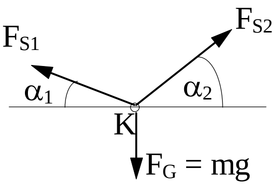
Lösungen für Aufgaben zur Technischen Mechanik – Statik -

Lösung 2.1.8



$\rightarrow$: $-F_{S1}\cos\alpha_1 + F_{S2}\cos\alpha_2 = 0$ $\qquad F_{S2} = \dfrac{F_{S1}\cos\alpha_1}{\cos\alpha_2}$

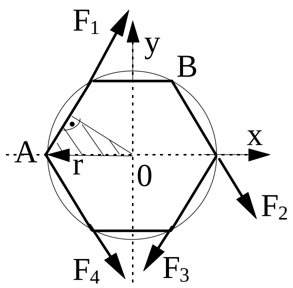
$\uparrow$: $F_{S1}\sin\alpha_1 + F_{S2}\sin\alpha_2 - mg = 0$ $\qquad F_{S1}(\sin\alpha_1 + \tan\alpha_2 \cos\alpha_1) = mg$

$$\cos\alpha_1 = \frac{b}{l} = \frac{4}{4{,}5} = 0{,}8889 \quad \alpha_1 = 27{,}3°$$

$$\tan\alpha_2 = \frac{a+\sqrt{l^2-b^2}}{c-b} = \frac{4{,}062}{6} = 0{,}677 \quad \alpha_2 = 34{,}1°$$

Ergebnis: $F_{S1} = 277\text{N}$, $F_{S2} = 297\text{N}$

Lösung 2.2.3



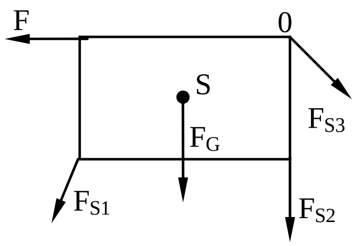
$$F_{Rx} = \Sigma F_{ix} = F_1\cos60° + F_2\cos60° - F_3\cos60° + F_4\cos60°$$
$$= 2F\cos60° = F$$

$$F_{Ry} = \Sigma F_{iy} = F_1\sin60° - F_2\sin60° - F_3\sin60° - F_4\sin60°$$
$$= -2F\sin60° = -F\sqrt{3}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{F^2 + 3F^2} = 2F$$

$$\tan\alpha_R = \frac{F_{Ry}}{F_{Rx}} = -\tan60° \quad \alpha_R = 300°$$

Lösung 2.2.7



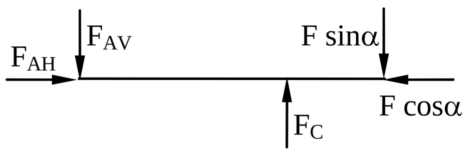
$\leftarrow$: $F + F_{S1}\cos 60° - F_{S3}\cos 45° = 0$

$\uparrow$: $-F_{S2} - F_{S1}\sin 60° - F_{S3}\sin 45° - F_G = 0$

$\curvearrowleft 0$: $2F_{S1}a\sin60° - F_{S1}a\cos60° + F_G\, a = 0$

Ergebnisse: $F_{S1} = -\ 812$ N....Druckstab
$F_{S2} = -\ 891$ N....Druckstab
$F_{S3} = \ 840$ N....Zugstab

Lösung 3.1.2



$\rightarrow$: $F_{AH} - F\cos\alpha = 0$

$\uparrow$: $-F_{AV} - F\sin\alpha + F_C = 0$

$\curvearrowleft A$: $-F\sin\alpha \cdot 3a + F_C \cdot 2a = 0$

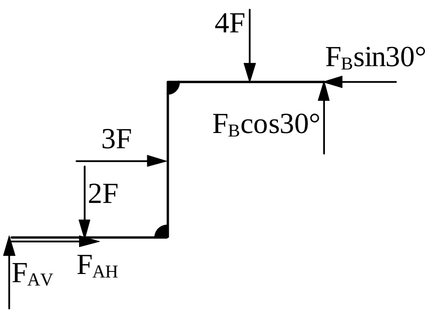
Ergebnis:

$$F_C = \frac{3}{2}F\sin\alpha = \frac{3}{4}\sqrt{3}F = 39kN$$

$$F_{AH} = F\cos\alpha = \frac{1}{2}F = 15kN$$

$$F_{AV} = \frac{1}{2}F\sin\alpha = \frac{\sqrt{3}}{4}F = 13kN$$

Lösung 3.1.12



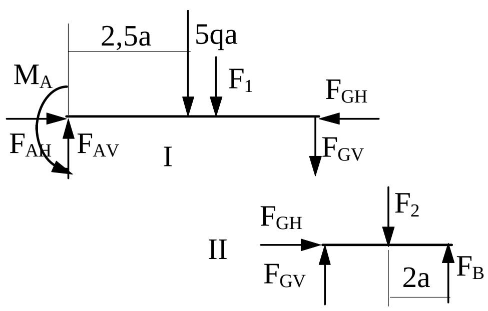
$\rightarrow$: $F_{AH} + 3F - F_B \sin 30° = 0$

$\uparrow$: $F_{AV} + F_B \cos 30° - 2F - 4F = 0$

$\curvearrowleft A$: $F_B \cos 30° \cdot 4a + F_B \sin 30° \cdot 2a - 2F \cdot a - 3F \cdot a - 4F \cdot \mathbf{3}a = 0$

Ergebnis: $F_B = 3{,}82F$; $F_{AH} = -1{,}09F$; $F_{AV} = 2{,}7F$

Lösung 3.2.3



Teil I:

$\rightarrow$: $F_{AH} - F_{GH} = 0$

$\uparrow$: $F_{AV} - F_{GV} - F_1 - 5qa = 0$

$\curvearrowleft A$: $M_A - F_1 \cdot 3a - 5qa \cdot 2{,}5a - F_{GV} \cdot 5a = 0$

Teil II:

$\rightarrow$: $F_{GH} = 0$

$\uparrow$: $F_B + F_{GV} - F_2 = 0$

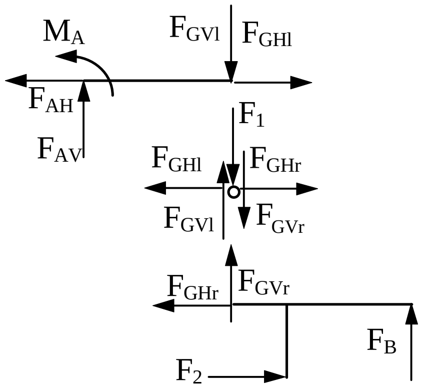
$\curvearrowleft G$: $-F_2 \cdot 3a + F_B \cdot 5a = 0$

Ergebnis:

$$F_B = \frac{3}{5}F_2 = 960\,\mathrm{N} \quad F_{GH} = 0 \quad F_{GV} = \frac{2}{5}F_2 = 640\,\mathrm{N} \quad F_{AH} = 0 \quad F_{AV} = F_1 + \frac{2}{5}F_2 + 5qa = 2440\,\mathrm{N}$$

$$M_A = 3F_1 a + 2F_2 a + \frac{25}{2}qa^2 = 8100\,\mathrm{Nm}$$

Lösung 3.2.9



$\rightarrow$: $-F_{AH} + F_{GHl} = 0$
$\uparrow$: $F_{AV} - F_{GVl} = 0$
A : $-F_{GVl}\,a + M_A = 0$

$\rightarrow$: $-F_{GHl} + F_{GHr} = 0$
$\uparrow$: $-F_{GVr} + F_{GVl} - F_1 = 0$

