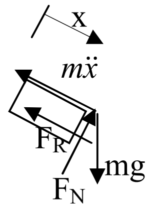
$$\frac{\partial L}{\partial\dot{x}}=\left(m_1+m_2\right)\dot{x}+m_2 l\dot{\varphi}\cos\varphi\qquad\frac{\partial L}{\partial x}=0$$

$$\left(\frac{\partial L}{\partial\dot{x}}\right)^{\bullet}=\left(m_1+m_2\right)\ddot{x}+m_2 l\ddot{\varphi}\cos\varphi-m_2 l\dot{\varphi}^2\sin\varphi$$

$$\boxed{\ddot{x}\cos\varphi+l\ddot{\varphi}+g\sin\varphi=0\qquad\left(m_1+m_2\right)\ddot{x}+m_2 l\ddot{\varphi}\cos\varphi-m_2 l\dot{\varphi}^2\sin\varphi=0}$$

Lösung 4.22





$$mgl\sin\alpha-\mu mg\left(l\cos\alpha+k\right)=0$$

$$mgl\left(\sin\alpha-\mu\cos\alpha\right)=\mu mgk$$

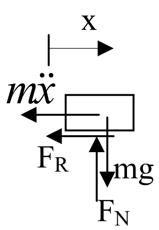
$$k=\frac{1}{\mu}\left(\sin\alpha-\mu\cos\alpha\right)$$

$$m\ddot{x}+\mu mg\cos\alpha-mg\sin\alpha=0$$

$$\ddot{x}=g\left(\sin\alpha-\mu\cos\alpha\right)=C$$

$$\dot{x}=Ct+C_1\qquad x=\frac{1}{2}Ct^2+C_1t+C_2\qquad t=0:\dot{x}=0,x=0\Rightarrow C_1=0,C_2=0$$

$$\dot{x}=Ct\qquad x=\frac{1}{2}Ct^2\qquad t=t_1:\quad x=l,\dot{x}=v_1\quad\Rightarrow\quad t_1=\sqrt{\frac{2l}{C}}\qquad v_1=\sqrt{2lC}$$



$$m\ddot{x}+\mu mg=0\qquad\ddot{x}=-\mu g\qquad\dot{x}=-\mu gt+C_3\qquad x=-\frac{1}{2}\mu gt^2+C_3t+C_4$$

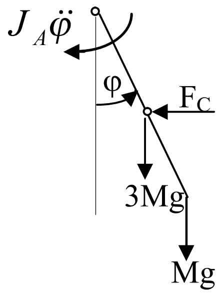
$$t=0:\quad x=0\quad\dot{x}=v_1\quad\Rightarrow\quad C_4=0\quad C_3=v_1$$

$$t=t_E:\qquad\dot{x}=0\quad\Rightarrow\quad t_E=\frac{v_1}{\mu g}\qquad\left(x=k\right):zur\ Best.\,von\ k$$

$$t^* = t_1 + t_E \quad t^* = \sqrt{\frac{2l}{C}} + \frac{\sqrt{2lC}}{\mu g} = \sqrt{\frac{2l}{g(\sin\alpha - \mu\cos\alpha)}} + \frac{1}{\mu}\sqrt{\frac{2l}{g}(\sin\alpha - \mu\cos\alpha)}$$

Schwingungen

Lösung 5.1



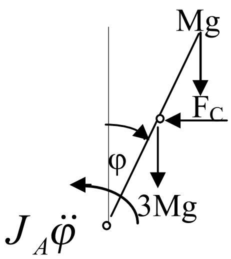
$$J_A = M(2a)^2 + \frac{1}{12}(3M)(2a)^2 + 3Ma^2 = 8Ma^2$$

$$J_A\ddot{\varphi} + F_C a\cos\varphi + 3Mga\sin\varphi + Mg2a\sin\varphi = 0 \quad F_C = ca\sin\varphi$$

$$J_A\ddot{\varphi} + (ca\cos\varphi + 5Mg)a\sin\varphi = 0$$

$$\textit{kleine Ausschläge} \quad \Rightarrow \quad \cos\varphi \approx 1, \quad \sin\varphi \approx \varphi$$

$$\ddot{\varphi} + \frac{ca+5Mg}{8Ma}\varphi = 0 \quad \omega^2 = \frac{ca+5Mg}{8Ma} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{8Ma}{ca+5Mg}}$$



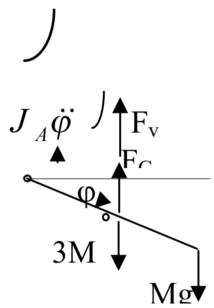
$$J_A\ddot{\varphi} + ca^2\sin\varphi\cos\varphi - 3Mga\sin\varphi - Mg2a\sin\varphi = 0$$

$$J_A\ddot{\varphi} + (ca\cos\varphi - 5Mg)a\sin\varphi = 0$$

$$\textit{kleine Ausschläge} \quad \Rightarrow \quad \cos\varphi \approx 1, \quad \sin\varphi \approx \varphi$$

$$\ddot{\varphi} + \frac{ca-5Mg}{8Ma}\varphi = 0 \quad \omega^2 = \frac{ca-5Mg}{8Ma} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{8Ma}{ca-5Mg}}$$

Für $ca \le 5Mg$ ist keine Schwingung möglich.



$$J_A\ddot{\varphi} + ca\sin\varphi a\cos\varphi + F_v a\cos\varphi - 3Mga\cos\varphi - Mg2a\cos\varphi = 0$$

$$J_A\ddot{\varphi} + ca^2\cos\varphi\sin\varphi + F_v a\cos\varphi - 5Mga\cos\varphi = 0$$

$$\textit{kleine Ausschläge} \quad \Rightarrow \quad \cos\varphi \approx 1, \quad \sin\varphi \approx \varphi$$

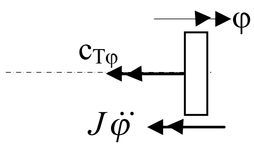
$$J_A\ddot{\varphi} + ca^2\varphi + F_v a - 5Mga = 0$$

$$\textit{Statisches Gleichgewicht}: \quad F_v a - 5Mga = 0 \quad \Rightarrow$$

$$\ddot{\varphi} + \frac{ca}{8Ma}\varphi = 0 \quad \omega^2 = \frac{ca}{8Ma} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{8Ma}{ca}}$$

Man erhält Lösung b.) aus a.), wenn man 5Mg durch –5Mg ersetzt, c.) ohne Schwerkraft.

Lösung 5.2



$$J\ddot{\varphi} + c_T\varphi = 0 \quad \ddot{\varphi} + \frac{c_T}{J}\varphi = 0 \quad \omega_0 = \sqrt{\frac{c_T}{J}} \quad T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{J}{c_T}}$$

Bestimmung von c_T:

Fall a.): Federn parallel

$$\varphi_1 = \varphi_2 = \varphi \qquad M_1 + M_2 = M \qquad M = c_T \varphi$$

$$\varphi_1 = \frac{M_1 l_1}{G I_{p1}} \quad \varphi_2 = \frac{M_2 l_2}{G I_{p2}} \qquad c_T = \frac{M}{\varphi} = \frac{M_1 + M_2}{\varphi} = \frac{G I_{p1}}{l_1} + \frac{G I_{p2}}{l_2} \qquad I_p = \frac{\pi}{32} d^4$$

$$c_T = \frac{G\pi}{32}\left(\frac{d_1^4}{l_1} + \frac{d_2^4}{l_2}\right) \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{G\pi}{32} \cdot \frac{\left(\frac{d_1^4}{l_1} + \frac{d_2^4}{l_2}\right)}{J}}$$

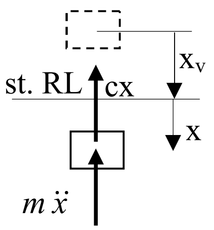
Fall b.) Federn hintereinander

$$\varphi_1 + \varphi_2 = \varphi \qquad M_1 = M_2 = M \qquad M = c_T \varphi$$

$$\varphi_1 = \frac{M_1 l_1}{G I_{p1}} \quad \varphi_2 = \frac{M_2 l_2}{G I_{p2}} \qquad c_T = \frac{M}{\varphi} = \frac{M}{\varphi_1 + \varphi_2} = \frac{1}{\frac{l_1}{G I_{p1}} + \frac{l_2}{G I_{p2}}} \qquad I_p = \frac{\pi}{32} d^4$$

$$c_T = \frac{G\pi}{32}\left(\frac{1}{\frac{l_1}{d_1^4} + \frac{l_2}{d_2^4}}\right) \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{G\pi}{32} \cdot \frac{1}{J\left(\frac{l_1}{d_1^4} + \frac{l_2}{d_2^4}\right)}}$$

Lösung 5.3



$$c \cdot x_v - mg = 0 \quad stat.\, Gleichgewicht$$

$$m\ddot{x} + cx = 0 \quad \Rightarrow \quad \ddot{x} + \frac{c}{m}x = 0 \quad \omega^2 = \frac{c}{m}$$

$$Federkraft\ F = F_c = c \cdot x = c \cdot \Delta l = c \cdot \frac{Fl}{EA} \quad \Rightarrow \quad c = \frac{EA}{l}$$

Lösung der Differentialgleichung:

$$x = A\cos\omega t + B\sin\omega t \qquad \dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$$

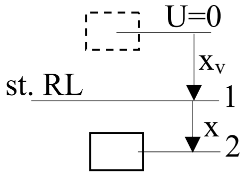
$$AB: \quad t = 0 \quad x(0) = 0 \quad 0 = A \cdot 1 \quad \Rightarrow \quad A = 0 \quad \dot{x}(0) = v \quad v = B\omega \quad \Rightarrow \quad B = \frac{v}{\omega}$$

$$x(t) = \frac{v}{\omega}\sin\omega t$$

$$F_S = c(x + x_v) = mg + c \cdot \frac{v}{\omega}\sin\omega t \quad F_{S\max} = c(x_{\max} + x_v) = mg + c \cdot \frac{v}{\omega} = mg + v\sqrt{c \cdot m}$$

$$F_{S\max} = mg\left(1 + \sqrt{\frac{EAv^2}{mg^2 l}}\right) = 49{,}81 \cdot 10^3 N = 49{,}81 kN$$

Lösung mit Hilfe des Energiesatzes:



$$T_1 = \frac{1}{2}mv^2 \quad T_2 = \frac{1}{2}m\dot{x}^2 \quad U_1 = \frac{1}{2}cx_v^2 - mgx_v$$

$$U_2 = \frac{1}{2}c\left(x_v + x\right)^2 - mg\left(x_v + x\right)$$

$$T_2 + U_2 = konst. \quad \frac{d}{dt}\left(T_2 + U_2\right) = 0 \quad \Rightarrow \quad Dgl.$$

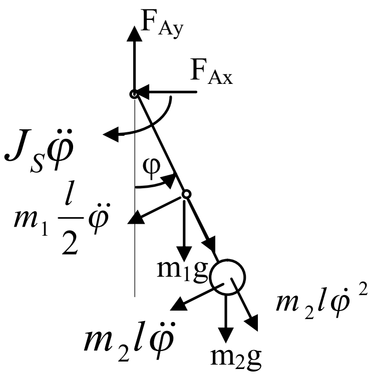
$$für\ x = x_{\max} \quad ist \quad \dot{x} = 0 \quad d.h. \quad T_2 = 0$$

$$\frac{1}{2}mv^2 + \frac{1}{2}cx_v^2 - mgx_v = \frac{1}{2}c\left(x_v + x_{\max}\right)^2 - mg\left(x_v + x_{\max}\right)$$

$$\frac{1}{2}mv^2 - \frac{1}{2}cx_{\max}^2 = x_{\max}\left(cx_v - mg\right) = 0 \quad x_{\max} = v\sqrt{\frac{m}{c}}$$

$$F_{S\max} = mg + cx_{\max} = mg + v\sqrt{mc}$$

Lösung 5.4



$$\uparrow: F_{Ay} - (m_1 + m_2)g - \left(\frac{1}{2}m_1 + m_2\right)l\dot{\varphi}^2\cos\varphi - \left(\frac{1}{2}m_1 + m_2\right)l\ddot{\varphi}\sin\varphi = 0$$

$$\leftarrow: F_{Ax} - \left(\frac{1}{2}m_1 + m_2\right)l\dot{\varphi}^2\sin\varphi + \left(\frac{1}{2}m_1 + m_2\right)l\ddot{\varphi}\cos\varphi = 0$$

$$A: J_S\ddot{\varphi} + \left(\frac{1}{4}m_1 + m_2\right)l^2\ddot{\varphi} + \left(\frac{1}{2}m_1 + m_2\right)gl\sin\varphi = 0 \quad oder$$

$$J_A\ddot{\varphi} + \left(\frac{1}{2}m_1 + m_2\right)gl\sin\varphi = 0 \quad mit \quad J_A = J_S + m_1\left(\frac{l}{2}\right)^2 + m_2 l^2$$

$$J_A = \left(\frac{1}{3}m_1 + m_2\right)l^2 \qquad \ddot{\varphi} + \frac{\left(\frac{1}{2}m_1 + m_2\right)g}{\left(\frac{1}{3}m_1 + m_2\right)l}\sin\varphi = 0$$

$$Dgl.: \quad \ddot{\varphi} + \omega_0^2\sin\varphi = 0 \quad mit \quad \omega_0^2 = \frac{g}{l}\cdot\frac{\left(\frac{1}{2}m_1 + m_2\right)}{\left(\frac{1}{3}m_1 + m_2\right)}$$

$$\ddot{\varphi} = -\omega_0^2\sin\varphi = \frac{d\dot{\varphi}}{dt} = \frac{d\dot{\varphi}}{d\varphi}\frac{d\varphi}{dt} = \dot{\varphi}\frac{d\dot{\varphi}}{d\varphi} \quad \Rightarrow \quad \ddot{\varphi}\,d\varphi = \dot{\varphi}\,d\dot{\varphi} = -\omega_0^2\sin\varphi\,d\varphi$$

$$\frac{1}{2}\dot{\varphi}^2 = -\omega_0^2(-\cos\varphi) + C^* \qquad \dot{\varphi}^2 = 2\omega_0^2\cos\varphi + C \quad t = 0: \quad \varphi = \varphi_0 \quad \dot{\varphi} = 0$$

$$0 = 2\omega_0^2\cos\varphi_0 + C \quad \Rightarrow \quad C = -2\omega_0^2\cos\varphi_0 \qquad \dot{\varphi}^2 = 2\omega_0^2(\cos\varphi - \cos\varphi_0)$$

Damit erhält man die Auflagerreaktionen:

$$F_{Ax} = \left(\frac{1}{2}m_1 + m_2\right)l\omega_0^2\sin\varphi[3\cos\varphi - 2\cos\varphi_0]$$

$$F_{Ay} = (m_1 + m_2)g + \left(\frac{1}{2}m_1 + m_2\right)l\omega_0^2\left[2\cos^2\varphi - 2\cos\varphi_0\cos\varphi - \sin^2\varphi\right]$$

$$mit \quad -\sin^2\varphi = \cos^2\varphi - 1$$

$$F_{Ay} = (m_1 + m_2)g + \left(\frac{1}{2}m_1 + m_2\right)l\omega_0^2\left[3\cos^2\varphi - 2\cos\varphi_0\cos\varphi - 1\right]$$

$$\text{für kleine Winkel } \varphi \text{ gilt}: \quad \sin\varphi \approx \varphi, \cos\varphi \approx 1 \quad \Rightarrow$$

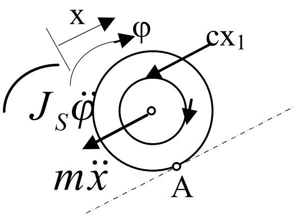
$$F_{Ax} = \left(\frac{1}{2}m_1 + m_2\right)l\omega_0^2\varphi[3 - 2\cos\varphi_0] \quad F_{Ay} = (m_1 + m_2)g + 2\left(\frac{1}{2}m_1 + m_2\right)l\omega_0^2[1 - \cos\varphi_0]$$

Lösung 5.5

Federkonstante der Blattfeder: $c_2 = \frac{3EI}{l^3} = \frac{3EI}{(b+d)^3}$

Reihenschaltung: $c = \frac{c_1 \cdot c_2}{c_1 + c_2} = \frac{3EIc_1}{c_1(b+d)^3 + 3EI}$

Unter der Voraussetzung, daß das stat. Gleichgewicht erfüllt ist, gilt:



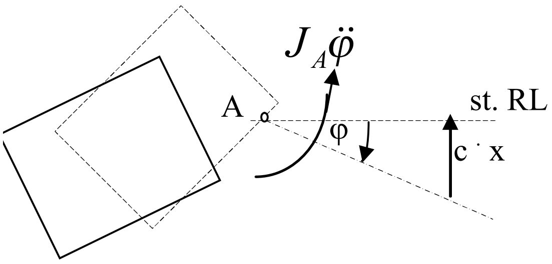
ZB: $x = b\varphi \qquad x_1 = (b+d)\varphi$

$$\sum M_A = 0: \quad \Rightarrow \quad J_S\ddot{\varphi} + m\ddot{x}b + cx_1(b+d) = 0$$

$$\left(J_S + mb^2\right)\ddot{\varphi} + c(b+d)^2\varphi = 0$$

$$\ddot{\varphi} + \frac{c(b+d)^2}{\left(J_S + mb^2\right)}\varphi = 0 \quad \omega^2 = \frac{c(b+d)^2}{J_S + mb^2} \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{(b+d)}\sqrt{\frac{J_S + mb^2}{c}}$$

Lösung 5.6

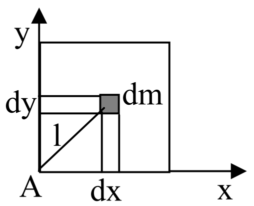


$$J_A\ddot{\varphi} + c \cdot x \cdot a\sqrt{2}\cos\varphi = 0 \qquad x = a\sqrt{2}\sin\varphi \approx a\sqrt{2}\varphi$$

$$J_A = J_S + \frac{1}{2}ma^2 = \frac{1}{6}ma^2 + \frac{1}{2}ma^2 = \frac{2}{3}ma^2$$

$$\frac{2}{3}ma^2\ddot{\varphi} + 2ca^2\varphi = 0 \quad \ddot{\varphi} + \frac{3c}{m}\varphi = 0 \quad \omega^2 = \frac{3c}{m} \quad T = 2\pi\sqrt{\frac{m}{3c}}$$

Ermittlung von J_A:

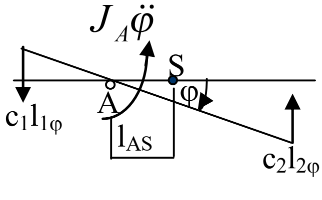


$$m = \rho ba^2 \qquad J_A = \int_{(m)} l^2 dm \qquad dm = \rho b dx dy \quad l^2 = x^2 + y^2$$

$$J_A = \iint\left(x^2 + y^2\right)\rho b dx dy = \rho b\left\{\int_0^a\int_0^a x^2 dx dy + \int_0^a\int_0^a y^2 dx dy\right\}$$

$$= \rho b\left\{\frac{1}{3}a^4 + \frac{1}{3}a^4\right\} = \frac{2}{3}\rho ba^4 = \frac{2}{3}ma^2$$

Lösung 5.7



$$l_S = \frac{1}{2}(l_1 + l_2) \quad l_{AS} = l_2 - l_S = \frac{1}{2}(l_2 - l_1)$$

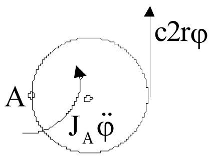
$$J_A = J_S + ml_{AS}^2 = \frac{1}{12}m(l_1 + l_2)^2 + \frac{1}{4}m(l_2 - l_1)^2$$

$$= \frac{1}{3}m\left(l_1^2 + l_2^2 - l_1 l_2\right)$$

$$J_A\ddot{\varphi} + c_2 l_2^2\varphi + c_1 l_1^2\varphi = 0 \quad \ddot{\varphi} + \frac{c_1 l_1^2 + c_2 l_2^2}{J_A}\varphi = 0$$

$$\omega_0^2 = \frac{c_1 l_1^2 + c_2 l_2^2}{J_A} \quad \omega_0 = \sqrt{\frac{3}{m} \cdot \frac{c_1 l_1^2 + c_2 l_2^2}{l_1^2 + l_2^2 - l_1 l_2}} \qquad T = \frac{2\pi}{\omega_0} \qquad f_0 = \frac{\omega_0}{2\pi}$$

Lösung 5.8

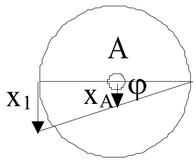


$$J_A\ddot{\varphi} + 4cr^2\varphi = 0 \qquad J_A = J_S + mr^2 = \frac{3}{2}mr^2$$

$$\ddot{\varphi} + \frac{4cr^2}{J_A}\varphi = 0 \qquad \ddot{\varphi} + \frac{8c}{3m}\varphi = 0$$

$$\boxed{\omega = 2\sqrt{\frac{2c}{3m}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{1}{\pi}\sqrt{\frac{2c}{3m}} \qquad T = \pi\sqrt{\frac{3m}{2c}}}$$

Lösung 5.9



$$\text{ZB:} \quad r\varphi = x_A \quad 2r\varphi = x_1 \quad x_A = \frac{1}{2}x_1 \quad \varphi = \frac{1}{2}\frac{x_1}{r}$$

$$L = T - U = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}J_A\dot{\varphi}^2 + \frac{1}{2}M\dot{x}_A^2 - \frac{1}{2}cx_A^2$$

$$= \frac{1}{2}\dot{x}_1^2\left(m + \frac{J_A}{4r^2} + \frac{1}{4}M\right) - \frac{1}{2}c\frac{1}{4}x_1^2 \qquad J_A = \frac{1}{2}Mr^2$$

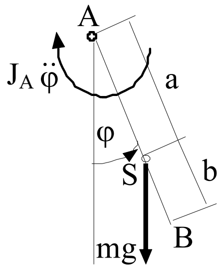
$$\left(\frac{\partial L}{\partial \dot{x}_1}\right)^{\bullet} - \frac{\partial L}{\partial x_1} = 0 \qquad \ddot{x}_1\left(m + \frac{3}{8}M\right) + \frac{1}{4}cx_1 = 0 \qquad \boxed{\ddot{x}_1 + \frac{c}{\left(4m + \frac{3}{2}M\right)}x_1 = 0}$$

$$\omega^2 = \frac{c}{\left(4m + \frac{3}{2}M\right)} \qquad x_1(t) = A\sin\omega t + B\cos\omega t \quad \dot{x}_1 = A\omega\cos\omega t - B\omega\sin\omega t$$

$$\text{AB:} \quad t = 0 \quad x_1 = 0 \quad 0 = B \qquad \dot{x}_1 = v_0 \qquad v_0 = A\omega \qquad A = \frac{v_0}{\omega}$$

$$\boxed{x_1(t) = \frac{v_0}{\omega}\sin\omega t}$$

Lösung 5.10



$$\text{A}: \quad J_A\ddot{\varphi} + mga\sin\varphi = 0 \quad \text{analog} \quad \text{B}: \quad J_B\ddot{\varphi} + mgb\sin\varphi = 0$$

$$\text{linearisiert}: \quad \ddot{\varphi} + \frac{mga}{J_A}\varphi = 0 \quad \text{mit} \quad \omega_A^2 = \frac{mga}{J_A} \quad T_A^2 = \frac{4\pi^2 J_A}{mga}$$

$$\ddot{\varphi} + \frac{mgb}{J_B}\varphi = 0 \quad \text{mit} \quad \omega_B^2 = \frac{mgb}{J_B} \quad T_B^2 = \frac{4\pi^2 J_B}{mgb}$$

$$J_A = J_S + ma^2 \qquad J_B = J_S + mb^2 \qquad a + b = l \qquad b = l - a$$

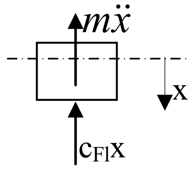
$$\frac{mgaT_A^2}{4\pi^2} = J_S + ma^2 \qquad \frac{mgbT_B^2}{4\pi^2} = J_S + mb^2$$

$$\frac{mg}{4\pi^2}\left(aT_A^2 - bT_B^2\right) = m\left(a^2 - b^2\right) \quad \frac{g}{4\pi^2}\left(aT_A^2 - (l-a)T_B^2\right) = a^2 - l^2 - a^2 + 2al$$

$$a\left[\left(T_A^2 + T_B^2\right)g - 8\pi^2 l\right] = \left(T_B^2 g - 4\pi^2 l\right)l \qquad a = l \cdot \frac{T_B^2 g - 4\pi^2 l}{\left(T_A^2 + T_B^2\right)g - 8\pi^2 l}$$

$$\boxed{J_S = \frac{mgaT_A^2}{4\pi^2} - ma^2}$$

Lösung 5.11



$$m\ddot{x} + c_{Fl}x + \left(c_{Fl}x_0 - mg\right) = 0 \qquad c_{Fl}x_0 - mg = 0$$

Federkonstante der Flüssigkeit : $c_{Fl} = \rho_{Fl} g A$

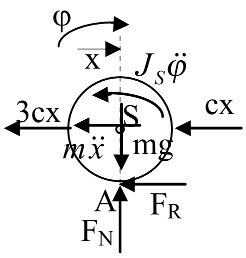
$$\ddot{x} + \frac{c_{Fl}}{m}x = 0 \qquad \omega_0^2 = \frac{\rho_{Fl} g A}{m} \qquad T = 2\pi\sqrt{\frac{m}{\rho_{Fl} g A}}$$

Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \qquad L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}c_{Fl}\left(x_0 + x\right)^2 + mg\left(x_0 + x\right) \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}$$

$$\frac{\partial L}{\partial x} = -c_{Fl}\left(x_0 + x\right) + mg \quad \Rightarrow \quad m\ddot{x} + c_{Fl}\left(x_0 + x\right) - mg = 0 \qquad \ddot{x} + \frac{c_{Fl}}{m}x = 0$$

Lösung 5.12



$$ZB: \quad \varphi = \frac{x}{r}$$

$$A: \quad J_S\ddot{\varphi} + m\ddot{x}r + 4cxr = 0 \quad \Rightarrow \quad \ddot{x}\left(m + \frac{J_S}{r^2}\right) + 4cx = 0$$

$$J_S = \frac{1}{2}mr^2 \quad \ddot{x} + \frac{8c}{3m}x = 0 \quad \omega = 2\sqrt{\frac{2c}{3m}} = 40\frac{1}{s} \quad T = \frac{2\pi}{\omega} = 0{,}1571s$$

Lösungsansatz:

$$x = A\sin\omega t + B\cos\omega t \quad \dot{x} = A\omega\cos\omega t - B\omega\sin\omega t \quad \ddot{x} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$$

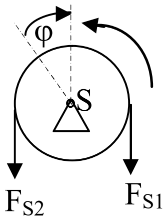
$$AB: \quad t = 0: \quad x = x_0 \quad \Rightarrow \quad B = x_0 \qquad \dot{x} = 0 \quad \Rightarrow \quad A = 0$$

$$x = x_0\cos\omega t \qquad \dot{x} = -x_0\omega\sin\omega t \quad \Rightarrow \quad \dot{x}_{\max} = x_0\omega = 0{,}4\frac{m}{s}$$

$$\ddot{x} = -x_0\omega^2\cos\omega t \quad \Rightarrow \quad \ddot{x}_{\max} = x_0\omega^2 = 16\frac{m}{s^2}$$

Lösung 5.13

Statisches Gleichgewicht: $m_1 g - cx_{st} = 0$



$$F_{S1} = m_1 g - m_1\ddot{x} \qquad F_{S2} = c\left(x_{st} + x\right) \qquad \varphi = \frac{x}{R}$$

$$S: \quad J_S\ddot{\varphi} + F_{S2}R - F_{S1}R = 0$$

$$J_S\frac{\ddot{x}}{R^2} + c\left(x_{st} + x\right) - m_1 g + m_1\ddot{x} = 0 \quad \Rightarrow \quad \ddot{x} + \frac{2c}{2m_1 + m_2}x = 0$$

$$\omega_0 = \sqrt{\frac{2c}{2m_1 + m_2}} \qquad f_0 = \frac{\omega_0}{2\pi} \qquad T = \frac{1}{f_0}$$

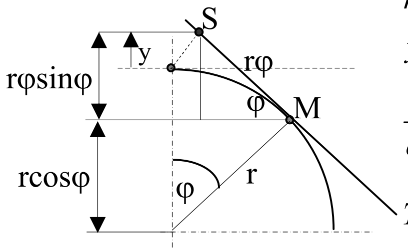
$$L = T - U = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} J_S \dot{\varphi}^2 - \frac{1}{2} c(x_{st} + x)^2 + m_1 g(x_{st} + x)$$

$$x = A \sin \omega_0 t + B \cos \omega_0 t \qquad \dot{x} = A\omega_0 \cos \omega_0 t - B\omega_0 \sin \omega_0 t$$

$$t = 0: \quad x = x_0 \quad \Rightarrow \quad B = x_0 \qquad \dot{x} = v_0 \quad \Rightarrow \quad A = \frac{v_0}{\omega_0}$$

$$x(t) = \frac{v_0}{\omega_0} \sin \omega_0 t + x_0 \cos \omega_0 t$$

Lösung 5.14



$$r\cos\varphi + r\varphi \sin\varphi = r + y \quad \Rightarrow$$

$$y = r(\varphi \sin\varphi + \cos\varphi - 1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0 \qquad L = T - U$$

$$T = \frac{1}{2} J_M \dot{\varphi}^2 \qquad J_M = J_S + m(r\varphi)^2$$

$$U = mgy = mgr(\varphi \sin\varphi + \cos\varphi - 1)$$

$$L = \frac{1}{2} J_S \dot{\varphi}^2 + \frac{1}{2} m r^2 \varphi^2 \dot{\varphi}^2 - mgr(\varphi \sin\varphi + \cos\varphi - 1)$$

$$\frac{\partial L}{\partial \dot{\varphi}} = J_S \dot{\varphi} + m r^2 \varphi^2 \dot{\varphi} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = J_S \ddot{\varphi} + m r^2 \left(2\varphi \dot{\varphi}^2 + \varphi^2 \ddot{\varphi}\right)$$

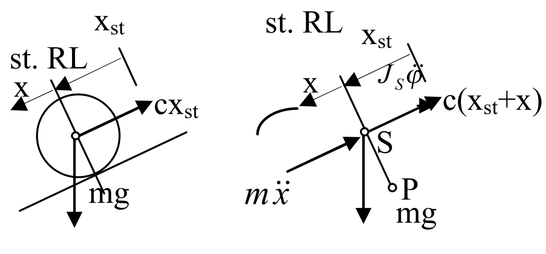
$$\frac{\partial L}{\partial \varphi} = m r^2 \varphi \dot{\varphi}^2 - mgr(\sin\varphi + \varphi \cos\varphi - \sin\varphi) = m r^2 \varphi \dot{\varphi}^2 - mgr\varphi \cos\varphi$$

$$\ddot{\varphi}\left[J_S + m r^2 \varphi^2\right] + \varphi\left[m r^2 \dot{\varphi}^2 + mgr \cos\varphi\right] = 0 \quad \textit{nichtlineare Dgl. 2. Ordnung}$$

Linearisierung für kleine Ausschläge: $\varphi << 1 \quad \Rightarrow \quad \cos\varphi \approx 1, \quad \varphi^2 \approx 0, \quad \dot{\varphi}$

$$\ddot{\varphi} + \frac{mgr}{J_S} \varphi = 0 \qquad J_S = \frac{1}{12} m l^2 \quad \Rightarrow \quad \omega_0^2 = \frac{12 g r}{l^2} \qquad T = \frac{2\pi}{\omega_0} = \frac{\pi l}{\sqrt{3gr}}$$

Lösung 5.15



Statisches Gleichgewicht:

$$mg \sin\alpha \cdot r - c x_{st} \cdot r = 0$$

$$J_S \ddot{\varphi} + m\ddot{x} r - mg \sin\alpha \cdot r + c(x_{st} + x) \cdot r = 0 \qquad J_S = \frac{1}{2} m r^2 \qquad \varphi = \frac{x}{r}$$

$$\ddot{x}\left(m + \frac{1}{2} m\right) - mg \sin\alpha + c(x_{st} + x) = 0 \quad \Rightarrow \quad \ddot{x} + \frac{2c}{3m} x = 0 \qquad \omega = \sqrt{\frac{2c}{3m}} = 10 \frac{1}{s}$$

$$x = A\sin\omega t + B\cos\omega t \quad \dot{x} = A\omega\cos\omega t - B\omega\sin\omega t \quad \ddot{x} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$$

$$AB: \quad t = 0: \quad x = x_0 \quad \Rightarrow \quad B = x_0 \qquad \dot{x} = 0 \quad \Rightarrow \quad A = 0$$

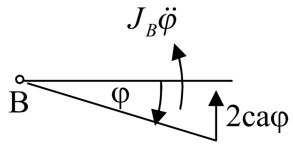
$$x = x_0\cos\omega t \qquad \dot{x} = -x_0\omega\sin\omega t \quad \Rightarrow \quad \dot{x}_{\max} = x_0\omega = 10\frac{cm}{s}$$

$$\ddot{x} = -x_0\omega^2\cos\omega t \quad \Rightarrow \quad \ddot{x}_{\max} = x_0\omega^2 = 100\frac{cm}{s^2} \qquad T = \frac{2\pi}{\omega} = 0{,}628s$$

$$L = T - U = \frac{1}{2}J_S\dot{\varphi}^2 + \frac{1}{2}m\dot{x}^2 - \frac{1}{2}cx^2 \quad oder \quad mit\ dem\ st.\ Gleichgewicht:$$

$$L = \frac{1}{2}J_S\dot{\varphi}^2 + \frac{1}{2}m\dot{x}^2 - \frac{1}{2}c(x_{st} + x)^2 - mg(x_{st} + x)\sin\alpha$$

Lösung 5.16



Stat. und dyn. Gleichgewicht (Momente um B):

$$m_2 g\cdot 2a - c\cdot 2a\varphi_0\cdot 2a = 0$$

$$J_B\ddot{\varphi} - m_2 g\cdot 2a + c\cdot 2a(\varphi + \varphi_0)\cdot 2a = 0 \quad \Rightarrow$$

$$\ddot{\varphi} + \frac{4ca^2}{J_B}\varphi = 0 \qquad J_B = \frac{1}{12}m_1\left(a^2 + 4a^2\right) + m_2(2a)^2$$

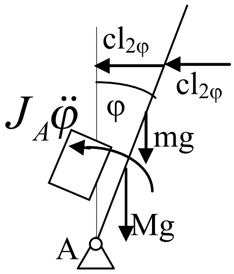
$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \frac{\partial L}{\partial\varphi} = 0 \quad L = T - U \quad T = \frac{1}{2}J_B\dot{\varphi}^2 \quad U = \frac{1}{2}c[2a(\varphi_0 + \varphi)]^2 - m_2 g 2a(\varphi_0 + \varphi)$$

$$L = \frac{1}{2}J_B\dot{\varphi}^2 - \frac{1}{2}c[2a(\varphi_0 + \varphi)]^2 + m_2 g 2a(\varphi_0 + \varphi) \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) = J_B\ddot{\varphi}$$

$$\frac{\partial L}{\partial\varphi} = -4ca^2(\varphi_0 + \varphi) + 2m_2 ga \qquad J_B\ddot{\varphi} + 4ca^2\varphi + \left(4ca^2\varphi_0 - 2m_2 ga = 0\right) = 0$$

$$\ddot{\varphi} + \frac{4ca^2}{J_B}\varphi = 0 \qquad J_B = \left(\frac{5}{12}m_1 + 4m_2\right)a^2 \qquad \omega_0 = \sqrt{\frac{4c}{\frac{5}{12}m_1 + 4m_2}} \quad T = 2\pi\sqrt{\frac{\frac{5}{12}m_1 + 4m_2}{4c}}$$

Lösung 5.17

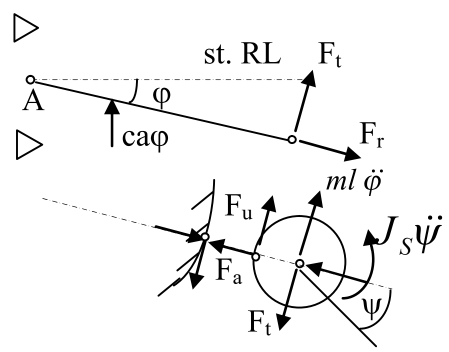


$$\sum M_A = 0: \quad J_A\ddot{\varphi} + 2cl_2^2\varphi - mg\frac{l_1}{2}\varphi - Mgl_3\varphi = 0$$

$$\ddot{\varphi} + \frac{2cl_2^2 - mg\frac{l_1}{2} - Mgl_3}{J_A}\varphi = 0 \quad \omega_0 = \sqrt{\frac{2cl_2^2 - mg\frac{l_1}{2} - Mgl_3}{J_A}}$$

$$T = \frac{2\pi}{\omega_0} \quad J_A = \frac{1}{3}ml_1^2 + Ml_3^2 \qquad T = 2\pi\sqrt{\frac{\frac{1}{3}ml_1^2 + Ml_3^2}{2cl_2^2 - mg\frac{l_1}{2} - Mgl_3}}$$