$\rightarrow$: $-F_{GHr} + F_2 = 0$
$\uparrow$: $F_B + F_{GVr} = 0$
G : $F_2\,c + F_B\,a = 0$

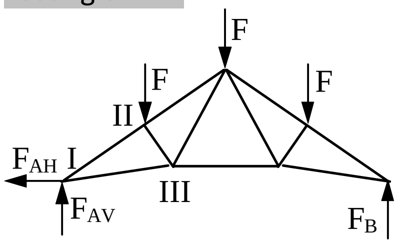
Ergebnis:

$$F_{AH} = F_2; \qquad F_{GHl} = F_{GHr} = F_2; \qquad F_B = -F_2 \cdot \frac{c}{a}$$

$$F_{AV} = F_1 + F_2 \cdot \frac{c}{a}; \qquad F_{GVl} = F_1 + F_2 \cdot \frac{c}{a};$$

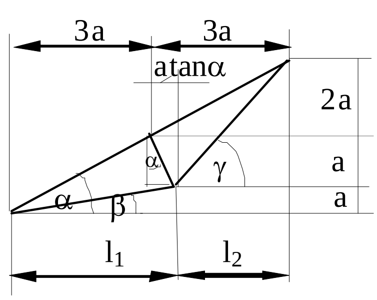
$$M_A = F_1 \cdot a + F_2 \cdot c; \qquad F_{GVr} = F_2 \cdot \frac{c}{a};$$

Lösung 3.2.22



Symmetrie:
$F_{AH} = 0$; $\quad F_{AV} = F_B = 1{,}5F = 30\,\mathrm{kN}$

Zur Geometrie:

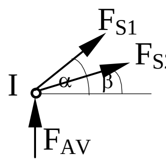


$$l_1 = 3a + a\tan\alpha; \quad \tan\alpha = \frac{4a}{6a} = \frac{2}{3}; \quad l_1 = \frac{11}{3}a$$

$$l_2 = 6a - l_1 = \frac{7}{3}a$$

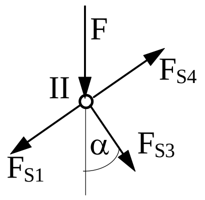
$$\tan\alpha = \frac{2}{3} \rightarrow \alpha = 33{,}7° \quad \tan\beta = \frac{a}{l_1} = \frac{3}{11} \rightarrow \beta = 15{,}25°$$

$$\tan\gamma = \frac{3a}{l_2} = \frac{9}{7} \rightarrow \gamma = 52{,}15°$$



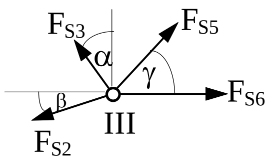
$\rightarrow$: $F_{S1}\cos\alpha + F_{S2}\cos\beta = 0$

$\uparrow$: $F_{AV} + F_{S1}\sin\alpha + F_{S2}\sin\beta = 0 \quad F_{S1} = -91{,}4\,\mathrm{kN} \quad F_{S2} = 78{,}8\,\mathrm{kN}$



$\nearrow$: $-F_{S1} + F_{S4} - F\sin\alpha = 0$

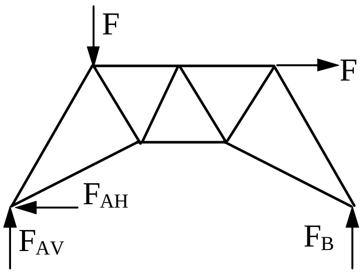
$\nwarrow$: $-F_{S3} - F\cos\alpha = 0$ $\qquad F_{S3} = -16{,}6\,\text{kN} \quad F_{S4} = -80{,}4\,\text{kN}$



$\rightarrow$: $F_{S6} - F_{S3}\sin\alpha + F_{S5}\cos\gamma - F_{S2}\cos\beta = 0$

$\uparrow$: $F_{S5}\sin\gamma + F_{S3}\cos\alpha - F_{S2}\sin\beta = 0 \qquad F_{S5} = 43{,}9\,\text{kN} \quad F_{S6} = 40\,\text{kN}$

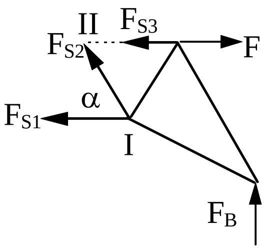
Lösung 3.2.25



$\rightarrow$: $F - F_{AH} = 0$ $\qquad F_{AH} = F$

$\uparrow$: $F_B - F + F_{AV} = 0$ $\qquad F_{AV} = 0{,}25F$

$\curvearrowleft A$: $-F \cdot a - F \cdot 2a + F_B \cdot 4a = 0$ $\qquad F_B = 0{,}75\,F$



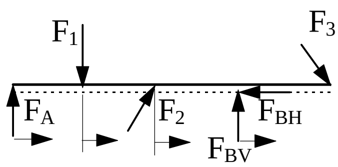
$\curvearrowleft$ I : $F_{S3} \cdot a - F \cdot a + F_B \cdot \frac{3}{2} a = 0 \qquad F_{S3} = -\frac{1}{8} F$

$\curvearrowleft$ II : $-F_{S1} \cdot a + F_B \cdot 2a = 0 \qquad F_{S1} = \frac{3}{2} F$

$\uparrow$: $F_{S2}\sin\alpha + F_B = 0 \qquad F_{S2} = -\frac{3F}{4\sin\alpha}$

aus $\tan\alpha = 2$ folgt $\alpha = 63{,}4°$ $F_{S2} = -0{,}84F$

Lösung 3.3.2

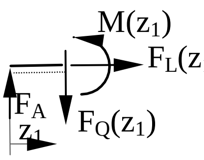


$\rightarrow$: $-F_{BH} + F_2\cos 60° + F_3\cos 60° = 0$

$\uparrow$: $F_A + F_2\sin 60° - F_1 - F_3\sin 60° + F_{BV} = 0$

$\curvearrowleft A$: $-F_1 \cdot a + F_2\sin 60° \cdot 2a - F_3\sin 60° \cdot 4a + F_{BV} \cdot 3a = 0$

Ergebnis: $F_A = 756$ N; $F_{BH} = 1000$ N; $F_{BV} = 1244$ N



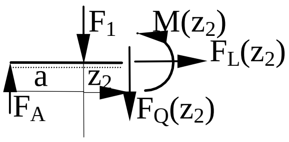
$0 \le z_1 \le a$

$\rightarrow$: $F_L(z_1) = 0$; $\quad \uparrow$: $F_Q(z_1) = F_A = 756$ N

$\curvearrowleft S_1$: $M(z_1) = F_A z_1$;

$z_1 = 0$: $M(0) = 0$ $\quad z_1 = a$: $M(a) = 756$ Nm

$0 \le z_2 \le a$

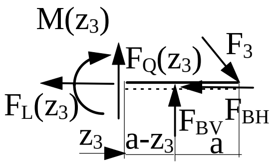


$\rightarrow$: $F_L(z_2) = 0$; $\uparrow$: $F_Q(z_2) = F_A - F_1 = -1244$ N

S_2 : $M(z_2) = F_A\,(a + z_2) - F_1\, z_2$;

$z_2 = 0$: $M_2(0) = 756$ Nm $\quad z_2 = a$: $M_2(a) = -488$ Nm

$0 \le z_3 \le a$



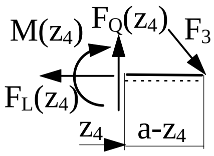
$\rightarrow$: $F_L(z_3) = - F_{BH} + F_3 \cos 60° = - 500$ N;