Lösung 5.18



Schnittskizze:

Fall a): freie Koord.: φ, ψ

ZB: $l\varphi = r\psi$

Fall b): $\psi = 0$

$$\begin{aligned} &Hebel: \quad J_A\ddot{\varphi} + ca^2\varphi + F_t l = 0 \quad mit \quad J_A = 0 \\ &Rad: \quad J_S\ddot{\psi} - F_u r = 0 \quad F_t - F_u - ml\ddot{\varphi} = 0 \end{aligned}$$

$$F_u = \frac{J_S\ddot{\psi}}{r} \qquad F_t = ml\ddot{\varphi} + \frac{J_S\ddot{\psi}}{r} \qquad ca^2\varphi + \left(ml\ddot{\varphi} + \frac{J_S\ddot{\psi}}{r}\right)l = 0$$

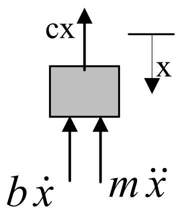
$$Fall\ a): \quad \ddot{\psi} = \frac{l}{r}\ddot{\varphi} \qquad \ddot{\varphi}\left(ml^2 + J_S\frac{l^2}{r^2}\right) + ca^2\varphi = 0$$

$$\ddot{\varphi} + \frac{ca^2}{ml^2 + J_S\frac{l^2}{r^2}}\varphi = 0 \quad T = 2\pi\sqrt{\frac{ml^2}{ca^2}}\cdot\sqrt{1 + \frac{J_S}{mr^2}} = 4\pi\sqrt{\frac{m}{c}}\sqrt{\frac{7}{5}}$$

$$Fall\ b): \quad \ddot{\psi} = 0 \qquad ml^2\ddot{\varphi} + ca^2\varphi = 0 \quad \ddot{\varphi} + \frac{ca^2}{ml^2}\varphi = 0 \quad T = 2\pi\sqrt{\frac{ml^2}{ca^2}} = 4\pi\sqrt{\frac{m}{c}}$$

Lösung 5.19

Bewegungsgleichung:



$$m\ddot{x} + b\dot{x} + cx = 0 \quad \ddot{x} + \frac{b}{m}\dot{x} + \frac{c}{m}x = 0 \quad mit \quad \omega_0^2 = \frac{c}{m},\ 2\delta = \frac{b}{m} \quad \Rightarrow$$

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0 \quad mit\ der\ Lösung$$

$$x = e^{-\delta t}\left(A\cos\omega t + B\sin\omega t\right) = e^{-\delta t}\cdot C\sin\left(\omega t + \varphi_0\right)$$

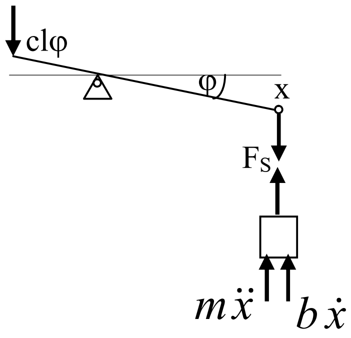
$$\omega = \omega_0\sqrt{1-\vartheta^2} \quad \vartheta = \frac{\delta}{\omega_0}$$

$$\frac{x(t)}{x(t+zT)} = \frac{e^{-\delta t}}{e^{-\delta(t+zT)}} = e^{\delta\, zT} \quad z - Anzahl\ der\ Schwingungen$$

$$\ln\frac{x(t)}{x(t+zT)} = \delta\, zT = \frac{2\pi z\delta}{\omega} = \frac{2\pi\delta}{\omega_0}z\frac{1}{\sqrt{1-\vartheta^2}} \quad \Rightarrow \quad z = \frac{\sqrt{1-\vartheta^2}}{2\pi\delta}\sqrt{\frac{c}{m}}\cdot\ln\frac{x(t)}{x(t+zT)}$$

$$\ln\frac{x(t)}{x(t+zT)} = \ln 10 \approx 2{,}3 \quad \Rightarrow \quad \delta\, zT = 2{,}3 \quad zT = \frac{2{,}3}{\delta} = \frac{2{,}3\cdot 2m}{b} = 1{,}96\,s \quad z \approx 40\,Schw.$$

Lösung 5.20



$$2l\varphi = x \quad \Rightarrow \quad \varphi = \frac{x}{2l}$$

$$m\ddot{x} + b\dot{x} + F_S = 0 \quad F_S \cdot 2l - cl^2\varphi = 0 \quad \Rightarrow \quad F_S = \frac{cl}{2}\varphi$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{c}{4m}x = 0 \qquad \ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0$$

$$x = e^{-\delta t}\left(A\cos\omega t + B\sin\omega t\right)$$

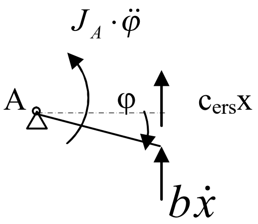
$$t = 0: \quad x = x_0 \quad \Rightarrow \quad A = x_0$$

$$\dot{x} = 0 \quad \Rightarrow \quad B = \frac{x_0\delta}{\omega}$$

$$x = x_0 e^{-\frac{b}{2m}t}\left[\cos\omega t + \frac{b}{2m\omega}\sin\omega t\right] \quad mit \quad \omega = \frac{1}{2m}\sqrt{cm - b^2} \quad T = \frac{2\pi}{\omega}$$

$$Zahlenwerte: \quad \omega = 15s^{-1} \quad T = 0{,}419s$$

Lösung 5.21



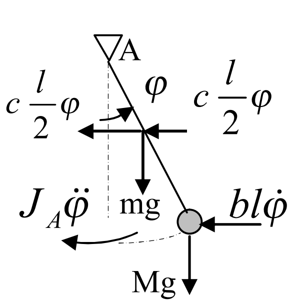
$$c_{ers} = \frac{3EI}{l^3} \qquad x = a\varphi$$

$$J_A\ddot{\varphi} + ba^2\dot{\varphi} + c_{ers}a^2\varphi = 0 \qquad \ddot{\varphi} + \frac{ba^2}{J_A}\dot{\varphi} + \frac{3EIa^2}{J_A l^3}\varphi = 0$$

$$\delta = \frac{ba^2}{2J_A} \qquad \omega_0^2 = \frac{3EIa^2}{J_A l^3} \qquad \omega = \sqrt{\omega_0^2 - \delta^2} = \sqrt{\frac{3EIa^2}{J_A l^3} - \frac{b^2a^4}{4J_A^2}}$$

$$\omega = \frac{a}{2J_A}\sqrt{\frac{12EI \cdot J_A}{l^3} - b^2a^2} \qquad T = \frac{2\pi}{\omega} = \frac{4\pi J_A}{a\sqrt{\frac{12EI \cdot J_A}{l^3} - b^2a^2}}$$

Lösung 5.22



$$\sum M_A = 0 \quad J_A\ddot{\varphi} + 2c\frac{l}{2}\varphi\cdot\frac{l}{2} + bl^2\dot{\varphi} + mg\frac{l}{2}\varphi + Mgl\varphi = 0$$

$$\ddot{\varphi} + \frac{bl^2}{J_A}\dot{\varphi} + \frac{l}{J_A}\left(\frac{cl}{2} + \frac{mg}{2} + Mg\right)\varphi = 0$$

$$\ddot{\varphi} + 2\delta\dot{\varphi} + \omega_0^2\varphi = 0$$

$$\omega_0^2 = \frac{l}{J_A}\left(\frac{cl}{2} + \frac{mg}{2} + Mg\right) \qquad \delta = \frac{bl^2}{2J_A} \qquad J_A = \frac{1}{3}ml^2 + Ml^2$$

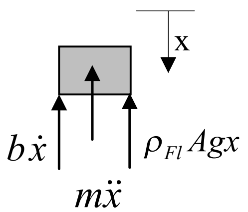
Lösung der Dgl. für schwache Dämpfung mit den AB. t = 0: $\varphi = 0, \quad \dot{\varphi} = \frac{v_0}{l}$:

$$\varphi(t) = e^{-\delta t}\cdot\frac{v_0}{l\omega}\sin\omega t \qquad \omega = \sqrt{\omega_0^2 - \delta^2} = \sqrt{\frac{l}{J_A}\left(\frac{cl}{2} + \frac{mg}{2} + Mg\right) - \frac{b^2l^4}{4J_A^2}}$$

$$\omega = \sqrt{\frac{\frac{c}{2} + \frac{g}{l}\left(\frac{m}{2} + M\right)}{M + \frac{m}{3}} - \left(\frac{b}{2\left(M + \frac{m}{3}\right)}\right)^2} \qquad T = \frac{2\pi}{\omega}$$

$$\varphi_{\max}\ \textit{für} \quad t = \frac{T}{4} = \frac{\pi}{2\omega} \qquad \left|\varphi_{\max}\right| = \frac{v_0}{l\omega}\cdot e^{-\delta\frac{\pi}{2\omega}}$$

Lösung 5.23

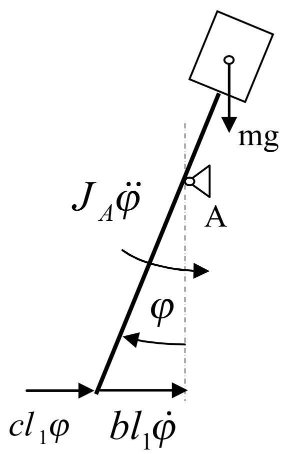


$$m\ddot{x} + b\dot{x} + \rho_{Fl}Agx = 0 \qquad \ddot{x} + \frac{b}{m}\dot{x} + \frac{\rho_{Fl}Ag}{m}x = 0$$

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0 \quad \omega_0 = \sqrt{\frac{\rho_{Fl}Ag}{m}} \quad \omega = \sqrt{\omega_0^2 - \delta^2} = \omega_0\sqrt{1-\vartheta^2} \quad \Rightarrow$$

$$\vartheta = \sqrt{1 - \frac{\omega^2}{\omega_0^2}} \quad mit \quad \omega = \frac{2\pi}{T} \qquad \vartheta = \sqrt{1 - \frac{m}{\rho_{Fl}Ag}\left(\frac{2\pi}{T}\right)^2}$$

Lösung 5.24



$$\sum M_A = 0$$

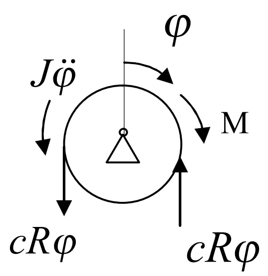
$$J_A\ddot{\varphi} + bl_1^2\dot{\varphi} + \left(cl_1^2 - mgl_2\right)\varphi = 0 \quad \ddot{\varphi} + \frac{bl_1^2}{J_A}\dot{\varphi} + \frac{\left(cl_1^2 - mgl_2\right)}{J_A}\varphi = 0$$

$$2\delta = \frac{bl_1^2}{J_A} \quad \omega_0 = \sqrt{\frac{\left(cl_1^2 - mgl_2\right)}{J_A}} \quad J_A = J_S + ml_2^2 \quad \vartheta = \frac{\delta}{\omega_0}$$

$$b = \frac{2\delta J_A}{l_1^2} = \frac{2\vartheta\omega_0 J_A}{l_1^2} = \frac{2\vartheta}{l_1^2}\sqrt{\left(cl_1^2 - mgl_2\right)\left(J_S + ml_2^2\right)}$$

Zahlenwert: b = $1{,}454\dfrac{Ns}{cm} = 145{,}4\dfrac{Ns}{m}$

Lösung 5.25



$$J\ddot{\varphi} + 2cR^2\varphi - M = 0 \qquad \ddot{\varphi} + \frac{2cR^2}{J}\varphi = \frac{M}{J} \qquad \omega_0^2 = \frac{2cR^2}{J}$$

$$\ddot{\varphi} + \omega_0^2\varphi = \frac{\hat{M}}{J}\sin\Omega t$$

$$\textit{Lösungsansatz}: \quad \varphi(t) = \varphi_{\max}\sin\Omega t \quad \ddot{\varphi}(t) = -\varphi_{\max}\Omega^2\sin\Omega t$$

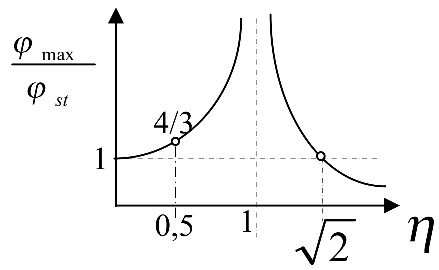
$$-\varphi_{\max}\Omega^2\sin\Omega t + \omega_0^2\varphi_{\max}\sin\Omega t = \frac{\hat{M}}{J}\sin\Omega t \quad \textit{Koeff.-Vergl.} \Rightarrow$$

$$\varphi_{\max}\omega_0^2\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right)=\frac{\hat{M}}{J} \quad mit \quad \eta=\frac{\Omega}{\omega_0} \quad \Rightarrow \quad \varphi_{\max}=\frac{\hat{M}}{J\omega_0^2}\cdot\frac{1}{1-\eta^2}=\frac{\hat{M}J}{J2cR^2}\cdot\frac{1}{1-\eta^2}=\varphi_{st}\cdot\frac{1}{1-\eta^2}$$

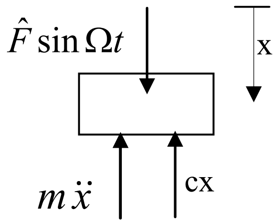
$$\varphi_{st}=\frac{\hat{M}}{2cR^2}=\frac{1}{200}=0{,}005$$

$$a.) \quad \eta=0{,}5 \quad \Rightarrow \quad V=\left|\frac{1}{1-\eta^2}\right|=\frac{4}{3} \quad \Rightarrow \quad \varphi_{\max}=\frac{1}{200}\cdot\frac{4}{3}=0{,}0067$$

$$b.) \quad \eta=\sqrt{2} \quad \Rightarrow \quad V=\left|\frac{1}{1-\eta^2}\right|=1 \quad \Rightarrow \quad \varphi_{\max}=\varphi_{st}=0{,}005$$



Lösung 5.26



$$m = m_1 + m_2 \qquad c = 4c_F$$

$$\ddot{x} + \frac{c}{m}x = \frac{\hat{F}}{m}\sin\Omega t = \frac{\hat{F}}{c}\cdot\frac{c}{m}\sin\Omega t = x_{st}\omega_0^2\sin\Omega t$$

$$L.-Ansatz: \quad x_p = x_{\max}\sin\Omega t \quad \ddot{x}_p = -x_{\max}\Omega^2\sin\Omega t$$

$$-x_{\max}\Omega^2\sin\Omega t + \omega_0^2 x_{\max}\sin\Omega t = x_{st}\omega_0^2\sin\Omega t$$

$$x_{\max}\omega_0^2\left(1-\left(\frac{\Omega}{\omega_0}\right)^2\right) = x_{st}\omega_0^2 \quad\Rightarrow\quad x_{\max} = \frac{x_{st}}{1-\eta^2} \quad \eta = \frac{\Omega}{\omega_0} \qquad F_{B\max} = cx_{\max} = c\cdot\frac{x_{st}}{1-\eta^2}$$

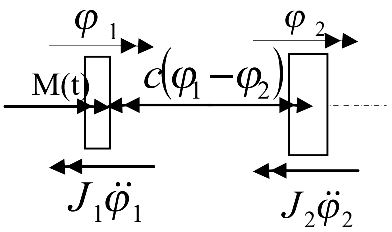
$$1.\,\text{Forderung}: \quad cx_{\max} = \pm 0{,}03\hat{F} \quad\Rightarrow\quad x_{\max} = \pm 0{,}03 x_{st} \quad \frac{1}{1-\eta^2} = \pm 0{,}03 \quad\Rightarrow$$

$$1{,}03 = 0{,}03\eta^2 \quad \eta^2 = \frac{1{,}03}{0{,}03} = \frac{\Omega^2}{\omega_0^2} \quad\Rightarrow\quad \omega_0^2 = \frac{0{,}03}{1{,}03}\Omega^2 = \frac{0{,}03}{1{,}03}(2\pi n)^2 = 2872 s^{-2}$$

$$2.\,\text{Forderung}: \quad x_{\max} = \pm 2\cdot 10^{-4}cm \quad c = \left|\frac{0{,}03\hat{F}}{x_{\max}}\right| = 15\cdot 10^4\,\frac{N}{cm}$$

$$\omega_0^2 = \frac{c}{m_1+m_2} \quad\Rightarrow\quad m_1\omega_0^2 + m_2\omega_0^2 = c \quad m_2 = \frac{c}{\omega_0^2} - m_1 = 3223 kg$$

Lösung 5.27



$$J_1\ddot{\varphi}_1 + c(\varphi_1-\varphi_2) = \hat{M}\sin\Omega t$$

$$J_2\ddot{\varphi}_2 - c(\varphi_1-\varphi_2) = 0$$

$$Lösungsansatz: \quad \varphi_1 = A_1\sin\Omega t \quad \ddot{\varphi}_1 = -A_1\Omega^2\sin\Omega t$$

$$\varphi_2 = A_2\sin\Omega t \quad \ddot{\varphi}_2 = -A_2\Omega^2\sin\Omega t$$

$$-J_1\Omega^2 A_1 + cA_1 - cA_2 = \hat{M}$$

$$-J_2\Omega^2 A_2 + cA_2 - cA_1 = 0 \quad \Rightarrow \quad \begin{vmatrix} \left(c - J_1\Omega^2\right) & -c \\ -c & \left(c - J_2\Omega^2\right) \end{vmatrix} \cdot \begin{vmatrix} A_1 \\ A_2 \end{vmatrix} = \begin{vmatrix} \hat{M} \\ 0 \end{vmatrix}$$

$$D = \begin{vmatrix} \left(c - J_1\Omega^2\right) & -c \\ -c & \left(c - J_2\Omega^2\right) \end{vmatrix} = \left(c - J_1\Omega^2\right) \cdot \left(c - J_2\Omega^2\right) - c^2 = \Omega^2\left[J_1 J_2 \Omega^2 - c\left(J_1 + J_2\right)\right]$$

$$D_1 = \begin{vmatrix} \hat{M} & -c \\ 0 & \left(c - J_2\Omega^2\right) \end{vmatrix} = \hat{M}\left(c - J_2\Omega^2\right) \qquad D_2 = \begin{vmatrix} \left(c - J_1\Omega^2\right) & \hat{M} \\ -c & 0 \end{vmatrix} = \hat{M}c$$

$$A_1 = \frac{D_1}{D} = \frac{\hat{M}\left(c - J_2\Omega^2\right)}{\Omega^2\left[J_1 J_2 \Omega^2 - c\left(J_1 + J_2\right)\right]} \qquad A_2 = \frac{D_2}{D} = \frac{\hat{M}c}{\Omega^2\left[J_1 J_2 \Omega^2 - c\left(J_1 + J_2\right)\right]}$$

$$\text{Rel.} - \text{Amplitude:} \quad A_2 - A_1 = \Delta A = \frac{\hat{M} J_2}{J_1 J_2 \Omega^2 - c\left(J_1 + J_2\right)}$$