$\uparrow$: $F_Q(z_3) = - F_{BV} + F_3 \sin 60° = -378$ N;

S_3 : $M(z_3) = F_{BV}\,(a - z_3) - F_3 \sin 60° \cdot (2a - z_3)$;

$z_3 = 0$: $M_3(0) = - 488$ Nm $\quad z_3 = a$: $M_3(a) = -866$ Nm

$0 \le z_4 \le a$

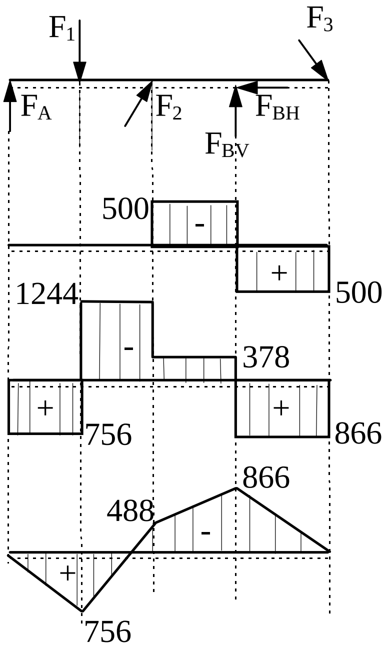


$\rightarrow$: $F_L(z_4) = F_3 \cos 60° = 500$ N;

$\uparrow$: $F_Q(z_4) = F_3 \sin 60° = 866$ N;

S_4 : $M(z_4) = - F_3 \sin 60° \cdot (a - z_4)$;

$z_4 = 0$: $M_4(0) = - 866$ Nm $\quad z_4 = a$: $M_4(a) = 0$

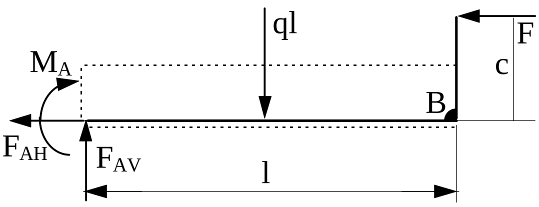


F_L in N

F_Q in N

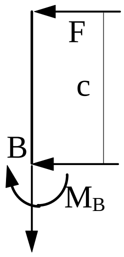
M in Nm $\quad |M_{max}| = 866\,\text{Nm}$

Lösung 3.3.6



A: $M_A = Fc - \frac{1}{2}ql^2 = ql\left(c - \frac{l}{2}\right)$

$$M_A \text{ ist } \begin{cases} \text{positiv} & \text{für } c > \frac{l}{2} \\ 0 & \text{für } c = \frac{l}{2} \\ \text{negativ} & \text{für } c < \frac{l}{2} \end{cases}$$

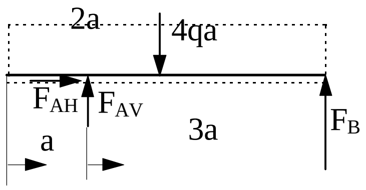


B: $M_B = Fc = qlc$

M_B ist positiv für beliebige c, bei $c \geq \frac{l}{2}$ ist $M_B > M_A$, $|M_A| = |M_B|$ ist nur für negatives M_A möglich.

$- M_A = M_B$ oder $-ql\left(c - \frac{l}{2}\right) = qlc \quad c = \frac{l}{4}$

Lösung 3.3.7

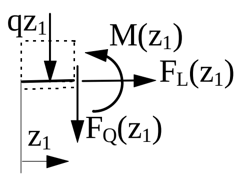


$\rightarrow$: $F_{AH} = 0$

$\uparrow$: $- 4qa + F_{AV} + F_B = 0$

A: $-4qa \cdot a + F_B \cdot 3a = 0$

Ergebnis: $F_{AH} = 0$; $\quad F_{AV} = \frac{8}{3}qa = 26{,}7kN$; $\quad F_B = \frac{4}{3}qa = 13{,}3kN$;



$0 \leq z_1 \leq a$

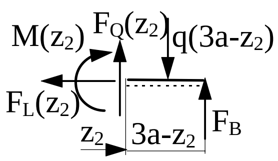
$\rightarrow$: $F_L(z_1) = 0$

$\uparrow$: $F_Q(z_1) = - qz_1$

$z_1 = 0$: $F_Q(0) = 0 \quad z_1 = a$: $\quad F_Q(a) = -qa = -10kN$

S_1: $M(z_1) = - \frac{1}{2}qz_1^2$;

$z_1 = 0$: $M(0) = 0 \quad z_1 = a$: $M(a) = - 0{,}5\, qa^2 = - 5kNm$



$0 \leq z_2 \leq 3a$

$\rightarrow$: $F_L(z_2) = 0$

$\uparrow$: $F_Q(z_2) = - F_B + q(3a - z_2)$

$z_2 = 0$: $F_Q(0) = \frac{5}{3}qa = 16{,}7kN$;

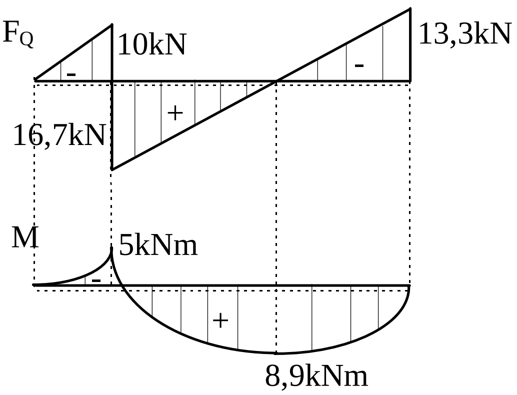
$z_2 = 3a$: $F_Q(a) = -\frac{4}{3}qa = - 13{,}3kN$

S_2: $M(z_2) = F_B \cdot (3a-z_2) - \frac{1}{2}q(3a - z_2)^2$;

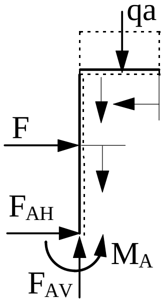
$$z_2 = 0:\ M_2(0) = -\frac{1}{2}qa^2 = -5\text{kNm} \qquad z_2 = 3a:\ M_2(3a) = 0$$

Besondere Werte:

- Querkraft-Nullstelle: $\bar{z}_2:\quad F_Q(\bar{z}_2) = 0 \qquad \bar{z}_2 = \frac{5}{3}a = 1{,}66m$
- Extremwert für M: $M(\bar{z}_2) = \frac{8}{9}qa^2 = 8{,}9\text{kNm} = M_{max}$