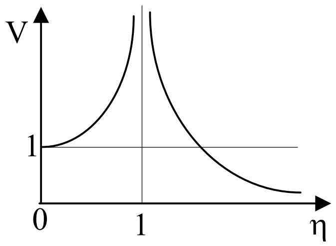
$$f_0 = \frac{\omega_0}{2\pi} \quad \omega_0 \quad \text{aus} \quad D = 0 \quad \text{für} \quad \Omega = \omega_0 \quad \omega_0^2\left[J_1 J_2 \omega_0^2 - c\left(J_1 + J_2\right)\right] = 0$$

$$\omega_0 = 0 \quad \text{(starre Drehung)}, \quad \left[J_1 J_2 \omega_0^2 - c\left(J_1 + J_2\right)\right] = 0 \quad \Rightarrow \omega_0 = \sqrt{\frac{c\left(J_1 + J_2\right)}{J_1 J_2}}$$

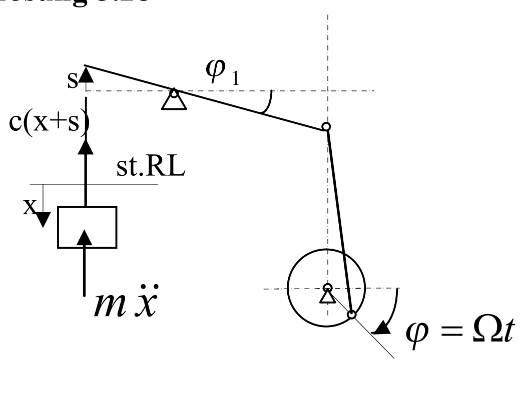
$$\text{auch} \quad L = T - U = \frac{1}{2} J_1 \dot{\varphi}_1^2 + \frac{1}{2} J_2 \dot{\varphi}_2^2 - \frac{1}{2} c\left(\varphi_1 - \varphi_2\right)^2 \quad \text{gen. Koor.:}\ \varphi_1, \varphi_2 \quad W^* = M(t) \cdot \varphi_1$$

$$\text{mit} \quad \Delta\varphi = \varphi_1 - \varphi_2 \quad \Delta\ddot{\varphi} = \ddot{\varphi}_1 - \ddot{\varphi}_2 \quad \Rightarrow \quad \Delta\ddot{\varphi} + \frac{c\left(J_1 + J_2\right)}{J_1 J_2}\Delta\varphi = \frac{\hat{M}}{J_1}\,\mathbf{sin}\,\Omega t$$

$$\Delta\varphi = -\Delta A \sin\Omega t = \frac{\hat{M} J_2}{c\left(J_1 + J_2\right)} \cdot \frac{1}{1 - \eta^2} \sin\Omega t = \Delta\varphi_{\max} \sin\Omega t \quad \Rightarrow \quad V = \frac{1}{1 - \eta^2} = \frac{\Delta\varphi_{\max}}{\dfrac{\hat{M} J_2}{c\left(J_1 + J_2\right)}}$$



Lösung 5.28



$$\varphi_1 = \frac{s}{a} = \frac{r \sin\Omega t}{b} \quad \Rightarrow \quad s = \frac{a}{b} r \sin\Omega t$$

Gleichgewicht an der Masse m:

$$m\ddot{x} + c\left(x + s\right) = 0 \quad m\ddot{x} + cx = -cs$$

$$\ddot{x} + \frac{c}{m}x = -\frac{c}{m}\frac{a}{b} r \sin\Omega t \qquad \omega_0^2 = \frac{c}{m}$$

$$\textit{Ansatz}: x_p = A \sin\Omega t \quad \ddot{x}_p = -A\Omega^2 \sin\Omega t$$

$$-A\Omega^2 \sin\Omega t + \omega_0^2 A \sin\Omega t = -\omega_0^2 \frac{a}{b} r \sin\Omega t$$

$$\left(-\Omega^2+\omega_0^2\right)A=-\omega_0^2\frac{a}{b}r \quad \Rightarrow \quad A=\frac{a}{b}\cdot\frac{r}{\eta^2-1}=\pm x_{\max} \quad mit \quad \eta=\frac{\Omega}{\omega_0}$$

$$stat.\ Auslenkung: \quad x_0=\frac{mg}{c}$$

$$\frac{mg}{c}=+\frac{a}{b}\cdot\frac{r}{\eta^2-1} \quad \Rightarrow \quad c=\frac{b}{a}\cdot\frac{mg}{r}\left(\frac{\Omega^2 m}{c}-1\right)\Big|\cdot c \quad \Rightarrow$$

$$c^2+\frac{b}{a}\cdot\frac{mg}{r}\cdot c-\frac{b}{a}\cdot\frac{m^2 g}{r}\Omega^2=0 \quad c_{1,(2)}=-\frac{b}{a}\cdot\frac{mg}{2r}\left[1\pm\sqrt{1+\frac{4ar\Omega^2}{bg}}\right]$$

$$n=150\min^{-1} \quad \Rightarrow \quad \Omega=2\pi n=15{,}708s^{-1}$$

$$c_1=21{,}23\frac{N}{cm} \quad \Rightarrow \quad \omega_{01}=\sqrt{\frac{c_1}{m}}=14{,}57s^{-1} \quad \eta_1=\frac{\Omega}{\omega_{01}}=1{,}078\ (\ddot{u}berkritisch)$$

$$x_{01}=\frac{mg}{c_1}=4{,}62cm$$

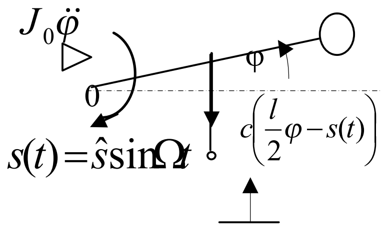
$$\frac{mg}{c}=-\frac{a}{b}\cdot\frac{r}{\eta^2-1} \quad \Rightarrow \quad c_{2,3}=\frac{b}{a}\cdot\frac{mg}{2r}\left[1\pm\sqrt{1-\frac{4ar\Omega^2}{bg}}\right]=\frac{mg}{r}\left[1\pm\sqrt{1-\frac{2r\Omega^2}{g}}\right]$$

$$c_2=97{,}8\frac{N}{cm} \quad \Rightarrow \quad \omega_{02}=\sqrt{\frac{c_2}{m}}=31{,}27s^{-1} \quad \eta_2=\frac{\Omega}{\omega_{02}}=0{,}5023\ (unterkritisch)$$

$$c_3=33\frac{N}{cm} \quad \Rightarrow \quad \omega_{03}=\sqrt{\frac{c_3}{m}}=18{,}16s^{-1} \quad \eta_3=\frac{\Omega}{\omega_{03}}=0{,}8649\ (unterkritisch)$$

x_{02} = 1,003 cm x_{03} = 2,973 cm

Lösung 5.29



$$J_0\ddot{\varphi} + c\left(\frac{l}{2}\varphi - s(t)\right)\cdot\frac{l}{2} = 0 \qquad J_0 = ml^2$$

$$\ddot{\varphi} + \frac{c}{4m}\varphi = \frac{c}{2ml}\hat{s}\sin\Omega t \qquad \omega_0 = \frac{1}{2}\sqrt{\frac{c}{m}}$$

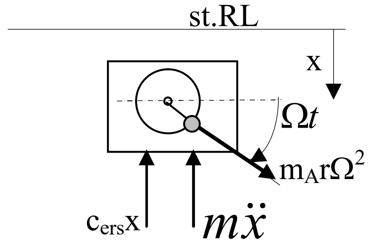
$$Ansatz: \quad \varphi_p = A\sin\Omega t \quad \ddot{\varphi}_p = -A\Omega^2\sin\Omega t$$

$$-A\Omega^2\sin\Omega t + \omega_0^2 A\sin\Omega t = \frac{c}{2ml}\hat{s}\sin\Omega t \quad \Rightarrow \quad A = \frac{c\hat{s}}{2ml\left(\omega_0^2 - \Omega^2\right)} = \frac{2\hat{s}}{l}\cdot\frac{1}{1-\eta^2} = \varphi_{\max}$$

$$\varphi(t) = \varphi_{\max}\sin\Omega t = \frac{2\hat{s}}{l}\cdot\frac{1}{1-\eta^2}\sin\Omega t$$

Lösung 5.30

Federzahl einer Blattfeder: $c = \frac{3EI}{l^3}$, $c_{ers} = 2c$



$$m\ddot{x} + c_{ers}x = m_A r\Omega^2\sin\Omega t \quad c_{ers} = \frac{6EI}{l^3}$$

$$\ddot{x} + \frac{6EI}{l^3 m}x = \frac{m_A r\Omega^2}{m}\sin\Omega t \quad \omega_0 = \sqrt{\frac{6EI}{l^3 m}}$$

$$Ansatz: \quad x_p = A\sin\Omega t \quad \ddot{x}_p = -A\Omega^2\sin\Omega t$$

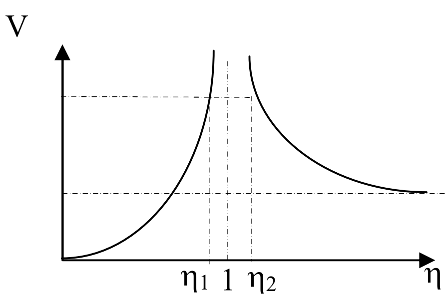
$$-A\Omega^2 + \omega_0^2 A = \frac{m_A r\Omega^2}{m} \quad \Rightarrow \quad A = \hat{x} = \frac{m_A r\Omega^2}{m\omega_0^2}\left|\frac{1}{1-\eta^2}\right|$$

$$\hat{x} = \frac{m_A r\Omega^2}{c_{ers}}\left|\frac{1}{1-\frac{\Omega^2}{\omega_0^2}}\right| = \frac{m_A r\Omega^2}{\left|c_{ers} - m\Omega^2\right|} \quad \Rightarrow \quad \left|c_{ers} - m\Omega^2\right| = \frac{m_A r\Omega^2}{\hat{x}} \quad \Rightarrow$$

$$c_{ers1} = \Omega^2\left[m + \frac{m_A r}{\hat{x}}\right] \qquad c_{ers2} = \Omega^2\left[m - \frac{m_A r}{\hat{x}}\right] \qquad \Omega = 2\pi n \quad c_{ers} = \frac{6EI}{l^3}$$

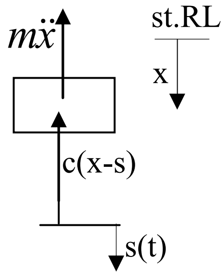
$$I_{1,2} = \frac{l^3\Omega^2}{6E}\left[m \pm \frac{m_A r}{\hat{x}}\right] \qquad \Omega = 157{,}08 s^{-1} \quad I_1 = 8466 cm^4 \quad I_2 = 8188 cm^4 \quad \Rightarrow$$

$$\omega_{01} = 158{,}2 s^{-1} \quad \Rightarrow \quad \eta_1 = 0{,}9929 \qquad \omega_2 = 155{,}95 s^{-1} \quad \Rightarrow \quad \eta_2 = 1{,}007$$



$$\boxed{\hat{x} = \frac{m_A}{m}r\left|\frac{\Omega^2}{\omega_0^2 - \Omega^2}\right| \qquad V = \frac{\eta^2}{1-\eta^2}}$$

Lösung 5.31



$$m\ddot{x}+c(x-s)=0 \quad m\ddot{x}+cx=cs$$

$$\ddot{x}+\frac{c}{m}x=\frac{c}{m}\hat{s}\,\mathbf{sin}\,\Omega t \qquad \omega_0^2=\frac{c}{m}$$

$$Ansatz: x_p=A\,\mathbf{sin}\,\Omega t \quad \ddot{x}_p=-A\Omega^2\,\mathbf{sin}\,\Omega t$$

$$-A\Omega^2\,\mathbf{sin}\,\Omega t+\omega_0^2 A\,\mathbf{sin}\,\Omega t=\omega_0^2\hat{s}\,\mathbf{sin}\,\Omega t$$

$$A=\frac{\hat{s}}{1-\eta^2} \qquad x(t)=\frac{\hat{s}}{1-\eta^2}\mathbf{sin}\,\Omega t \qquad |\hat{x}|=\left|\frac{\hat{s}}{1-\eta^2}\right|$$

1. $c_1=1000\dfrac{N}{cm} \quad \Rightarrow \quad \omega_0=\sqrt{\dfrac{c_1}{m}}=10^2\sqrt{10}s^{-1}$

   $\Omega=2\pi f_{err}=157.08s^{-1} \quad \eta^2=0{,}2457 \quad |\hat{x}|=1{,}3276mm$

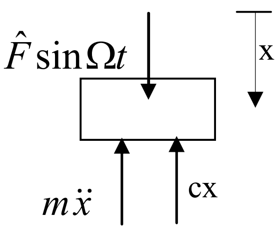
2. $|\hat{x}|=\dfrac{\hat{s}}{3} \quad \Rightarrow \quad \dfrac{\hat{s}}{3}=\left|\dfrac{\hat{s}}{1-\eta^2}\right| \quad \eta^2-1=3 \quad \eta=2=\dfrac{\Omega}{\omega_0} \quad \Rightarrow \quad \omega_0^2=\left(\dfrac{\Omega}{2}\right)^2=\dfrac{c_2}{m}$

   $c_2=m\left(\dfrac{\Omega}{2}\right)^2=61{,}68\dfrac{N}{cm}$

3. $F_{c\max}=c(A-\hat{s})+mg=c\left(\dfrac{\hat{s}}{1-\eta^2}-\hat{s}\right)+mg=c\hat{s}\left(\dfrac{\eta^2}{1-\eta^2}\right)+mg$

   $F_{c\max 1}=42{,}57N \qquad F_{c\max 2}=18{,}03N$

Lösung 5.32



$$\ddot{x}+\frac{c}{m}x=\frac{\hat{F}}{m}\sin\Omega t$$

$$L-Ansatz: \quad x_p=x_{\max}\sin\Omega t \quad \ddot{x}_p=-x_{\max}\Omega^2\sin\Omega t$$

$$-x_{\max}\Omega^2\sin\Omega t+\omega_0^2 x_{\max}\sin\Omega t=\frac{\hat{F}}{m}\sin\Omega t$$

$$x_{\max}=\frac{\hat{F}}{m(\omega_0^2-\Omega^2)}=\frac{\hat{F}}{c}\frac{1}{1-\eta^2}=\hat{x}$$

1. $c_1=4\cdot 10^3\dfrac{N}{cm} \quad \Rightarrow \omega_0^2=4\cdot 10^4 s^{-2} \quad \Omega=2\pi n=157s^{-1} \quad \Omega^2=2{,}467\cdot 10^4 s^{-2} \quad \eta^2=0{,}6169$

   $\hat{x}=0{,}0652cm$

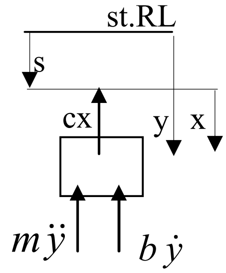
2. $F_{dyn}=cx=\dfrac{\hat{F}}{1-\eta^2}\sin\Omega t \quad F_{Fund.}=mg+F_{dyn} \quad \left(\dfrac{F_{dyn\max}}{\hat{F}}\right)=\dfrac{1}{1-\eta^2}$

   $\eta>1 \quad |\eta^2-1|=\left|\dfrac{\hat{F}}{F_{dyn\max}}\right| \quad \eta=\dfrac{\Omega}{\omega_0}, \quad \omega_0^2=\dfrac{c}{m} \quad \Rightarrow \quad c=\dfrac{m\Omega^2}{\left|\dfrac{\hat{F}}{F_{dyn\max}}\right|+1}=2{,}74\cdot 10^2\dfrac{N}{cm}$

3. $mit\ c_1: \quad F_{dyn\max}=c_1\hat{x}=260{,}8N \quad \Rightarrow \quad F_{Fund}=358{,}8N$

   $mit\ c: \quad F_{dyn\max}=\dfrac{1}{8}\hat{F}=12{,}5N \quad \Rightarrow \quad F_{Fund}=110{,}6N$

Lösung 5.33



$c=\frac{3EI}{l^3}$ $\quad y=x+s \quad \dot{y}=\dot{x}+\dot{s} \quad \ddot{y}=\ddot{x}+\ddot{s} \quad x=y-s$

y – Absolutbewegung der Masse m

x – Relativbewegung der Masse m

$m\ddot{y}+b\dot{y}+c(y\text{-}s)=0 \qquad \ddot{y}+\frac{b}{m}\dot{y}+\frac{c}{m}y=\frac{c}{m}\cdot\hat{s}\,\mathbf{sin}\,\Omega t$ oder

$\ddot{x}+\frac{b}{m}\dot{x}+\frac{c}{m}x=\hat{s}\Omega^2\,\mathbf{sin}\,\Omega t-\frac{b}{m}\hat{s}\Omega\,\mathbf{cos}\,\Omega t$

Ansatz für die partikuläre Lösung:

$x=A\sin\Omega t+B\cos\Omega t \quad \dot{x}=A\Omega\cos\Omega t-B\Omega\sin\Omega t \quad \ddot{x}=-A\Omega^2\sin\Omega t-B\Omega^2\cos\Omega t$

in die Dgl. eingesetzt und Koeffizientenvergleich:

$A\left(\frac{c}{m}-\Omega^2\right)-\frac{b}{m}\Omega B=\hat{s}\Omega^2 \quad und \quad \frac{b}{m}\Omega A+B\left(\frac{c}{m}-\Omega^2\right)=-\frac{b}{m}\hat{s}\Omega$

$mit \quad \frac{c}{m}=\omega_0^2 \quad \frac{b}{m}=2\delta \quad \frac{\Omega}{\omega_0}=\eta \quad \frac{\delta}{\omega_0}=\vartheta$ folgt

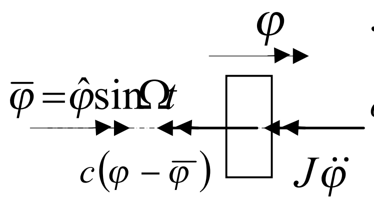
$\mathrm{A}=\hat{s}\eta^2\frac{1-\eta^2-4\vartheta^2}{(1-\eta^2)^2+4\eta^2\vartheta^2} \qquad B=-2\vartheta\eta\hat{s}\frac{1}{(1-\eta^2)^2+4\eta^2\vartheta^2}$

$x(t)=A\sin\Omega t+B\cos\Omega t=C\sin(\Omega t-\varphi^*) \quad mit \quad C=\sqrt{A^2+B^2} \quad \tan\varphi^*=-\frac{B}{A}$

$C=\frac{\hat{s}\eta}{(1-\eta^2)^2+4\eta^2\vartheta^2}\sqrt{\eta^2(1-\eta^2-4\vartheta^2)^2+4\vartheta^2} \qquad \tan\varphi^*=\frac{2\vartheta}{\eta(1-\eta^2-4\vartheta^2)}$

φ^* ist der Phasenwinkel zwischen s und x.