Lösung 3.3.13

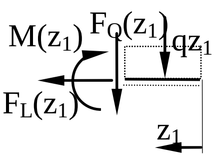


$\rightarrow:\ F_{AH} + F = 0$

$\uparrow:\ F_{AV} - qa = 0$

$A:\ M_A - F \cdot a - qa \cdot 0{,}5a = 0$

Ergebnis: $F_{AV} = qa$; $F_{AH} = -qa$; $M_A = 1{,}5qa^2$



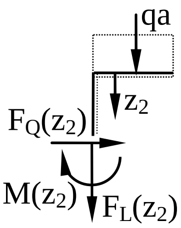
$0 \le z_1 \le a$

$\leftarrow:\ F_L(z_1) = 0$

$\uparrow:\ F_Q(z_1) = -qz_1 \qquad F_Q(0) = 0 \qquad F_Q(a) = -qa$

$S_1:\ M(z_1) = -0{,}5q\,z_1^2$

$z_1 = 0:\ M(0) = 0 \qquad z_1 = a:\ M(a) = -0{,}5qa^2$

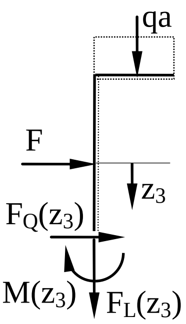


$0 \le z_2 \le a$

$\leftarrow:\ F_Q(z_2) = 0$

$\downarrow:\ F_L(z_2) = -qa$

$S_2:\ M(z_2) = -0{,}5qa^2$



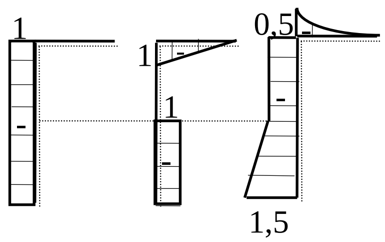
$0 \le z_3 \le a$

$\leftarrow$: $F_Q(z_3) = - F = -qa$

$\downarrow$: $F_L(z_2) = - qa$

S_2 : $M(z_2) = - 0{,}5qa^2 - Fz_3$

$z_3 = 0$: $M(0) = - 0{,}5qa^2$ $\quad$ $z_3 = a$: $M(a) = - 1{,}5qa^2$

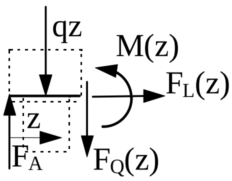


F_L in qa $\quad$ F_Q in $\quad$ M in qa^2 $\quad$ $|M_{max}| = 1{,}5\ qa^2$

Lösung 3.3.23

Aus Symmetriegründen im Teil I folgt $F_A = F_{GV} = \frac{1}{2}qa$

$0 \le z \le l$



$\uparrow$: $F_Q(z) = F_A - qz$

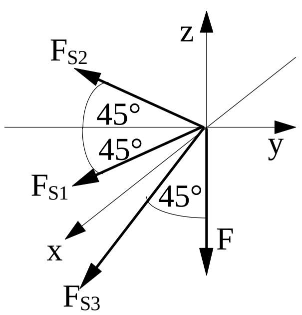
S : $M(z) = F_A \cdot z - \frac{1}{2}qz^2$

$M(0{,}5a) = 0{,}125qa^2$; $\quad$ $M(a) = 0$ $\quad$ $M(l) = - 0{,}5ql(l-a)$

$|M_{max}|_I = |M_{max}|_{II}$

$$\frac{1}{8}qa^2 = \frac{1}{2}ql(l-a) \quad a^2 = 4l^2 - 4la \quad a^2 + 4la - 4l^2 = 0 \quad a = 2l\left(\sqrt{2}-1\right) = 0{,}828\,l$$

Lösung 4.1.3



x: $-F_{S2}\sin45° + F_{S1}\sin45° = 0$ $\quad$ $F_{S1} = F_{S2}$

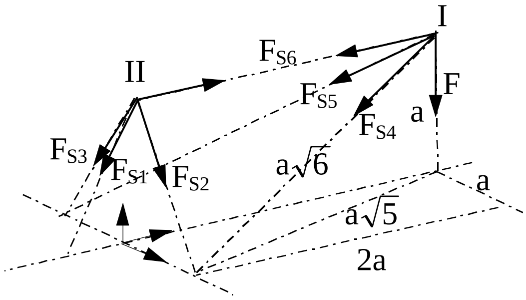
y: $-F_{S2}\cos45° - F_{S1}\cos45° - F_{S3}\sin45° = 0$

z: $-F - F_{S3}\cos45° = 0$

Ergebnis: $F_{S1} = F_{S2} = \frac{\sqrt{2}}{2}F = 707\text{N}$; $\quad F_{S3} = -\sqrt{2}F = -1414\text{N}$

Lösung 4.2.5

Aus Symmetriegründen folgt: $F_{S4} = F_{S5}$ und $F_{S2} = F_{S3}$



Knoten I:

$$\downarrow\textbf{:}\ F + 2F_{S5}\frac{1}{\sqrt{6}} = 0 \quad F_{S5} = F_{S4} = -\frac{\sqrt{6}}{2}F = -1{,}2F$$

$$\swarrow\textbf{:}\ F_{S6} + 2F_{S5}\frac{\sqrt{5}}{\sqrt{6}}\cdot\frac{2}{\sqrt{5}} = 0 \quad F_{S6} = -\frac{2\sqrt{6}}{3}F_{S5}$$

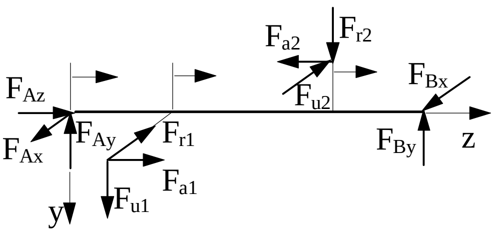
$$F_{S6} = 2F$$

Knoten II:

$$\swarrow\textbf{:}\ -F_{S6} + F_{S1}\frac{1}{\sqrt{2}} = 0 \quad F_{S1} = \sqrt{2}F_{S6} = 2\sqrt{2}F = 2{,}8F$$

$$\downarrow\textbf{:}\ \frac{\sqrt{2}}{2}F_{S1} + 2\frac{\sqrt{2}}{2}F_{S2} = 0 \quad F_{S2} = F_{S3} = -\frac{1}{2}F_{S1} = -1{,}4F$$

Lösung 4.3.1



Nach dem Erstarrungsprinzip gilt:

$\twoheadrightarrow F_{u1} \cdot r_{01} - F_{u2} \cdot r_{02} = 0$ oder $F_{u2} = F_{u1} \cdot \frac{r_{01}}{r_{02}}$

$F_{u1} = 4500N$ $\quad F_{a1} = 1640N$ $\quad F_{r1} = 1740N$

$F_{u2} = 10800N$ $\quad F_{a2} = 3930N$ $\quad F_{r2} = 4180N$

Lagerreaktionen:

$\swarrow$ $F_{Ax} - F_{r1} - F_{u2} + F_{Bx} = 0$

$\uparrow$: $F_{Ay} - F_{r2} - F_{u1} + F_{By} = 0$

$\rightarrow$: $F_{Az} + F_{a1} - F_{a2} = 0$

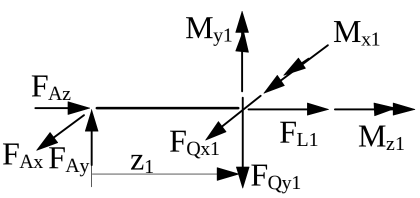
$\swarrow\swarrow$ A: $-F_{u1}l_1 - F_{r2}(l_1 + l_2) + F_{a2}r_{02} + F_{By}(l_1 + l_2 + l_3) = 0$

$\downarrow$ A: $-F_{r1}l_1 - F_{u2}(l_1 + l_2) - F_{a1}r_{01} + F_{Bx}(l_1 + l_2 + l_3) = 0$