Lösung 5.34



$$J\ddot{\varphi} + c(\varphi - \overline{\varphi}) = 0 \qquad \ddot{\varphi} + \frac{c}{J}\varphi = \frac{c\hat{\varphi}}{J}\sin\Omega t \quad mit$$

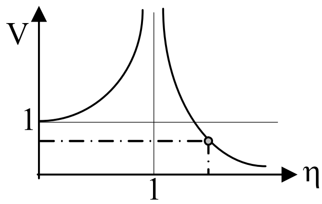
$$\omega_0^2 = \frac{c}{J} \quad J = \frac{1}{2}MR^2 \quad c = \frac{c_1 c_2}{c_1 + c_2}$$

$$c_1 = \frac{G\pi d_1^4}{32l} \quad c_2 = \frac{G\pi d_2^4}{32l} \quad \Rightarrow \quad c = \frac{G\pi}{32l}\cdot\frac{d_1^4 d_2^4}{\left(d_1^4 + d_2^4\right)} \qquad \omega_0 = \sqrt{\frac{G\pi}{16l}\frac{d_1^4 d_2^4}{\left(d_1^4 + d_2^4\right)MR^2}}$$

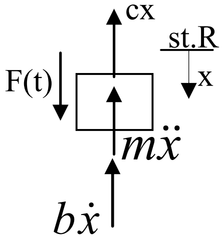
$$\varphi_p = A\sin\Omega t \quad \ddot{\varphi}_p = -A\Omega^2\sin\Omega t \quad \Rightarrow \quad -A\Omega^2 + A\omega_0^2 = \omega_0^2\hat{\varphi} \quad A = \frac{\hat{\varphi}}{1-\eta^2} \quad \eta = \frac{\Omega}{\omega}$$

$$\varphi_p = \hat{\varphi}\frac{1}{1-\eta^2}\sin\Omega t = \hat{\varphi}_p\sin\Omega t \quad V = \frac{\hat{\varphi}_p}{\hat{\varphi}} = \frac{1}{1-\eta^2}$$

Zahlenwerte: $\omega_0 = 32{,}39 s^{-1} \quad \eta = 1{,}544 \quad V(\eta) = 0{,}723$



Lösung 5.35



$$m\ddot{x} + b\dot{x} + cx = F(t)$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{c}{m}x = \frac{\hat{F}}{m}\sin\Omega t \qquad \ddot{x} + 2\delta\dot{x} + \omega_0^2 x = \overline{Q}\sin\Omega t$$

Partikuläre Lösung:

$$x(t) = \frac{\overline{Q}}{\omega_0^2}\frac{1}{\sqrt{\left(1-\eta^2\right)^2 + 4\vartheta^2\eta^2}}\sin\left(\Omega t - \varphi_0\right)$$

$$x(t) = \hat{x}\sin\left(\Omega t - \varphi_0\right) \quad mit \quad \hat{x} = \frac{\hat{F}}{c}\cdot\frac{1}{\sqrt{\left(1-\eta^2\right)^2 + 4\vartheta^2\eta^2}} = \hat{x}(\eta)$$

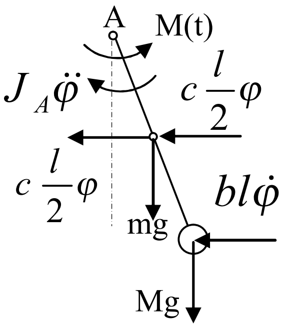
$$Maximalausschlag: \quad \frac{d\hat{x}}{d\eta} = 0 \quad \Rightarrow \quad \eta^* = \sqrt{1-2\vartheta^2} \quad und \quad \hat{x}_{\max} = \frac{\hat{F}}{c}\cdot\frac{1}{2\vartheta\sqrt{1-\vartheta^2}} \quad mit$$

$$\vartheta = \frac{\delta}{\omega_0} \quad \eta = \frac{\Omega}{\omega_0} \quad \delta = \frac{b}{2m} \quad \Rightarrow \quad \vartheta = \frac{b}{2m\sqrt{\frac{c}{m}}} \qquad \hat{x}_{\max} = \frac{\hat{F}m}{bc}\frac{\sqrt{\frac{c}{m}}}{\sqrt{1-\frac{b^2}{4cm}}}$$

Aufgelöst nach c: $c = \frac{m\hat{F}^2}{\hat{x}_{\max}^2 b^2} + \frac{b^2}{4m} = 320{,}31\frac{N}{cm} \quad \Rightarrow \quad \omega_0 = 40{,}02\,s^{-1}$

$$\eta^* = \sqrt{1-2\vartheta^2} = \frac{\Omega_1}{\omega_0} \quad \Rightarrow \quad \Omega_1 = \omega_0\sqrt{1-2\vartheta^2} = 39{,}98\,s^{-1}$$

Lösung 5.36



$$J_A\ddot{\varphi} + 2c\left(\frac{l}{2}\right)^2\varphi + mg\frac{l}{2}\varphi + Mgl\varphi + bl^2\dot{\varphi} = M(t)$$

$$\ddot{\varphi} + \frac{bl^2}{J_A}\dot{\varphi} + \frac{l\left(c\frac{l}{2} + \frac{1}{2}mg + Mg\right)}{J_A}\varphi = \frac{\hat{M}}{J_A}\sin\Omega t$$

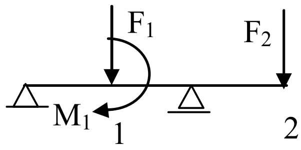
$$\ddot{\varphi} + 2\delta\dot{\varphi} + \omega_0^2\varphi = \overline{Q}\sin\Omega t \quad J_A = Ml^2 + \frac{1}{3}ml^2 = l^2\left(M + \frac{1}{3}m\right)$$

$$\varphi_p = \frac{\overline{Q}}{\omega_0^2}\cdot\frac{1}{\sqrt{\left(1-\eta^2\right)^2 + 4\vartheta^2\eta^2}}\sin\left(\Omega t - \varphi_0\right) \qquad \varphi_0 = \arctan\frac{2\vartheta\eta}{1-\eta^2}$$

$$\vartheta = \frac{\delta}{\omega_0} \quad \eta = \frac{\Omega}{\omega_0} \qquad \varphi_p = \frac{\hat{M}}{J_A\omega_0^2\sqrt{\left(1-\eta^2\right)^2 + 4\vartheta^2\eta^2}}\sin\left(\Omega t - \varphi_0\right) = \hat{\varphi}\sin\left(\Omega t - \varphi_0\right)$$

$$\left|\hat{\varphi}\right| = \left|\frac{\hat{M}}{J_A\omega_0^2\sqrt{\left(1-\eta^2\right)^2 + 4\vartheta^2\eta^2}}\right| = \left|\frac{\hat{M}}{\left(\frac{1}{2}cl^2 + \frac{1}{2}mgl + Mgl\right)\sqrt{\left(1-\eta^2\right)^2 + 4\vartheta^2\eta^2}}\right| \quad \vartheta = \frac{bl^2}{2J_A\omega_0}$$

Lösung 5.37



Mit Hilfe der Einflußzahlen gilt:

$$y_1 = \alpha_{11}F_1 + \alpha_{12}F_2 + \gamma_{11}M_1$$
$$y_2 = \alpha_{21}F_1 + \alpha_{22}F_2 + \gamma_{21}M_1$$
$$\varphi_1 = \delta_{11}F_1 + \delta_{12}F_2 + \beta_{11}M_1$$
$$F_1 = -m_1\ddot{y}_1 \qquad F_2 = -m_2\ddot{y}_2 \qquad M_1 = -J_1\ddot{\varphi}_1$$

Ansatz für die harmonische Schwingung:

$$y_1 = A_1 \sin\omega t \quad \ddot{y}_1 = -A_1\omega^2 \sin\omega t \quad y_2 = A_2 \sin\omega t \quad \ddot{y}_2 = -A_2\omega^2 \sin\omega t$$
$$\varphi_1 = A_3 \sin\omega t \quad \ddot{\varphi}_1 = -A_3\omega^2 \sin\omega t \quad \text{eingesetzt:}$$
$$A_1\left(\alpha_{11}m_1\omega^2 - 1\right) + A_2\alpha_{12}m_2\omega^2 + A_3\gamma_{11}J_1\omega^2 = 0$$
$$A_1\alpha_{21}m_1\omega^2 + A_2\left(\alpha_{22}m_2\omega^2 - 1\right) + A_3\gamma_{21}J_1\omega^2 = 0$$
$$A_1\delta_{11}m_1\omega^2 + A_2\delta_{12}m_2\omega^2 + A_3\left(\beta_{11}J_1\omega^2 - 1\right) = 0$$

Koeffizientendeterminante:

$$\begin{vmatrix} \left(\alpha_{11}m_1\omega^2 - 1\right) & \alpha_{12}m_2\omega^2 & \gamma_{11}J_1\omega^2 \\ \alpha_{21}m_1\omega^2 & \left(\alpha_{22}m_2\omega^2 - 1\right) & \gamma_{21}J_1\omega^2 \\ \delta_{11}m_1\omega^2 & \delta_{12}m_2\omega^2 & \left(\beta_{11}J_1\omega^2 - 1\right) \end{vmatrix} = 0$$

Für $J_1 = 0$ folgt:

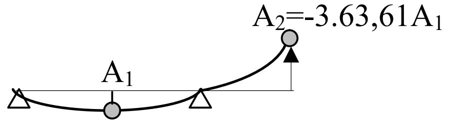
$$\begin{vmatrix} \left(\alpha_{11}m_1\omega^2 - 1\right) & \alpha_{12}m_2\omega^2 \\ \alpha_{21}m_1\omega^2 & \left(\alpha_{22}m_2\omega^2 - 1\right) \end{vmatrix} = 0 = \left(\alpha_{11}m_1\omega^2 - 1\right)\left(\alpha_{22}m_2\omega^2 - 1\right) - \alpha_{12}m_2\omega^2\alpha_{21}m_1\omega^2$$

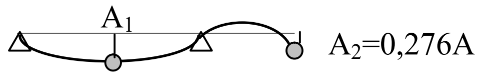
$$\alpha_{12} = \alpha_{21}$$
$$\omega^4\left[m_1m_2\left(\alpha_{11}\alpha_{22} - \alpha_{12}^2\right)\right] - \omega^2\left(\alpha_{11}m_1 + \alpha_{22}m_2\right) + 1 = 0$$
$$\omega^4 - \frac{\left(\alpha_{11}m_1 + \alpha_{22}m_2\right)}{\left[m_1m_2\left(\alpha_{11}\alpha_{22} - \alpha_{12}^2\right)\right]}\omega^2 + \frac{1}{\left[m_1m_2\left(\alpha_{11}\alpha_{22} - \alpha_{12}^2\right)\right]} = 0$$
$$\omega_{1,2}^2 = \frac{\left(\alpha_{11}m_1 + \alpha_{22}m_2\right)}{2\left[m_1m_2\left(\alpha_{11}\alpha_{22} - \alpha_{12}^2\right)\right]} \pm \sqrt{\left\{\frac{\left(\alpha_{11}m_1 + \alpha_{22}m_2\right)}{2\left[m_1m_2\left(\alpha_{11}\alpha_{22} - \alpha_{12}^2\right)\right]}\right\}^2 - \frac{1}{\left[m_1m_2\left(\alpha_{11}\alpha_{22} - \alpha_{12}^2\right)\right]}}$$
$$m_1 = m_2 = m$$
$$\omega_{1,2}^2 = \frac{1}{2m\left(\alpha_{11}\alpha_{22} - \alpha_{12}^2\right)}\left\{\left(\alpha_{11} + \alpha_{22}\right) \pm \sqrt{\left(\alpha_{11} - \alpha_{22}\right)^2 + 4\alpha_{12}^2}\right\}$$

Einflußzahlen: $\alpha_{11} = \dfrac{a^3}{6EI} \qquad \alpha_{22} = \dfrac{a^3}{EI} \qquad \alpha_{12} = -\dfrac{a^3}{4EI}$

$$\omega_{1,2}^2 = \frac{4EI}{5ma^3}\left(7 \mp \sqrt{34}\right) \qquad \omega_1^2 = 0{,}9352\frac{EI}{ma^3} \quad \omega_1 = 0{,}9670\sqrt{\frac{EI}{ma^3}}$$
$$\omega_2^2 = 10{,}2648\frac{EI}{ma^3} \quad \omega_2 = 3{,}2039\sqrt{\frac{EI}{ma^3}}$$

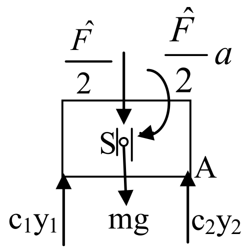
$$\frac{A_2}{A_1} = -\frac{\alpha_{11}m_1\omega^2 - 1}{\alpha_{12}m_2\omega^2} \qquad \omega_1^2 = 0{,}9352\frac{EI}{ma^3} \;\Rightarrow\; \frac{A_2}{A_1} = -3{,}6128$$
$$\omega_2^2 = 10{,}2648\frac{EI}{ma^3} \;\Rightarrow\; \frac{A_2}{A_1} = 0{,}277$$





Lösung 5.38

Erregerkraft: $F(t)=\hat{F}\cos^2\Omega t=\hat{F}\cdot\frac{1}{2}(1+\cos 2\Omega t)=\frac{1}{2}\hat{F}+\frac{1}{2}\hat{F}\cos 2\Omega t=F_{sta}+F_{dyn}(t)$ Statischer Anteil:

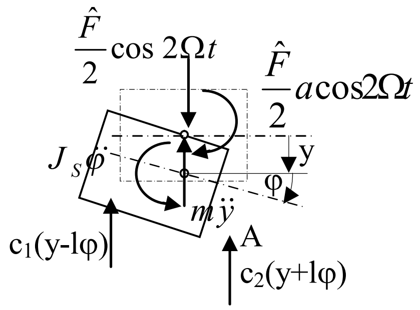


$$c_1y_1+c_2y_2-mg-\frac{\hat{F}}{2}=0 \quad (1)$$

$$\frac{\hat{F}}{2}a+c_1y_1l-c_2y_2l=0 \qquad (2) \quad \Rightarrow \quad c_1y_1=c_2y_2-\frac{\hat{F}}{2}\frac{a}{l} \quad in\ Gl.(1)$$

$$2c_2y_2-mg-\frac{\hat{F}}{2}\left(1+\frac{a}{l}\right)=0 \qquad y_2=v_{Asta}=\frac{mg}{2c_2}+\frac{\hat{F}}{4c_2}\left(1+\frac{a}{l}\right)$$

Dynamischer Anteil:



$$m\ddot{y}+c_1(y-l\varphi)+c_2(y+l\varphi)=\frac{\hat{F}}{2}\cos 2\Omega t$$

$$J_S\ddot{\varphi}+c_2(y+l\varphi)l-c_1(y-l\varphi)l=\frac{\hat{F}}{2}a\cos 2\Omega t$$

Lösungsansätze:

$$y=A_1\cos 2\Omega t \qquad \ddot{y}=-4A_1\Omega^2\cos 2\Omega t$$

$$\varphi=A_2\cos 2\Omega t \qquad \ddot{\varphi}=-4A_2\Omega^2\cos 2\Omega t \quad eingesetzt$$

$$A_1\left(c_1 + c_2 - 4m\Omega^2\right) + A_2 l\left(c_2 - c_1\right) = \frac{\hat{F}}{2} \quad und \quad A_1\left(c_2 - c_1\right) + A_2 l\left(c_1 + c_2 - 4J_S\frac{\Omega^2}{l^2}\right) = \frac{\hat{F}}{2}\frac{a}{l}$$

oder

$$\begin{vmatrix} \left(c_1 + c_2 - 4m\Omega^2\right) & \left(c_2 - c_1\right) \\ \left(c_2 - c_1\right) & \left(c_1 + c_2 - 4J_S\dfrac{\Omega^2}{l^2}\right) \end{vmatrix} \cdot \begin{vmatrix} A_1 \\ lA_2 \end{vmatrix} = \frac{\hat{F}}{2} \cdot \begin{vmatrix} 1 \\ \dfrac{a}{l} \end{vmatrix}$$

$$A_1 = \frac{D_1}{D} = \frac{\dfrac{\hat{F}}{2}\left\{c_1\left(1 + \dfrac{a}{l}\right) + c_2\left(1 - \dfrac{a}{l}\right) - 4J_S\dfrac{\Omega^2}{l^2}\right\}}{4\left\{c_1 c_2 + 4mJ_S\dfrac{\Omega^4}{l^2} - \left(c_1 + c_2\right)\Omega^2\left(m + \dfrac{J_S}{l^2}\right)\right\}}$$

$$A_2 l = \frac{D_2}{D} = \frac{\dfrac{\hat{F}}{2}\left\{c_1\left(1 + \dfrac{a}{l}\right) - c_2\left(1 - \dfrac{a}{l}\right) - 4m\Omega^2\dfrac{a}{l}\right\}}{4\left\{c_1 c_2 + 4mJ_S\dfrac{\Omega^4}{l^2} - \left(c_1 + c_2\right)\Omega^2\left(m + \dfrac{J_S}{l^2}\right)\right\}}$$

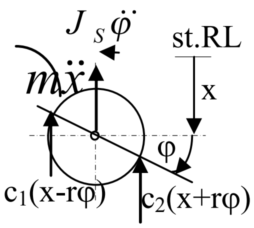
$$v_{Adyn} = y + l\varphi = \left(A_1 + lA_2\right)\cos 2\Omega t \qquad v_{Ages} = v_{Asta} + v_{Adyn}$$

Für $c_1 = c_2 = c$ entkoppeln sich die Bewegungsgleichungen.

$$m\ddot{y} + 2cy = \frac{\hat{F}}{2}\mathbf{cos}\,2\Omega t \quad und \quad J_S\ddot{\varphi} + 2cl^2\varphi = \frac{\hat{F}}{2}a\,\mathbf{cos}\,2\Omega t$$ und man erhält

$$v_{Ages} = \frac{\hat{F}}{4c}\left\{1 + \frac{a}{l} + \frac{2mg}{\hat{F}} + \left(\frac{1}{1 - 2\frac{m}{c}\Omega^2} + \frac{\frac{a}{l}}{1 - 2\frac{J_S\Omega^2}{cl^2}}\right)\mathbf{cos}\,2\Omega t\right\}$$

Lösung 5.39



$$m\ddot{x} + c_1(x - r\varphi) + c_2(x + r\varphi) = 0$$
$$J_S\ddot{\varphi} + c_2(x + r\varphi)r - c_1(x - r\varphi)r = 0 \quad oder$$
$$m\ddot{x} + (c_2 + c_1)x + (c_2 - c_1)r\varphi = 0$$
$$J_S\ddot{\varphi} + (c_2 - c_1)rx + (c_2 + c_1)r^2\varphi = 0$$
$$x = A\sin\omega t \quad \ddot{x} = -A\omega^2\sin\omega t \quad \varphi = B\sin\omega t \quad \ddot{\varphi} = -B\omega^2\sin\omega t$$

$$\begin{vmatrix} -m\omega^2 + (c_1 + c_2) & r(c_2 - c_1) \\ r(c_2 - c_1) & -J_S\omega^2 + r^2(c_1 + c_2) \end{vmatrix} \cdot \begin{vmatrix} A \\ B \end{vmatrix} = 0 \qquad \text{hom. Glsyst.} \qquad \text{Det}\,D = 0 \;\Rightarrow$$

$$\begin{vmatrix} -m\omega^2 + (c_1 + c_2) & r(c_2 - c_1) \\ r(c_2 - c_1) & -J_S\omega^2 + r^2(c_1 + c_2) \end{vmatrix} = 0$$

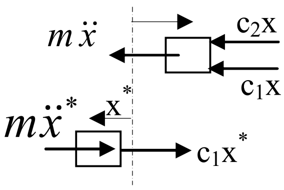
$$\left[-m\omega^2 + (c_1 + c_2)\right]\cdot\left[-J_S\omega^2 + r^2(c_1 + c_2)\right] - r^2(c_2 - c_1)^2 = 0 \;\Rightarrow$$

$$\omega^4 - \frac{(c_1 + c_2)(J_S + mr^2)}{mJ_S}\omega^2 + \frac{4r^2c_1c_2}{mJ_S} = 0$$

$$\omega_{1,2}^2 = \frac{(c_1 + c_2)(J_S + mr^2)}{2mJ_S}\left[1 \mp \sqrt{1 - \frac{16r^2c_1c_2mJ_S}{(c_1 + c_2)^2(J_S + mr^2)^2}}\right]$$

Speziell: $\omega_{1,2}^2 = \frac{9}{2}\frac{c}{m}\left[1 \mp \sqrt{1 - \frac{64}{81}}\right] = \frac{9}{2}\frac{c}{m}\left[1 \mp \frac{\sqrt{17}}{9}\right] \;\Rightarrow\; \omega_1 = 1{,}56\sqrt{\frac{c}{m}} \qquad \omega_2 = 2{,}56\sqrt{\frac{c}{m}}$

Lösung 5.40



$$m\ddot{x} + (c_1 + c_2)x = 0 \quad \ddot{x} + \frac{c_1 + c_2}{m}x = 0 \quad \ddot{x} + \omega_1^2x = 0$$
$$x = A\sin\omega_1 t + B\cos\omega_1 t \quad \dot{x} = A\omega_1\cos\omega_1 t - B\omega_1\sin\omega_1 t$$
$$AB: t = 0: \quad x = x_0 \quad\Rightarrow\quad B = x_0 \quad \dot{x} = 0 \quad\Rightarrow\quad A = 0$$
$$\Rightarrow: x(t) = x_0\cos\omega_1 t \quad \dot{x}(t) = -x_0\omega_1\sin\omega_1 t \quad \ddot{x}(t) = -\omega_1^2x(t)$$
$$\ddot{x}_{\max} = \omega_1^2x_0 \quad \omega_1 = \sqrt{\frac{c_1 + c_2}{m}} = 2\sqrt{\frac{c_1}{m}} \quad \dot{x}_{\max} = x_0\omega_1$$

$$t = t_1 \quad x = 0 \quad\Rightarrow\quad x_0\cos\omega_1 t_1 = 0 \quad \omega_1 t_1 = \frac{\pi}{2} \quad\Rightarrow\quad t_1 = \frac{\pi}{2\omega_1} \quad und\ \dot{x}(t_1) = -x_0\omega_1\sin\frac{\pi}{2} = -x_0\omega_1$$

Bewegung in Richtung x^*:

$$m\ddot{x}^* + c_1x^* = 0 \quad \ddot{x}^* + \frac{c_1}{m}x^* = 0 \quad \omega_2 = \sqrt{\frac{c_1}{m}} \quad x^* = C\sin\omega_2 t + D\cos\omega_2 t$$

Neue Zeitzählung:

$$t=0:\quad x^*=0 \quad\Rightarrow\quad D=0 \qquad \dot{x}^*=-\dot{x}(t_1) \quad\Rightarrow\quad C\omega_2=x_0\omega_1 \quad C=x_0\frac{\omega_1}{\omega_2}$$

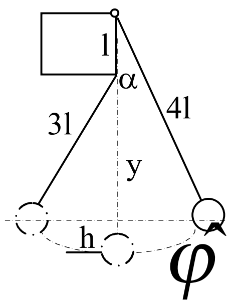
$$x^*(t)=x_0\frac{\omega_1}{\omega_2}\sin\omega_2 t \quad \dot{x}^*(t)=x_0\omega_1\cos\omega_2 t \quad \ddot{x}^*(t)=-x_0\omega_1\omega_2\sin\omega_2 t$$

$$\textit{linkes Ende}:\quad \dot{x}^*=0 \quad\Rightarrow\quad x_0\omega_1\cos\omega_2 t_2=0 \quad \omega_2 t_2=\frac{\pi}{2} \quad t_2=\frac{\pi}{2\omega_2} \quad \ddot{x}^*_{\max}=-x_0\omega_1\omega_2$$

$$T=\frac{1}{2}(T_1+T_2)=\pi\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)=\pi\left(\frac{1}{2}\sqrt{\frac{m}{c_1}}+\sqrt{\frac{m}{c_1}}\right)=\frac{3\pi}{2}\sqrt{\frac{m}{c_1}}=0{,}149\,s$$

$$\dot{x}_{\max}=-x_0 2\sqrt{\frac{c_1}{m}}=-0{,}632\frac{m}{s} \qquad \ddot{x}_{\max}=-x_0 4\frac{c_1}{m}=-40\frac{m}{s^2} \qquad \ddot{x}^*_{\max}=-x_0 2\frac{c_1}{m}=-20\frac{m}{s^2}$$

Lösung 5.41



$$T=\frac{T_1}{2}+\frac{T_2}{2} \quad T_1=\frac{2\pi}{\omega_1} \quad T_2=\frac{2\pi}{\omega_2} \quad \omega_1=\sqrt{\frac{g}{4l}} \quad \omega_2=\sqrt{\frac{g}{3l}}$$

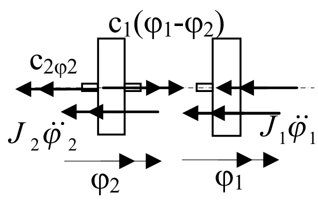
$$T=\pi\sqrt{\frac{l}{g}}\left(2+\sqrt{3}\right)=11{,}72\sqrt{\frac{l}{g}}$$

$$y=3l-h=3l\cos\hat{\varphi} \quad h=4l(1-\cos\alpha) \quad\Rightarrow$$

$$3l\cos\hat{\varphi}=3l-4l(1-\cos\alpha) \quad \cos\hat{\varphi}=\frac{4}{3}\cos\alpha-\frac{1}{3}$$

$$\hat{\varphi}=arc\cos\frac{4\cos\alpha-1}{3}$$

Lösung 5.42



$$J_1\ddot{\varphi}_1+c_1(\varphi_1-\varphi_2)=0$$

$$J_2\ddot{\varphi}_2-c_1(\varphi_1-\varphi_2)+c_2\varphi_2=0$$

$$\textit{Ansatz}:\varphi_1=A_1\sin\omega t \quad \varphi_2=A_2\sin\omega t \quad \ddot{\varphi}_i=-A_i\omega^2\sin\omega t$$

$$\left(c_1-J_1\omega^2\right)A_1-c_1A_2=0 \quad -c_1A_1+\left(c_1+c_2-J_2\omega^2\right)A_2=0$$

Ein homogenes Gleichungssystem hat nichttriviale Lösungen, wenn die Koeffizientendeterminante verschwindet.

$$\begin{vmatrix}\left(c_1-J_1\omega^2\right) & -c_1\\ -c_1 & \left(c_1+c_2-J_2\omega^2\right)\end{vmatrix}=0 \quad\Rightarrow\quad \left(c_1-J_1\omega^2\right)\left(c_1+c_2-J_2\omega^2\right)-c_1^2=0 \quad \text{oder}$$

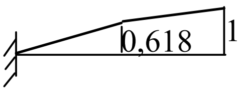
$$\omega^4-\frac{J_1(c_1+c_2)+J_2c_1}{J_1J_2}\omega^2+\frac{c_1c_2}{J_1J_2}=0 \quad \text{mit} \quad c_1=c_2=c \quad \text{und} \quad J_1=J_2=J \quad \text{folgt}$$

$$\omega^4-\frac{3c}{J}\omega^2+\frac{c^2}{J^2}=0 \qquad \omega_{1,2}^2=\frac{3c}{2J}\left[1\mp\sqrt{1-\frac{4}{9}}\right] \quad \omega_1^2=0{,}382\frac{c}{J} \quad \omega_2^2=2{,}62\frac{c}{J}$$

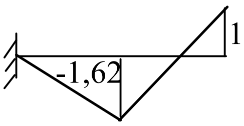
Schwingformen:

Aus der ersten Gleichung des Gleichungssystems folgt: $\dfrac{A_2}{A_1} = 1 - \dfrac{J\omega^2}{c}$

$\omega^2 = \omega_1^2: \quad \dfrac{A_2}{A_1} = 0{,}618 \qquad\qquad \omega^2 = \omega_2^2: \quad \dfrac{A_2}{A_1} = -1{,}62$

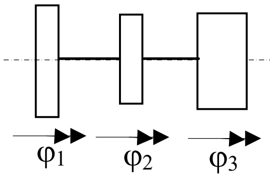


Grundschwingung



Oberschwingung

Lösung 5.43



$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_i}\right) - \frac{\partial L}{\partial \varphi_i} = 0 \qquad i = 1,2,3$$

$$L = T - U$$

$$T = \frac{1}{2}\left(J_1\dot{\varphi}_1^2 + J_2\dot{\varphi}_2^2 + J_3\dot{\varphi}_3^2\right) \quad U = \frac{1}{2}\left[c_1(\varphi_2 - \varphi_1)^2 + c_2(\varphi_3 - \varphi_2)^2\right]$$

$$L = \frac{1}{2}\left(J_1\dot{\varphi}_1^2 + J_2\dot{\varphi}_2^2 + J_3\dot{\varphi}_3^2\right) - \frac{1}{2}\left[c_1(\varphi_2 - \varphi_1)^2 + c_2(\varphi_3 - \varphi_2)^2\right]$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right) = J_1\ddot{\varphi}_1 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_2}\right) = J_2\ddot{\varphi}_2 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_3}\right) = J_3\ddot{\varphi}_3$$

$$\frac{\partial L}{\partial \varphi_1} = c_1(\varphi_2 - \varphi_1) \qquad \frac{\partial L}{\partial \varphi_2} = -c_1(\varphi_2 - \varphi_1) + c_2(\varphi_3 - \varphi_2) \qquad \frac{\partial L}{\partial \varphi_3} = -c_2(\varphi_3 - \varphi_2)$$

$$J_1\ddot{\varphi}_1 - c_1(\varphi_2 - \varphi_1) = 0 \quad J_2\ddot{\varphi}_2 + c_1(\varphi_2 - \varphi_1) - c_2(\varphi_3 - \varphi_2) = 0 \quad J_3\ddot{\varphi}_3 + c_2(\varphi_3 - \varphi_2) = 0$$

Eigenfrequenzen:

$$Ansatz: \quad \varphi_1 = Ae^{i\omega t} \qquad \varphi_2 = Be^{i\omega t} \qquad \varphi_3 = Ce^{i\omega t}$$

$$\begin{vmatrix} (c_1 - J_1\omega^2) & -c_1 & 0 \\ -c_1 & (c_1 + c_2 - J_2\omega^2) & -c_2 \\ 0 & -c_2 & (c_2 - J_3\omega^2) \end{vmatrix} \cdot \begin{vmatrix} A \\ B \\ C \end{vmatrix} = 0 \qquad \det(A) = 0 \quad \Rightarrow$$

$$\omega^6 - \left(\frac{c_1}{J_1} + \frac{c_1 + c_2}{J_2} + \frac{c_2}{J_3}\right)\omega^4 + c_1c_2\frac{J_1 + J_2 + J_3}{J_1J_2J_3}\omega^2 = 0$$

$$\omega_3^2 = 0 \quad \omega_{1,2}^2 = \frac{1}{2}\left(\frac{c_1}{J_1} + \frac{c_1 + c_2}{J_2} + \frac{c_2}{J_3}\right) \mp \sqrt{\frac{1}{4}\left(\frac{c_1}{J_1} + \frac{c_1 + c_2}{J_2} + \frac{c_2}{J_3}\right)^2 - c_1c_2\frac{J_1 + J_2 + J_3}{J_1J_2J_3}}$$

Bewegungsformen:

Amplitudenverhältnisse:

$$\beta_i = \left(\frac{B}{A}\right)_i = 1 - \omega_i^2 \frac{J_1}{c_1} \quad \gamma_i = \left(\frac{C}{A}\right)_i = \left(1 - \omega_i^2 \frac{J_1}{c_1}\right)\left(1 - \omega_i^2 \frac{J_2}{c_2}\right) - \omega_i^2 \frac{J_1}{c_2} \quad i = 1,2,3 \quad und$$

$$\varphi_1 = A_1 \sin\omega_1 t + A_2 \cos\omega_1 t + A_3 \sin\omega_2 t + A_4 \cos\omega_2 t + A_5 + A_6 t$$

$$\varphi_2 = \beta_1 A_1 \sin\omega_1 t + \beta_1 A_2 \cos\omega_1 t + \beta_2 A_3 \sin\omega_2 t + \beta_2 A_4 \cos\omega_2 t + A_5 + A_6 t$$

$$\varphi_3 = \gamma_1 A_1 \sin\omega_1 t + \gamma_1 A_2 \cos\omega_1 t + \gamma_2 A_3 \sin\omega_2 t + \gamma_2 A_4 \cos\omega_2 t + A_5 + A_6 t$$

Zahlenbeispiel:

Mit $J_1 = J \quad J_2 = 2J \quad J_3 \Rightarrow \infty \quad c_1 = 2c \quad c_2 = 4c$ folgt

$$\omega_1^2 = \frac{c}{J} \qquad \omega_2^2 = 4\frac{c}{J} \qquad \omega_3^2 = 0 \qquad \beta_1 = \frac{1}{2} \quad \beta_2 = -1 \quad \gamma_1 = 0 \quad \gamma_2 = 0 \quad \beta_3 = \gamma_3 = 1$$

und wegen $\varphi_3 = 0 = A_5 + A_6 t$ folgt

$$\varphi_1 = A_1 \sin\omega_1 t + A_2 \cos\omega_1 t + A_3 \sin\omega_2 t + A_4 \cos\omega_2 t$$

$$\varphi_2 = \frac{1}{2} A_1 \sin\omega_1 t + \frac{1}{2} A_2 \cos\omega_1 t - A_3 \sin\omega_2 t - A_4 \cos\omega_2 t$$

Für Schwingungen in der Grundschwingungsform mit ω_1 gilt:

$$A_3 = 0 \quad A_4 = 0$$

$$AB: \quad t = 0: \quad \varphi_1(0) = \varphi_0 \quad \dot{\varphi}_1(0) = 0 \quad \Rightarrow \quad A_2 = \varphi_0 \quad A_1 = 0$$

$$\varphi_1(t) = \varphi_0 \cos\omega_1 t \qquad \varphi_2(t) = \frac{1}{2}\varphi_0 \cos\omega_1 t \quad \Rightarrow \quad \varphi_2(0) = \frac{1}{2}\varphi_0 \quad \dot{\varphi}_2(0) = 0$$

Stoßvorgänge

Lösung 6.1

Impulserh. $m_1v_1 - m_2v_2 = (m_1 + m_2)v \qquad v = \dfrac{m_1v_1 - m_2v_2}{m_1 + m_2}$

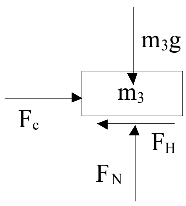
Energies. $(T + U + W^*)_I = (T + U)_{II}$ $\quad$ I – unmittelbar nach dem Stoß $\quad$ II – am Ende

$$\frac{1}{2}(m_1 + m_2)v^2 - F_R \cdot s = 0 \qquad F_R = \mu F_N \qquad F_N = (m_1 + m_2)g$$

$$\frac{1}{2}\frac{(m_1v_1 - m_2v_2)^2}{(m_1 + m_2)^2} - \mu gs = 0 \quad m_1^2v_1^2 - 2m_1m_2v_1v_2 + m_2^2v_2^2 - 2(m_1 + m_2)^2\mu gs = 0$$

$$v_1^2 - 2\left(\frac{m_2}{m_1}\right)v_2 \cdot v_1 + \left(\frac{m_2}{m_1}\right)^2 v_2^2 - 2\left(\frac{m_1 + m_2}{m_1}\right)^2 \mu gs = 0 \qquad \boxed{v_{1_{1,2}} = \left(\frac{m_2}{m_1}\right)v_2 \pm \sqrt{2\left(1 + \frac{m_2}{m_1}\right)^2 \mu gs}}$$