Ergebnis: $F_{Ax} = 3660N$ $\quad F_{Ay} = 5580N$ $\quad F_{Az} = 2290N$

$F_{Bx} = 8880N$ $\quad F_{By} = 3100N$

$0 \leq z_1 \leq l_1$



$F_{L1} = -F_{Az} = -2290N$

$F_{Qx1} = -F_{Ax} = -3360N$

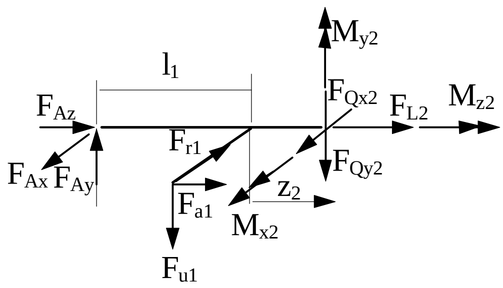
$F_{Qy1} = F_{Ay} = 5580N$

$M_{x1} = F_{Ay} \cdot z_1$ $\quad M_{x1}(0) = 0$ $\quad M_{x1}(l_1) = 307Nm$

$M_{y1} = -F_{Ax} \cdot z_1$ $\quad M_{y1}(0) = 0$ $\quad M_{y1}(l_1) = -201Nm$

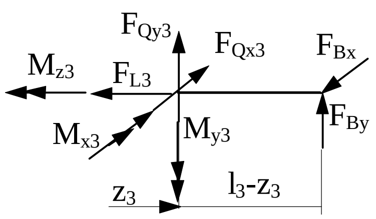
$M_{z1} = 0$

$0 \le z_2 \le l_2$



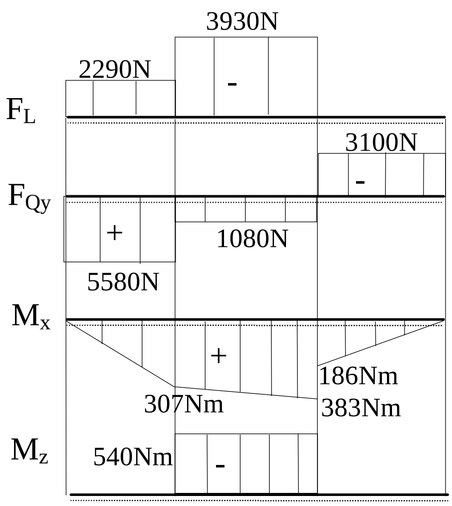
$F_{L2} = - F_{Az} - F_{a1} = - 3930N$
$F_{Qx2} = - F_{Ax} + F_{r1} = -1920N$
$F_{Qy2} = F_{Ay} - F_{u1} = 1080N$
$M_{x2} = F_{Ay} \cdot (l_1 + z_2) - F_{u1} \cdot z_2$
$M_{x2}(0) = 307Nm \quad M_{x2}(l_2) = 383Nm$
$M_{y2} = - F_{Ax} \cdot (l_1 + z_2) + F_{r1} \cdot z_2 - F_{a1} \cdot r_{01}$
$M_{y2}(0) = -398Nm \quad M_{y2}(l_2) = -533Nm$
$M_{z2} = -F_{u1} \cdot r_{01} = -540Nm$

$0 \le z_3 \le l_3$

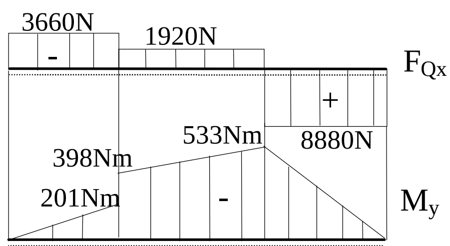


$F_{L3} = 0$
$F_{Qx3} = F_{Bx} = 8880N$
$F_{Qy3} = - F_{By} = -3100N$
$M_{x3} = F_{By} \cdot (l_3 - z_3) \qquad M_{x3}(0) = 186Nm \quad M_{x3}(l_3) = 0$
$M_{y3} = - F_{Bx} \cdot (l_3 - z_3) \qquad M_{y3}(0) = -533Nm \quad M_{y3}(l_3) = 0$
$M_{z3} = 0$

y-z-Ebene



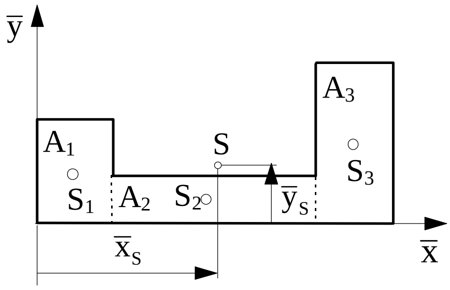
x-z-Ebene



M_{bmax} bei $z_2 = l_2 = 70mm$

$$M_{bmax} = \sqrt{M_{x2}^2(z_2 = l_2) + M_{y2}^2(z_2 = l_2)} = \sqrt{383^2 + 533^2}\,Nm = 656Nm$$

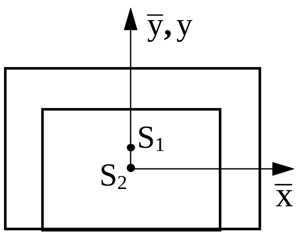
Lösung 5.4



i	$A_i\left[mm^2\right]$	$\bar{x}_{Si}[mm]$	$\bar{y}_{Si}[mm]$	$\bar{x}_{Si}A_i\left[mm^3\right]$	$\bar{y}_{Si}A_i\left[mm^3\right]$
1	600	10	15	6000	9000
2	900	50	7,5	45000	6750
3	1100	90	27,5	99000	30250
Σ	2600			150000	46000

Damit $\bar{x}_{SF} = \dfrac{150000}{2600}\,mm = 57{,}5mm;$ $\quad \bar{y}_{SF} = \dfrac{46000}{2600}\,mm = 17{,}7mm;$

Lösung 6.2



i	$A_i\left[h^2\right]$	$\bar{y}_{Si}[h]$	$\bar{y}_{Si}A_i\left[h^3\right]$	$I_{xxi}\left[h^4\right]$	$I_{yyi}\left[h^4\right]$	$\bar{y}_{Si}^2A_i\left[h^4\right]$
1	40	0,5	20	83,33	213,33	10
2	-24	0	0	-32	-72	0
Σ	16		20	51,33	141,33	10

$\bar{y}_S = \dfrac{20}{16}h = 1{,}25h;$ $\quad I_{\bar{x}\bar{x}} = 61{,}33h^4;$ $\quad I_{xx} = \left(61{,}33 - \dfrac{25}{16}\cdot 16\right)h^4 = 36{,}33h^4;$ $\quad I_{yy} = 141{,}33h^4$

$I_{xy} = 0;$ $\quad I_1 = I_{yy};$ $\quad I_2 = I_{xx}$

Lösung 6.6

i	$A_i\left[a^2\right]$	$\overline{x}_{Si}[a]$	$\overline{y}_{Si}[a]$	$\overline{x}_{Si}A_i\left[a^3\right]$	$\overline{y}_{Si}A_i\left[a^3\right]$
1	15	-1,5	2,5	-22,5	37,5
2	7,5	1	1,667	7,5	12,5
Σ	22,5			-15	50

$\overline{x}_S = -\frac{2}{3}a = -0{,}667a$; $\overline{y}_S = \frac{20}{9}a = 2{,}222a$;

Fortsetzung der Tabelle:

$I_{xxi}\left[a^4\right]$	$I_{yyi}\left[a^4\right]$	$I_{xyi}\left[a^4\right]$	$\overline{y}_{Si}^2A_i\left[a^4\right]$	$\overline{x}_{Si}^2A_i\left[a^4\right]$	$\overline{x}_{Si}\overline{y}_{Si}A_i\left[a^4\right]$
31,25	11,25	0	93,75	33,75	-56,25
10,42	3,75	3,125	20,84	7,5	12,5
41,67	15	3,125	114,59	41,25	-43,75

$I_{\overline{xx}} = (41{,}67 + 114{,}59)a^4 = 156{,}26a^4$; $I_{\overline{yy}} = (15 + 41{,}25)a^4 = 56{,}25a^4$