Lösung 6.2



$$m_1v_1 = (m_1 + m_2)v \qquad v = \frac{m_1}{m_1 + m_2}v_1 \quad \text{nach Stoß}$$

$$\rightarrow: \; F_c - F_H = 0 \qquad \uparrow: \; F_N - m_3g = 0 \quad F_H \le \mu_0 m_3 g$$

$$F_c = c \cdot x \qquad x = \frac{F_c}{c} \le \frac{\mu_0 m_3 g}{c} \qquad x_{\mathbf{max}} = \frac{\mu_0 m_3 g}{c}$$

Energiebilanz: $\dfrac{1}{2}(m_1 + m_2)v^2 = \dfrac{1}{2}cx_{\mathbf{max}}^2$

$$\frac{1}{2}\frac{m_1^2}{(m_1 + m_2)}v_1^2 = \frac{1}{2}\frac{(\mu_0 m_3 g)^2}{c}$$

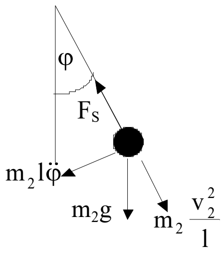
$$\boxed{v_1 = \frac{\mu_0 m_3 g}{m_1}\sqrt{\frac{(m_1 + m_2)}{c}}}$$

Lösung 6.3

Imp.: $m_1v_0 = m_2v_2 + m_1v_1 \qquad v_1 = \dfrac{m_1v_0 - m_2v_2}{m_1} = v_0 - \dfrac{m_2}{m_1}v_2$ $\quad$ **(*)**

Energie: $\dfrac{1}{2}m_1v_0^2 = \dfrac{1}{2}m_1v_1^2 + \dfrac{1}{2}m_2v_2^2 \qquad v_0^2 = v_1^2 + \dfrac{m_2}{m_1}v_2^2$ $\quad$ **(**)**

(*) in **(**)**: $2\dfrac{m_2}{m_1}v_0v_2 - \dfrac{m_2}{m_1}v_2^2\left(1 + \dfrac{m_2}{m_1}\right) = 0 \qquad v_2 = \dfrac{2m_1}{(m_1 + m_2)}v_0$ $\quad$ **(***)**



$\nwarrow$: $F_S - m_2 \frac{v_2^2}{l} - m_2 g\cos\varphi = 0 \quad F_S = m_2 \frac{v_2^2}{l} + m_2 g\cos\varphi$

Reißen: $F_S > F_{Smax} \quad F_{Smax}$ für $\varphi = 0 \quad F_{Smax} = m_2 \frac{v_2^2}{l} + m_2 g$

$$v_2^2 = l\left(\frac{F_{Smax}}{m_2} - g\right) = (***)^2 \qquad \boxed{v_0 > \frac{m_1 + m_2}{2m_1}\sqrt{l\left(\frac{F_{Smax}}{m_2} - g\right)}}$$

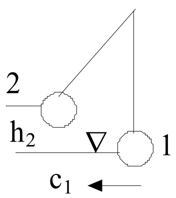
Lösung 6.4

Energie: $m_1 g h_1 = \frac{1}{2} m_1 v_1^2 \qquad v_1 = \sqrt{2gh_1} \qquad v_2 = 0$

Impuls: $m_1 v_1 + m_2 \cdot 0 = m_1 c_1 + m_2 c_2 \quad k = \frac{c_2 - c_1}{v_1 - v_2} = \frac{c_2 - c_1}{v_1} \quad kv_1 = c_2 - c_1$

$$m_1 v_1 = m_1 c_1 + m_2(kv_1 + c_1) \qquad v_1(m_1 - km_2) = c_1(m_1 + m_2) \qquad c_1 = \frac{m_1 - km_2}{m_1 + m_2} v_1 = -0{,}2 v_1$$

$$c_1 = -0{,}2\sqrt{2gh_1} \qquad c_2 = kv_1 + c_1 = v_1(k - 0{,}2) = 0{,}6\sqrt{2gh_1}$$



Energiebilanz für m_1: $\frac{1}{2} m_1 c_1^2 = m_1 g h_2$

$$h_2 = \frac{c_1^2}{2g} = 0{,}04 h_1$$

Lösung 6.5

Energie: $\frac{1}{2} m_1 v_0^2 - \mu_1 m_1 g s_1 = \frac{1}{2} m_1 v_1^2 \quad v_1^2 = v_0^2 - 2\mu_1 g s_1 \quad v_1$ – Aufprallgeschw.

Stoß: $m_1 v_1 = m_1 c_1 + m_2 c_2$ und $k = \frac{c_2 - c_1}{v_1}$ oder $c_1 = c_2 - kv_1$

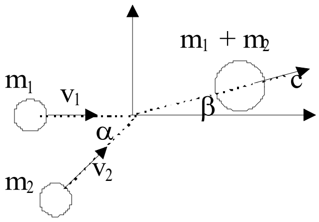
$$m_1 v_1 (1 + k) = c_2(m_1 + m_2) \qquad v_1 = \frac{1 + \frac{m_2}{m_1}}{1 + k} c_2$$

Energie: $\frac{1}{2} m_2 c_2^2 - \mu_2 m_2 g s_2 = 0 \qquad c_2^2 = 2\mu_2 g s_2$

Elimination von c_2 und v_1:

$$v_0^2 = v_1^2 + 2\mu_1 g s_1 = \left[\frac{1 + \frac{m_2}{m_1}}{1 + k}\right]^2 \cdot 2\mu_2 g s_2 + 2\mu_1 g s_1 = 429{,}19 \frac{m^2}{s^2} \qquad v_0 = 20{,}7 \frac{m}{s} = 74{,}58 \frac{km}{h}$$

Lösung 6.6



Impuls: $\rightarrow$: $m_1 v_1 + m_2 v_2 \cos\alpha = (m_1 + m_2) c \cdot \cos\beta$

$\uparrow$: $m_2 v_2 \sin\alpha = (m_1 + m_2) c \cdot \sin\beta$

quadr.+add.: $c = \dfrac{1}{m_1 + m_2}\sqrt{m_1^2 v_1^2 + 2 m_1 m_2 v_1 v_2 \cos\alpha + m_2^2 v_2^2}$

div.: $\tan\beta = \dfrac{m_2 v_2 \sin\alpha}{m_1 v_1 + m_2 v_2 \cos\alpha}$

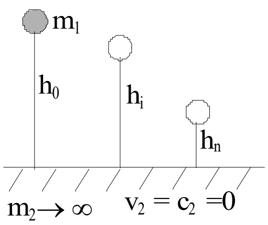
Energieverlust: $\Delta T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1 + m_2) c^2$

$$= \frac{1}{2}\left\{ m_1 v_1^2 + m_2 v_2^2 - \frac{1}{(m_1 + m_2)}\left(m_1^2 v_1^2 + 2 m_1 m_2 v_1 v_2 \cos\alpha + m_2^2 v_2^2 \right) \right\}$$

$$= \frac{m_1 m_2}{2(m_1 + m_2)}\left\{ v_1^2 + v_2^2 - 2 v_1 v_2 \cos\alpha \right\}$$

$\Delta T_{\mathbf{max}}$ für Frontalzusammenstoß $\alpha = \pi$

Lösung 6.7



Auftreffgeschwindigkeit: $v_1 = \sqrt{2 g h_0}$

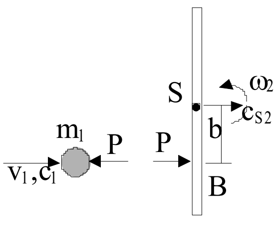
Stoßbed.: $k = -\dfrac{c_1}{v_1}$ $\qquad c_1 = -k v_1 = -k\sqrt{2 g h_0}$

Neue Steighöhe (Energiesatz):

$\dfrac{1}{2} m_1 c_1^2 = m_1 g h_1 \qquad h_1 = \dfrac{c_1^2}{2g} = k^2 h_0$

allgemein: $h_i = k^2 h_{i-1} \quad h_n = k^2 h_{n-1} = k^4 h_{n-2} = k^{2n} \cdot h_0 \quad k^{2n} = \dfrac{h_n}{h_0}$ $\boxed{k = \sqrt[2n]{\dfrac{h_n}{h_0}}}$

Lösung 6.8



Impuls und Drehimpuls:

$m_1 v_1 - p = m_1 c_1 \qquad p = m_2 c_{S2} \qquad p \cdot b = J_S \omega_2$

$m_1 v_1 - m_2 c_{S2} = m_1 c_1$ (1) $\qquad m_2 c_{S2} = J_S \dfrac{\omega_2}{b}$ (2) $\qquad J_S = \dfrac{1}{12} m_2 l^2$

$c_{2B} = c_{S2} + b\omega_2$ (3) $\quad k = 1 = -\dfrac{c_1 - c_{2B}}{v_1} \quad v_1 = c_{2B} - c_1$ (4)

$\omega_2 = \dfrac{c_{2B} - c_{S2}}{b}$ (3') $\quad c_{2B} = v_1 + c_1$ (4')

$m_2 c_{S2} = \dfrac{J_S}{b^2}(v_1 + c_1 - c_{S2})$ (2')

$$c_{S2}\left(\frac{m_2 b^2}{J_S}+1\right)=v_1+c_1 \quad c_{S2}=\frac{v_1+c_1}{\left(\frac{m_2 b^2}{J_S}+1\right)}=\frac{v_1+c_1}{1+12\left(\frac{b}{l}\right)^2}=\frac{4}{7}(v_1+c_1)$$

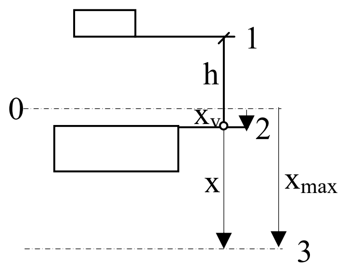
$$\textbf{(1')} \quad m_1 v_1 - m_2\frac{4}{7}(v_1+c_1)=m_1 c_1 \quad c_1\left(m_1+\frac{4}{7}m_2\right)=v_1\left(m_1-\frac{4}{7}m_2\right)$$

$$c_1=v_1\frac{m_1-\frac{4}{7}m_2}{m_1+\frac{4}{7}m_2}=v_1\frac{1-\frac{8}{7}}{1+\frac{8}{7}}=-\frac{1}{15}v_1 \quad \boxed{c_1=-\frac{1}{15}v_1 \quad c_{S2}=\frac{8}{15}v_1}$$

$$\text{aus } \mathbf{(2)} \quad \omega_2=\frac{bm_2}{J_S}c_{S2}=\frac{\frac{1}{4}m_2 l}{\frac{1}{12}m_2 l^2}\cdot\frac{8}{15}v_1=\frac{8}{5}\frac{v_1}{l} \qquad \boxed{\omega_2=\frac{8}{5}\frac{v_1}{l}}$$

Auch Energiesatz: $\frac{1}{2}m_1 v_1^2=\frac{1}{2}m_1 c_1^2+\frac{1}{2}m_2 c_{S2}^2+\frac{1}{2}J_S\omega^2$

Lösung 6.9



$$T_1+U_1=T_2+U_2 \quad\Rightarrow\quad m_1 gh=\frac{1}{2}m_1 v_1^2 \quad v_1=\sqrt{2gh}$$

$$\text{Plastischer Stoß:} \quad m_1 v_1=(m_1+m_2)v_2 \quad v_2=\frac{m_1}{m_1+m_2}\sqrt{2gh}$$

$$cx_v=m_2 g$$

$$T_2+U_2=T_3+U_3$$

$$\frac{1}{2}(m_1+m_2)v_2^2+\frac{1}{2}cx_v^2=\frac{1}{2}c(x_v+x)^2-(m_1+m_2)gx$$

$$\frac{1}{2}(m_1+m_2)v_2^2=\frac{1}{2}cx^2-m_1 gx$$

$$x^2-\frac{2m_1 g}{c}x-\frac{(m_1+m_2)}{c}\cdot\frac{m_1^2}{(m_1+m_2)^2}2gh=0$$

$$x_{1,2}=\frac{m_1 g}{c}\pm\sqrt{\left(\frac{m_1 g}{c}\right)^2+\frac{2ghm_1^2}{(m_1+m_2)c}}=\frac{m_1 g}{c}\left[1\pm\sqrt{1+\frac{2ch}{g(m_1+m_2)}}\right]$$

$$x_{max}=x_v+x_1=\frac{(m_1+m_2)g}{c}+\frac{m_1 g}{c}\sqrt{1+\frac{2ch}{g(m_1+m_2)}}$$

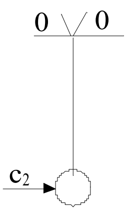
Zahlenwerte: $x_v = 0{,}613\text{cm}$ $x_{max} = 19{,}92\text{cm}$

Lösung 6.10

Energie: $\frac{1}{2}c\Delta l^2 - \mu m_1 g(a+\Delta l) = \frac{1}{2}m_1 v_1^2 \qquad v_1 = \sqrt{\frac{c\Delta l^2}{m_1} - 2\mu g(a+\Delta l)}$

elast. Stoß: $m_1 v_1 = m_1 c_1 + m_2 c_2 \qquad k = 1 = \frac{c_2 - c_1}{v_1} \quad c_1 = c_2 - v_1$

$$2m_1 v_1 = (m_1 + m_2)c_2 \qquad c_2 = \frac{2m_1 v_1}{(m_1+m_2)}$$



$$\frac{1}{2}m_2 c_2^2 - m_2 gh = m_2 gh \quad \frac{1}{2}m_2 c_2^2 = 2m_2 gh$$

$$c_2^2 = 4gh = \frac{4m_1^2}{(m_1+m_2)^2}\left(\frac{c\Delta l^2}{m_1} - 2\mu g(a+\Delta l)\right)$$

$$\Delta l^2 - 2\mu g\frac{m_1}{c}\Delta l - 2\mu g\frac{m_1}{c}a - \frac{(m_1+m_2)^2}{m_1}\frac{gh}{c} = 0$$

$$\boxed{\Delta l_{min} = \frac{\mu m_1 g}{c}\left[1 + \sqrt{1 + \frac{2ac}{\mu m_1 g} + \frac{(m_1+m_2)^2 hc}{m_1^3 \mu^2 g}}\right]}$$

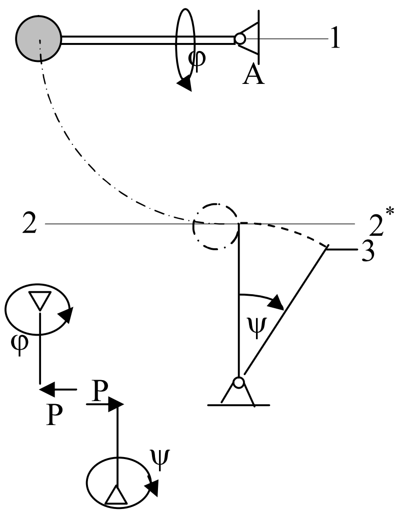
Lösung 6.11

$$J_A = Ml_1^2 + \frac{1}{3}m_1 l_1^2 = l_1^2\left(M + \frac{1}{3}m_1\right) \quad J_B = \frac{1}{3}m_2 l_2^2$$

1. $T_1 + U_1 = T_2 + U_2$

$$(M+m_1)gl_1 = \frac{1}{2}J_A\dot{\varphi}_2^2 + \frac{1}{2}m_1 g l_1$$

$$\dot{\varphi}_2 = \sqrt{3\frac{g}{l_1}\cdot\frac{2M+m_1}{3M+m_1}}$$



2. Stoß: $k = \frac{\dot{\psi}_2^* l_2 - \dot{\varphi}_2^* l_1}{\dot{\varphi}_2 l_1} = \frac{v_{2rel}^*}{v_2} \quad \dot{\varphi}_2^* = \dot{\psi}_2^*\frac{l_2}{l_1} - k\dot{\varphi}_2$

Impuls:

$$J_A\dot{\varphi}_2 = J_A\dot{\varphi}_2^* - Pl_1 \qquad 0 = J_B\dot{\psi}_2^* + Pl_2 \quad \Rightarrow \quad P = -J_B\frac{\dot{\psi}_2^*}{l_2}$$

$$J_A\dot{\varphi}_2 - J_A\dot{\varphi}_2^* = J_B\frac{l_1}{l_2}\dot{\psi}_2^* \quad \Rightarrow \quad J_A\dot{\varphi}_2 - J_A\frac{l_2}{l_1}\dot{\psi}_2^* + J_A k\dot{\varphi}_2 - J_B\frac{l_1}{l_2}\dot{\psi}_2^* = 0$$

$$\dot{\psi}_2^* = \frac{J_A(1+k)\dot{\varphi}_2}{J_A\frac{l_2}{l_1} + J_B\frac{l_1}{l_2}} = \frac{(1+k)}{\frac{l_2}{l_1} + \frac{J_B}{J_A}\frac{l_1}{l_2}}\cdot\dot{\varphi}_2 = \frac{(1+k)\frac{l_1}{l_2}}{\left(1 + \frac{m_2}{3M+m_1}\right)}\cdot\sqrt{3\frac{g}{l_1}\frac{2M+m_1}{3M+m_1}}$$

$$T_{2^*} + U_{2^*} = T_3 + U_3 \quad \Rightarrow \quad -\frac{1}{2}m_2 g l_2 + \frac{1}{2}J_B\dot{\psi}_2^{*2} = -\frac{1}{2}m_2 g l_2(2 - \cos\psi) + \frac{1}{2}J_B\dot{\psi}^2$$

$$\dot{\psi} = \sqrt{\dot{\psi}_2^{*2} + \frac{m_2 g l_2}{J_B}(1-\cos\psi)} = \sqrt{\dot{\psi}_2^{*2} + \frac{3g}{l_2}(1-\cos\psi)}$$

$$\dot{\psi}_{180^\circ} = \sqrt{\dot{\psi}_2^{*2} + \frac{6g}{l_2}} = \sqrt{\frac{6g}{l_2} + \frac{3(1+k)^2 g(2M+m_1)\left(\frac{l_1}{l_2}\right)^2}{l_1(3M+m_1)\left(1+\frac{m_2}{3M+m_1}\right)^2}}$$

Dreidimensionale Bewegung des Starren Körpers

Lösung 7.1

$$J_{xx}^{(A)} = \frac{1}{3}ml^2 + \frac{1}{12}ml^2 + m\left[l^2 + \left(\frac{1}{2}\right)^2\right] = \frac{5}{3}ml^2$$

$$J_{yy}^{(A)} = \frac{1}{3}ml^2 + \frac{1}{12}ml^2 + m\left[l^2 + \left(\frac{1}{2}\right)^2\right] + m\left(\sqrt{2}\,l\right)^2 = \frac{11}{3}ml^2$$

$$J_{zz}^{(A)} = \frac{1}{3}ml^2 + ml^2 + \frac{1}{12}ml^2 + m\left[l^2 + \left(\frac{1}{2}\right)^2\right] = \frac{8}{3}ml^2$$

$$J_{xy}^{(A)} = -m\left[\frac{1}{2}\cdot 0 + 1\cdot 0 + 1\cdot\frac{1}{2}\right] = -\frac{1}{2}ml^2$$

$$J_{yz}^{(A)} = -m\left[0\cdot 0 + 0\cdot\left(-\frac{1}{2}\right) + \frac{1}{2}\cdot(-1)\right] = \frac{1}{2}ml^2$$

$$J_{zx}^{(A)} = -m\left[0 + \left(-\frac{1}{2}\right)\cdot 1 + (-1)\cdot 1\right] = \frac{3}{2}ml^2$$

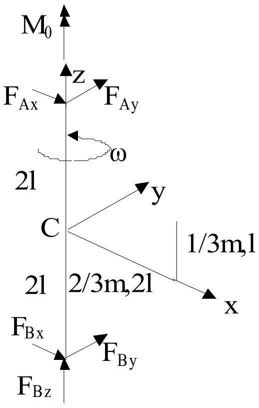
$$\boxed{J^{(A)} = \frac{ml^2}{6}\begin{bmatrix} 10 & -3 & 9 \\ -3 & 22 & 3 \\ 9 & 3 & 16 \end{bmatrix}} \qquad \mathbf{M}^{(A)} = \frac{d\mathbf{L}}{dt} = J^{(A)}\dot{\vec{\omega}} + \vec{\omega}\times\mathbf{L}^{(A)} \qquad \mathbf{L}^{(A)} = J^{(A)}\cdot\vec{\omega} \qquad \dot{\vec{\omega}} = 0$$

$$\vec{\omega} = \begin{bmatrix} -\frac{1}{2}\sqrt{2}\omega \\ 0 \\ \frac{1}{2}\sqrt{2}\omega \end{bmatrix} = \frac{1}{2}\sqrt{2}\omega\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{L}^{(A)} = J^{(A)}\cdot\vec{\omega} = \frac{ml^2}{6}\begin{bmatrix} 10 & -3 & 9 \\ -3 & 22 & 3 \\ 9 & 3 & 16 \end{bmatrix}\cdot\frac{1}{2}\sqrt{2}\omega\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \frac{\sqrt{2}}{12}ml^2\omega\begin{bmatrix} -1 \\ 6 \\ 7 \end{bmatrix}$$

$$\vec{\omega}\times\mathbf{L}^{(A)} = \frac{1}{12}ml^2\omega^2\begin{bmatrix} x & y & z \\ -1 & 0 & 1 \\ -1 & 6 & 7 \end{bmatrix} = \frac{1}{12}ml^2\omega^2\begin{bmatrix} -6 \\ 6 \\ -6 \end{bmatrix} \qquad \boxed{\mathbf{M}^{(A)} = \frac{1}{2}ml^2\omega^2\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}}$$

Lösung 7.2



$$J_x^{(C)} = \frac{1}{3}\left(\frac{1}{3}m\right)l^2 = \frac{1}{9}ml^2$$

$$J_y^{(C)} = \frac{1}{3}\left(\frac{2}{3}m\right)(2l)^2 + \frac{1}{12}\left(\frac{1}{3}m\right)l^2 + \frac{1}{3}m\left[(2l)^2 + \left(\frac{1}{2}l\right)^2\right] = \frac{7}{3}ml^2$$

$$J_z^{(C)} = \frac{1}{3}\left(\frac{2}{3}m\right)(2l)^2 + \frac{1}{3}m(2l)^2 = \frac{20}{9}ml^2$$

$$J_{xy}^{(C)} = 0; \quad J_{yz}^{(C)} = 0; \quad J_{zx}^{(C)} = 1\cdot 0 - (2l)\cdot\left(\frac{1}{2}\right)\cdot\frac{1}{3}m = -\frac{1}{3}ml^2$$

$$J^{(C)} = \begin{bmatrix} \frac{1}{9}ml^2 & 0 & -\frac{1}{3}ml^2 \\ 0 & \frac{7}{3}ml^2 & 0 \\ -\frac{1}{3}ml^2 & 0 & \frac{20}{9}ml^2 \end{bmatrix} = \frac{1}{9}ml^2\begin{bmatrix} 1 & 0 & -3 \\ 0 & 21 & 0 \\ -3 & 0 & 20 \end{bmatrix}$$

$$\vec{\omega} = \omega_z\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \mathbf{L}^{(C)} = J^{(C)}\cdot\vec{\omega} = \frac{1}{9}ml^2\omega_z\begin{bmatrix} -3 \\ 0 \\ 20 \end{bmatrix}$$

$$\vec{\omega}\times\mathbf{L}^{(C)} = \frac{1}{9}ml^2\omega_z^2\begin{bmatrix} x & y & z \\ 0 & 0 & 1 \\ -3 & 0 & 20 \end{bmatrix} = -\frac{1}{9}ml^2\omega_z^2\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

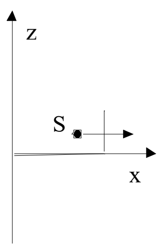
$$\mathbf{M}^{(C)} = \frac{1}{9}ml^2\dot{\omega}\begin{bmatrix} -3 \\ 0 \\ 20 \end{bmatrix} - \frac{1}{9}ml^2\omega^2\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2lF_{By} - 2lF_{Ay} \\ 2lF_{Ax} - 2lF_{Bx} \\ M_0 \end{bmatrix} \quad \text{oder}$$

$$2l\left(F_{By} - F_{Ay}\right) = -\frac{1}{3}ml^2\dot{\omega} \qquad (1)$$

$$2l\left(F_{Ax} - F_{Bx}\right) = -\frac{1}{3}ml^2\omega^2 \qquad (2)$$

$$M_0 = \frac{20}{9}ml^2\dot{\omega} \qquad (3) \qquad \boxed{\dot{\omega} = \frac{9}{20}\frac{M_0}{ml^2} \quad \text{Beweg.-Gl.}}$$

d.Alemb.: Schwerpunkt reine Translation auf Kreisbahn



$$x_S = \frac{\sum l_i x_i}{l_{ges.}} = \frac{2l\cdot l + l\cdot 2l}{3l} = \frac{4}{3}l \qquad a_n = x_S\omega^2 \quad a_t = x_S\dot{\omega}$$

$$\rightarrow: \quad F_{Ax} + F_{Bx} + mx_S\omega^2 = 0 \qquad (4)$$

$$\nearrow: \quad F_{Ay} + F_{By} - mx_S\dot{\omega} = 0 \qquad (5)$$

$$(1')\quad F_{By}-F_{Ay}=-\frac{1}{6}ml\dot{\omega} \qquad (1')+(5'):\quad 2F_{By}=\frac{7}{6}ml\dot{\omega}=\frac{21}{40}\frac{M_0}{l}$$

$$\boxed{F_{By}=\frac{21}{80}\frac{M_0}{l}}$$

$$(2')\quad F_{Ax}-F_{Bx}=-\frac{1}{6}ml\omega^2$$

$$(4')\quad F_{Ax}+F_{Bx}=-\frac{4}{3}ml\omega^2 \qquad (2')+(4'):\quad 2F_{Ax}=-\frac{3}{2}ml\omega^2$$

$$\boxed{F_{Ax}=-\frac{3}{4}ml\omega^2}$$

$$(5')\quad F_{By}+F_{Ay}=\frac{4}{3}ml\dot{\omega}$$

aus (5'): $\boxed{F_{Ay}=\frac{3}{4}ml\dot{\omega}=\frac{27}{80}\frac{M_0}{l}}$

aus (4'): $\boxed{F_{Bx}=-\frac{7}{12}ml\omega^2}$

Lösung 7.3

$$\vec{\omega}_{Stab}=\begin{bmatrix}0\\0\\\dot{\varphi}\end{bmatrix}\quad \vec{\omega}_{Zyl.}=\begin{bmatrix}\Omega\\0\\\dot{\varphi}\end{bmatrix}\quad J_x^{(A)}=\frac{1}{2}mr^2\quad J_y^{(A)}=J_z^{(A)}=\frac{1}{12}m\left(3r^2+(3r)^2\right)+ml^2=m\left(r^2+l^2\right)$$

$$J^{(A)}=\begin{bmatrix}\frac{1}{2}mr^2 & 0 & 0\\ 0 & m\left(r^2+l^2\right) & 0\\ 0 & 0 & m\left(r^2+l^2\right)\end{bmatrix}\quad \text{Drehimp. } \mathbf{L}^{(a)}=J^{(A)}\cdot\vec{\omega}_{Zyl.}=\begin{bmatrix}\frac{1}{2}mr^2\Omega\\0\\m\left(r^2+l^2\right)\dot{\varphi}\end{bmatrix}$$

$$\mathbf{M}^{(A)}=\dot{\mathbf{L}}^{(A)}+\vec{\omega}_{Stab}\times\mathbf{L}^{(A)}$$

$$\mathbf{M}^{(A)}=\begin{bmatrix}0\\M_y\\-mgl\sin\varphi\end{bmatrix}=\begin{bmatrix}\frac{1}{2}mr^2\dot{\Omega}\\0\\m\left(r^2+l^2\right)\ddot{\varphi}\end{bmatrix}+\begin{bmatrix}x & y & z\\0 & 0 & \dot{\varphi}\\\frac{1}{2}mr^2\Omega & 0 & m\left(r^2+l^2\right)\dot{\varphi}\end{bmatrix}$$

in Komponenten:

X: $\quad 0=\frac{1}{2}mr^2\dot{\Omega}\qquad \dot{\Omega}=0\qquad \boxed{\Omega(t)=C=\Omega_0}$

Y: $\quad M_y=0+\frac{1}{2}mr^2\Omega\dot{\varphi}\qquad M_y=\frac{1}{2}mr^2\Omega_0\dot{\varphi}$

Z: $\quad -mgl\sin\varphi=m\left(r^2+l^2\right)\ddot{\varphi}+0\qquad \ddot{\varphi}+\frac{gl}{r^2+l^2}\sin\varphi=0$ oder linearisiert $\ddot{\varphi}+\omega^2\varphi=0$

$\omega^2=\frac{gl}{r^2+l^2}$ Lösung: $\varphi(t)=A\sin\omega t+B\cos\omega t$

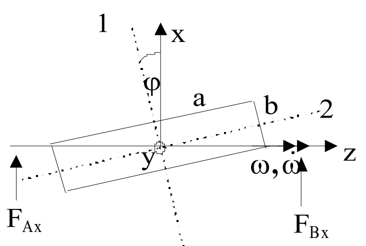
AB: $t=0$: $\dot{\varphi}(0)=0$, d.h. $A=0$

$\varphi(0)=\varphi_0$, $\qquad B=\varphi_0$

$\varphi(t)=\varphi_0\cos\omega t\qquad \dot{\varphi}(t)=-\varphi_0\omega\sin\omega t$ und damit

$$\boxed{M_y=-\frac{1}{2}mr^2\Omega_0\varphi_0\omega\sin\omega t}$$

Lösung 7.4 (zu aufwendige Lösung, geht viel einfacher !!!)



Momentensatz:

$$\begin{bmatrix}M_x\\M_y\\M_z\end{bmatrix}=\dot{\omega}\begin{bmatrix}J_{xz}\\J_{yz}\\J_z\end{bmatrix}+\omega^2\begin{bmatrix}-J_{yz}\\J_{xz}\\0\end{bmatrix}\quad \text{mit}\quad \dot{\omega}=0,\quad J_{yz}=0$$

für die dünne Scheibe gilt:

$$J_z = \int\left(x^2+y^2\right)dm = \rho t\int x^2 dA = \rho t I_z \qquad J_{xz} = -\int xz dm = -\rho t\int xz dA = \rho t I_{xz}$$

$$J_z = \frac{J_1+J_2}{2} - \frac{J_1-J_2}{2}\cos 2\varphi \qquad J_{xz} = -\frac{J_1-J_2}{2}\sin 2\varphi \quad J_1 = \rho t\cdot\frac{a^3 b}{12} = \frac{1}{12}ma^2 \quad J_2 = \frac{1}{12}mb^2$$

$$\cos\varphi = \frac{a}{\sqrt{a^2+b^2}};\quad \sin\varphi = \frac{b}{\sqrt{a^2+b^2}} \quad \sin 2\varphi = 2\sin\varphi\cos\varphi = \frac{2ab}{a^2+b^2} \quad \cos 2\varphi = 2\cos^2\varphi - 1$$

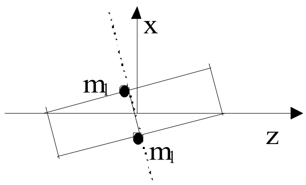
$$J_z = \frac{1}{24}m\left(a^2+b^2\right) - \frac{1}{24}m\left(a^2-b^2\right)\left(\frac{a^2-b^2}{a^2+b^2}\right) = \frac{ma^2b^2}{6\left(a^2+b^2\right)}$$

$$J_{xz} = -\frac{1}{24}m\left(a^2-b^2\right)\cdot\frac{2ab}{a^2+b^2} = -\frac{m\left(a^2-b^2\right)ab}{12\left(a^2+b^2\right)}$$

mit $\dot\omega = 0$: $\quad M_y = \omega^2 J_{xz} = -F_{Ax}l + F_{Bx}l \quad (1)$

$\uparrow$: $\quad F_{Ax} + F_{Bx} = 0 \quad$ (2) (keine Fliehkraft, da S auf z liegt)

(2') $\quad F_{Ax} = -F_{Bx} \qquad$ (1') $\quad 2F_{Bx} = \omega^2\dfrac{J_{xz}}{l}$ $\quad \boxed{F_{Bx} = \omega^2\frac{J_{xz}}{2l} = -\frac{m\omega^2\left(a^2-b^2\right)ab}{24\cdot l\left(a^2+b^2\right)} = -F_{Ax}}$



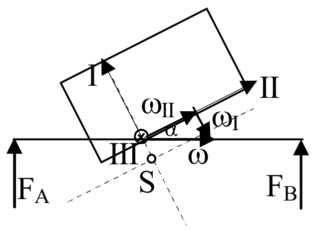
$$F_{Ax} = F_{Bx} = 0 \quad \text{für} \quad J^*_{xz} = 0$$

$$J^*_{xz} = J_{xz} + 2m_1\left(\frac{b}{2}\sin\varphi\right)\left(\frac{b}{2}\cos\varphi\right) = 0$$

$$-\frac{1}{12}m\frac{\left(a^2-b^2\right)ab}{a^2+b^2} + 2m_1\frac{b^2}{4}\frac{ab}{a^2+b^2} = 0$$

$$-\frac{1}{12}m\left(a^2-b^2\right) + \frac{1}{2}m_1 b^2 = 0 \qquad m_1 = \frac{1}{6}m\left(\frac{a^2}{b^2}-1\right)$$

Lösung 7.5 **(zu aufwendige Lösung, geht viel einfacher !!!)**



$$\omega_I = -\omega\sin\alpha \quad \omega_{II} = \omega\cos\alpha \qquad \omega_{III} = 0$$

$$\dot\omega_I = 0 \qquad \dot\omega_{II} = 0 \qquad \dot\omega_{III} = 0$$

$$M_I = J_I\dot\omega_I + \left(J_{III} - J_{II}\right)\omega_{II}\omega_{III} \quad \Rightarrow \quad M_I = 0$$

$$M_{II} = J_{II}\dot\omega_{II} + \left(J_I - J_{III}\right)\omega_I\omega_{III} \quad \Rightarrow \quad M_{II} = 0$$

$$M_{III} = J_{III}\dot\omega_{III} + \left(J_{II} - J_I\right)\omega_I\omega_{II} \quad \Rightarrow \quad M_{III} = \left(J_I - J_{II}\right)\omega^2\frac{\sin 2\alpha}{2}$$

$$J_I = \frac{1}{12}ml^2 + \frac{1}{16}md^2 = \frac{m}{4}\left(\frac{1}{3}l^2 + \frac{1}{4}d^2\right) = \frac{m}{4}d^2 \qquad J_{II} = \frac{1}{8}md^2 + m\left(\frac{e}{\cos\alpha}\right)^2$$

Überlagerung von Rotation und Translation durch Wahl des Koordinatensystems (Ursprung auf der Drehachse).

Lagerkraftanteil aus der Rotation:

$$F_{AR} = -F_{BR} = \frac{M_{III}}{2l} = \frac{(J_I - J_{II})}{3d} \cdot \omega^2 \frac{\sin 2\alpha}{2} = \frac{1}{48} md\omega^2 \sin 2\alpha - \frac{me^2\omega^2}{3d} \tan\alpha$$

Lagerkraftanteil aus der Translation des Schwerpunktes:

$me\omega^2$ wirkt im Schnittpunkt der Achse I mit der Drehachse

$$F_{BTr} \cdot 2l = me\omega^2 \cdot (l - e\tan\alpha) \quad \Rightarrow \quad F_{BTr} = me\omega^2 \frac{l - e\tan\alpha}{2l} \quad \text{(exakte Lösung)}$$

$$F_{ATr} + F_{BTr} = me\omega^2 \qquad F_{ATr} \approx F_{BTr} \quad \Rightarrow \quad F_{ATr} \approx F_{BTr} \approx \frac{1}{2} me\omega^2$$

Somit ergibt sich

$$F_A \approx \frac{1}{48} md\omega^2 \sin 2\alpha - \frac{me^2\omega^2}{3d} \tan\alpha + \frac{1}{2} me\omega^2 = m\omega^2 \left[e\left(\frac{1}{2} - \frac{e\tan\alpha}{3d} \right) + \frac{1}{48} d\sin 2\alpha \right]$$

$$F_B \approx -\frac{1}{48} md\omega^2 \sin 2\alpha + \frac{me^2\omega^2}{3d} \tan\alpha + \frac{1}{2} me\omega^2 = m\omega^2 \left[e\left(\frac{1}{2} + \frac{e\tan\alpha}{3d} \right) - \frac{1}{48} d\sin 2\alpha \right]$$