$I_{\overline{xy}} = (3{,}125 + 43{,}75) = 46{,}875a^4$

$I_{xx} = I_{\overline{xx}} - \overline{y}_S^2A = (156{,}26 - 111{,}09)a^4 = 45{,}17a^4$

$I_{yy} = I_{\overline{yy}} - \overline{x}_S^2A = (56{,}25 - 10)a^4 = 46{,}25a^4$

$I_{xy} = I_{\overline{xy}} + \overline{x}_S\overline{y}_SA = (46{,}875 - 33{,}317)a^4 = 13{,}56a^4$

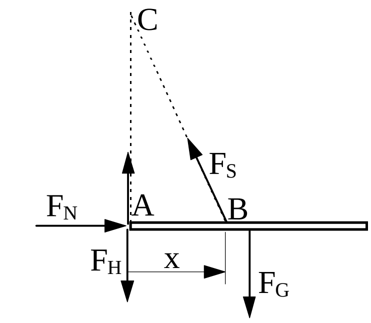
Hauptträgheitsmomente und Hauptachsen:

$$I_{1,2} = \left(45{,}71 \pm \sqrt{(-0{,}54)^2 + (13{,}56)^2}\right)a^4 = (45{,}71 \pm 13{,}57)a^4$$

$I_1 = 59{,}28a^4$; $I_2 = 32{,}14a^4$

$$\tan\varphi_{01} = \frac{I_1 - I_{xx}}{I_{xy}} = \frac{59{,}28 - 45{,}17}{13{,}56} = 1{,}04056 \qquad \varphi_{01} = 46{,}1°$$

Lösung 7.3



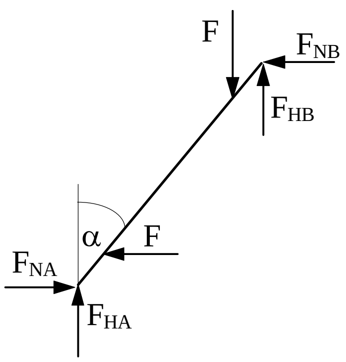
$$C{:}\quad F_G \cdot \frac{a}{2} - F_N \cdot a = 0 \qquad F_N = \frac{1}{2} F_G$$

$$B{:}\quad \pm F_H \cdot x - F_G \left(\frac{a}{2} - x \right) = 0$$

$$F_{Hmax} = \mu_0 F_N = \frac{1}{2} \mu_0 F_G$$

$$\pm \frac{1}{2} \mu_0 F_G \cdot x - F_G \left(\frac{a}{2} - x \right) = 0 \qquad \pm \frac{1}{2} \mu_0 \cdot x - \frac{a}{2} + x = 0$$

Lösung 7.5



$$\rightarrow{:}\quad F_{NA} - F - F_{NB} = 0$$

$$\uparrow{:}\quad F_{HA} - F + F_{HB} = 0$$

$$A{:}\quad F \cdot \frac{l}{8} \cos\alpha - F \cdot \frac{7}{8} l \sin\alpha + F_{NB} \cdot l \cos\alpha + F_{HB} \cdot l \sin\alpha = 0$$

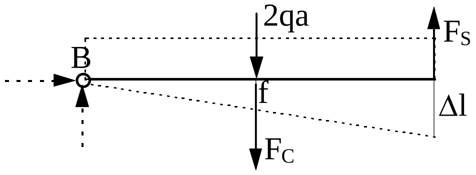
$$F_{HAmax} = \mu_0 F_{NA} \qquad F_{HBmax} = \mu_0 F_{NB}$$

$$\mu_0^2 + \frac{3}{4} (1 + \cot\alpha) \mu_0 - \cot\alpha = 0$$

$$\alpha = 45^\circ{:}\quad \mu_0^2 + \frac{3}{2} \mu_0 - 1 = 0 \qquad \mu_{01} = 0{,}5; (\mu_{02} = -2)$$

Lösungen für Aufgaben zur Technischen Mechanik – Festigkeitslehre -

Lösung 2.16



$$\rightarrow: F_{BH} = 0 \qquad (1)$$

$$\uparrow: F_{BV} = F_C - F_S + 2qa \qquad (2)$$

$$\curvearrowleft B: 2F_S a - 2qa^2 - F_C a = 0 \qquad (3)$$

$$\Delta l = -2f \qquad (4)$$

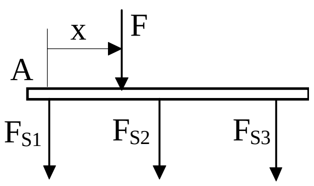
Für die lineare Feder gilt: $f = \dfrac{F_C}{c}$ und damit wird aus Gleichungen (3) und (4)

$$2F_S - F_C = 2qa$$

$$\frac{F_S l}{EA} + 2\frac{F_C}{c} = 0$$

$$F_S = \frac{qa}{1 + \dfrac{cl}{4EA}}; \quad F_C = -\frac{2qa\dfrac{cl}{4EA}}{1 + \dfrac{cl}{4EA}}; \quad F_{BV} = \frac{qa}{1 + \dfrac{cl}{4EA}}$$

Lösung 2.22



$$\downarrow: \; F_{S1} + F_{S2} + F_{S3} + F = 0 \qquad (1)$$

$$\curvearrowright A: \; F_{S2}\, a + 2\, F_{S3}\, a + F\, x = 0 \qquad (2)$$

$$w_i = \frac{F_{Si} l_i}{EA} \qquad (3), (4), (5)$$

1.) $w_1 = w_2 = w_3$

$$\frac{F_{S1}a}{EA} = \frac{3F_{S2}a}{2EA} = \frac{2F_{S3}a}{EA} \qquad F_{S2} = \frac{2}{3}F_{S1} \quad F_{S3} = \frac{1}{2}F_{S1} \qquad (6), (7)$$

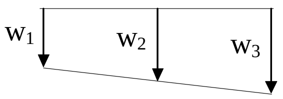
(6) und (7) in (1) : $\quad F_{S1} + \frac{2}{3}F_{S1} + \frac{1}{2}F_{S1} = -F \qquad F_{S1} = -\frac{6}{13}F$

(6) und (7) in (2) : $\quad \frac{2}{3}F_{S1}a + F_{S1}a = -Fx \qquad \boxed{x = -\frac{5}{3}\frac{F_{S1}}{F}a = \frac{10}{13}a}$

2.)

$$\boxed{\sigma_1 = \frac{F_{S1}}{A} = -\frac{6}{13}\frac{F}{A} \qquad \sigma_2 = \frac{F_{S2}}{A} = -\frac{4}{13}\frac{F}{A} \qquad \sigma_3 = \frac{F_{S3}}{A} = -\frac{3}{13}\frac{F}{A}}$$

3.)



$$w_2 = \frac{1}{2}\left(w_1 + w_3\right) \qquad (6')$$

(1) bis (5) und (6') liefern $\quad F_{S1} + F_{S2} + F_{S3} = -F \qquad (1')$

$$F_{S2}a + 2F_{S3}a = -\frac{2}{3}Fa \qquad (2')$$

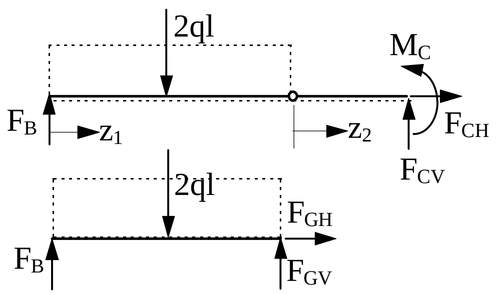
$$\frac{3}{2}F_{S2} = \frac{1}{2}F_{S1} + F_{S3} \qquad (6')$$

$F_{S1} + F_{S2} + F_{S3} = -F$ (I) $\quad F_{S1} - 3F_{S2} + 2F_{S3} = 0$ (II) $\quad F_{S2} + 2F_{S3} = -\frac{2}{3}F$ (III)

I - II : $4F_{S2} - F_{S3} = -F \qquad$ III + 2·(I - II) : $9F_{S2} = -\frac{8}{3}F$

$$\boxed{\begin{array}{lll} F_{S2} = -\frac{8}{27}F & F_{S3} = -\frac{5}{27}F & F_{S1} = -\frac{14}{27}F \\ w_2 = -\frac{4}{9}\frac{Fa}{EA} & w_3 = -\frac{10}{27}\frac{Fa}{EA} & w_1 = -\frac{14}{27}\frac{Fa}{EA} \end{array}}$$

Lösung 4.1



$$\rightarrow: F_{CH} = 0$$

$$\uparrow: F_B + F_{CV} - 2ql = 0$$

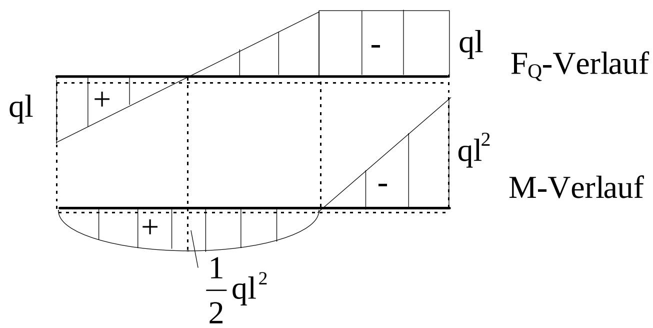
$$C: 3F_B l - 4ql^2 - M_C = 0$$

$$G: 2F_B l - 2ql^2 = 0$$

$$F_B = ql; \quad F_{CH} = 0; \quad F_{CV} = ql; \quad M_C = -ql^2$$

$$F_{GH} = 0; \quad F_{GV} = ql$$

$0 \le z_1 \le 2l$	$0 \le z_2 \le l$
$F_L(z_1) = 0$	$F_L(z_2) = 0$
$F_Q(z_1) = q(l - z_1)$	$F_Q(z_2) = -ql$
$M(z_1) = qlz_1 - \frac{1}{2}qz_1^2$	$M(z_2) = -qlz_2$



Aufgabe 6.2 (Statik): $\quad I_{xx} = 36{,}33h^4$

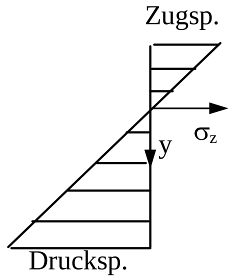
Maximalwerte für y: Unterseite $y = e_1 = 3{,}25h$
Oberseite $y = -e_2 = -1{,}75h$

Spannungsverteilung an der Einspannstelle (Maximales Moment)

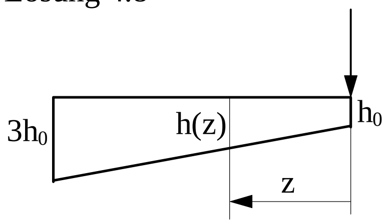
$$\sigma_z(y) = -\frac{ql^2}{I_{xx}} y$$

$$\sigma_z(y = e_1) = -0{,}0894 \frac{ql^2}{h^3} \Rightarrow \text{Druckspannung}$$

$$\sigma_z(y = -e_2) = 0{,}0482 \frac{ql^2}{h^3} \Rightarrow \text{Zugspannung}$$



Lösung 4.8



$$|\sigma(z)|_{max} = \frac{|M(z)|}{W_x(z)} \qquad |M(z)| = F \cdot z \quad W_x(z) = \frac{bh^2(z)}{6}$$

$$h(z) = h_0\left(1 + 2\frac{z}{l}\right) \quad W_x(z) = \frac{bh_0^2}{6}\left(1 + 2\frac{z}{l}\right)^2$$

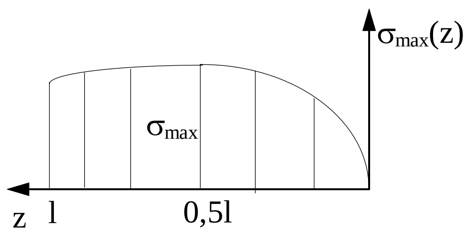
$$|\sigma(z)|_{max} = \frac{6F \cdot z}{bh_0^2\left(1 + 2\frac{z}{l}\right)^2}$$

Ort der max. Biegespannung entweder an der Einspannstelle (max. Moment) oder an der Stelle, an der $\frac{d\sigma_{max}(z)}{dz} = 0$ ist.

$$\frac{d\sigma_{max}(z)}{dz} = \frac{6F}{bh_0^2} \frac{\left(1 + 2\frac{z}{l}\right)^2 - 4\frac{z}{l}\left(1 + 2\frac{z}{l}\right)}{\left(1 + 2\frac{z}{l}\right)^4} = 0$$

$$\left(1 + 2\frac{z}{l}\right) - 4\frac{z}{l} = 0 \qquad \frac{z}{l} = \frac{1}{2}$$

$$\left|\sigma\left(\frac{l}{2}\right)\right|_{max} = \frac{3}{4}\frac{Fl}{bh_0^2} \qquad |\sigma(l)|_{max} = \frac{2}{3}\frac{Fl}{bh_0^2}$$



Die absolut größte Spannung beträgt $\sigma_{max} = \frac{3}{4}\frac{Fl}{bh_0^2}$ und tritt an der Stelle $z = \frac{l}{2}$ auf.

Lösung 4.18

Größte Beanspruchung an der Einspannstelle! Max. Spannung an einem Eckpunkt des Querschnittes, d.h.:

$$\left|\sigma_{\max}\right| = \frac{\left|M_{x\max}\right|}{\mathrm{I}_{xx}}\left|y_{\max}\right| + \frac{\left|M_{y\max}\right|}{\mathrm{I}_{yy}}\left|x_{\max}\right| = \frac{\left|M_{x\max}\right|}{\mathrm{W}_x} + \frac{\left|M_{y\max}\right|}{\mathrm{W}_y}$$

$$\left|M_{x\max}\right| = 2F \cdot 2l \qquad \mathrm{W}_x = \frac{bh^2}{6}$$

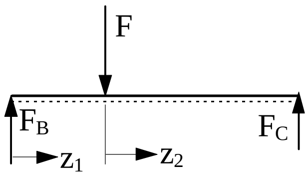
$$\left|M_{y\max}\right| = F \cdot l \qquad \mathrm{W}_y = \frac{hb^2}{6}$$

$$\left|\sigma_{\max}\right| = \frac{24Fl}{bh^2} + \frac{6Fl}{hb^2} = \sigma_{zul} \quad \Rightarrow \quad b^2 - \frac{24Fl}{\sigma_{zul}h^2} \cdot b - \frac{6Fl}{\sigma_{zul}h} = 0$$

$$b = \frac{12Fl}{\sigma_{zul}h^2}\left(1 \pm \sqrt{1 + \frac{\sigma_{zul} \cdot h^3}{24Fl}}\right) \qquad b = 58{,}7mm \qquad b_{gew} = 60mm$$

Nur das positive Vorzeichen ist physikalisch sinnvoll.

Lösung 4.33



$$\mathrm{F_B} = \frac{2}{3}\mathrm{F}; \qquad \mathrm{F_C} = \frac{1}{3}\mathrm{F}$$

$$0 \le \mathrm{z}_1 \le \mathrm{a} \qquad\qquad 0 \le \mathrm{z}_2 \le 2\mathrm{a}$$

$$\mathrm{M}(\mathrm{z}_1) = \mathrm{F_B z_1} = \frac{2}{3}\mathrm{F z_1} \qquad \mathrm{M}(\mathrm{z}_2) = \mathrm{F_C}(2\mathrm{a} - \mathrm{z}_2) = \frac{1}{3}\mathrm{F}(2\mathrm{a} - \mathrm{z}_2)$$

$$\mathrm{EI}v''(\mathrm{z}_1) = -\frac{2}{3}\mathrm{F z_1} \qquad\qquad \mathrm{EI}v''(\mathrm{z}_2) = -\frac{1}{3}\mathrm{F}(2\mathrm{a} - \mathrm{z}_2)$$

$$\mathrm{EI}v'(\mathrm{z}_1) = -\frac{1}{3}\mathrm{F z_1^2} + \mathrm{C}_1 \qquad\qquad \mathrm{EI}v'(\mathrm{z}_2) = \frac{1}{6}\mathrm{F}(2\mathrm{a} - \mathrm{z}_2)^2 + \mathrm{C}_3$$

$$\mathrm{EI}v(\mathrm{z}_1) = -\frac{1}{9}\mathrm{F z_1^3} + \mathrm{C_1 z_1} + \mathrm{C}_2 \qquad\qquad \mathrm{EI}v(\mathrm{z}_2) = -\frac{1}{18}\mathrm{F}(2\mathrm{a} - \mathrm{z}_2)^3 + \mathrm{C_3 z_2} + \mathrm{C}_4$$

$$\mathrm{RB/ÜB:} \quad v(\mathrm{z}_1 = 0) = 0 \qquad \Rightarrow \quad \mathrm{C}_2 = 0$$

$$v(\mathrm{z}_2 = 2\mathrm{a}) = 0 \qquad \Rightarrow \quad 2\mathrm{C_3 a} + \mathrm{C}_4 = 0$$

$$v(\mathrm{z}_1 = \mathrm{a}) = v(\mathrm{z}_2 = 0) \qquad \Rightarrow \quad -\frac{1}{9}\mathrm{Fa^3} + \mathrm{C_1 a} = -\frac{4}{9}\mathrm{Fa^3} + \mathrm{C}_4$$

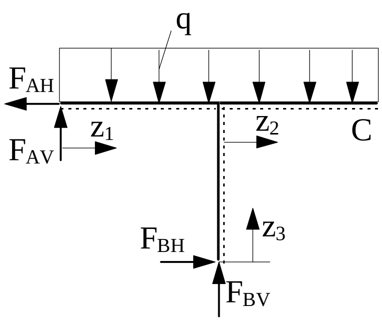
$$v'(\mathrm{z}_1 = \mathrm{a}) = v'(\mathrm{z}_2 = 0) \qquad \Rightarrow \quad -\frac{1}{3}\mathrm{Fa^2} + \mathrm{C}_1 = \frac{2}{3}\mathrm{Fa^2} + \mathrm{C}_3$$

$$\mathrm{C}_1 = \frac{5}{9}\mathrm{Fa^2} \qquad \mathrm{C}_2 = 0 \qquad \mathrm{C}_3 = -\frac{4}{9}\mathrm{Fa^2} \qquad \mathrm{C}_4 = \frac{8}{9}\mathrm{Fa^3}$$

$$v(\mathrm{z}_1) = \frac{\mathrm{Fa^3}}{9\mathrm{EI}}\left[5\frac{\mathrm{z}_1}{\mathrm{a}} - \left(\frac{\mathrm{z}_1}{\mathrm{a}}\right)^3\right] \qquad v(\mathrm{z}_2) = \frac{\mathrm{Fa^3}}{18\mathrm{EI}}\left[8 + 4\frac{\mathrm{z}_2}{\mathrm{a}} - 6\left(\frac{\mathrm{z}_2}{\mathrm{a}}\right)^2 + \left(\frac{\mathrm{z}_2}{\mathrm{a}}\right)^3\right]$$

$$v_\mathrm{F} = v(\mathrm{z}_1 = \mathrm{a}) = v(\mathrm{z}_2 = 0) = \frac{4\mathrm{Fa^3}}{9\mathrm{EI}}$$

Lösung 4.45



Die Aufgabe ist einfach statisch unbestimmt.
Gleichgewichtsbedingungen:

$$\leftarrow: \; F_{AH} - F_{BH} = 0$$
$$\uparrow: \; F_{AV} + F_{BV} - 2qa = 0$$
$$\curvearrowleft B: \; F_{AV} \cdot a - F_{AH} \cdot a = 0$$

$$M(z_1) = F_{AV} \cdot z_1 - \frac{1}{2} q z_1^2 \qquad M(z_2) = -\frac{1}{2} q (a - z_2)^2$$

$$EIv''(z_1) = \frac{1}{2} q z_1^2 - F_{AV} \cdot z_1 \qquad EIv''(z_2) = \frac{1}{2} q (a - z_2)^2$$

$$EIv'(z_1) = \frac{1}{6} q z_1^3 - \frac{1}{2} F_{AV} \cdot z_1^2 + C_1 \qquad EIv'(z_2) = -\frac{1}{6} q (a - z_2)^3 + C_3$$

$$EIv(z_1) = \frac{1}{24} q z_1^4 - \frac{1}{6} F_{AV} \cdot z_1^3 + C_1 z_1 + C_2 \qquad EIv(z_2) = \frac{1}{24} q (a - z_2)^4 + C_3 z_2 + C_4$$

$$M(z_3) = -F_{BH} \cdot z_3$$
$$EIv''(z_3) = F_{BH} \cdot z_3$$
$$EIv'(z_3) = \frac{1}{2} F_{BH} \cdot z_3^2 + C_5$$
$$EIv(z_3) = \frac{1}{6} F_{BH} \cdot z_3^3 + C_5 z_3 + C_6$$

RB/ÜB:

1. $v(z_1 = 0) = 0 \quad \Rightarrow \quad C_2 = 0$
2. $v(z_3 = 0) = 0 \quad \Rightarrow \quad C_6 = 0$
3. $v(z_1 = a) = 0 \quad \Rightarrow \quad \frac{1}{24} q a^4 - \frac{1}{6} F_{AV} a^3 + C_1 a = 0 \quad C_1 = \frac{1}{6} F_{AV} a^2 - \frac{1}{24} q a^3$
4. $v(z_2 = 0) = 0 \quad \Rightarrow \quad \frac{1}{24} q a^4 + C_4 = 0 \qquad C_4 = -\frac{1}{24} q a^4$
5. $v(z_3 = a) = 0 \quad \Rightarrow \quad \frac{1}{6} F_{BH} a^3 + C_5 a = 0 \qquad C_5 = -\frac{1}{6} F_{BH} a^2$
6. $v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad \frac{1}{6} q a^3 - \frac{1}{2} F_{AV} a^2 + C_1 = -\frac{1}{6} q a^3 + C_3$
7. $v'(z_3 = a) = v'(z_2 = 0) \quad \Rightarrow \quad \frac{1}{2} F_{BH} a^2 + C_5 = -\frac{1}{6} q a^3 + C_3 \quad C_3 = \frac{1}{3} F_{BH} a^2 + \frac{1}{6} q a^3$

Ergebnisse aus GGW und RB/ÜB:

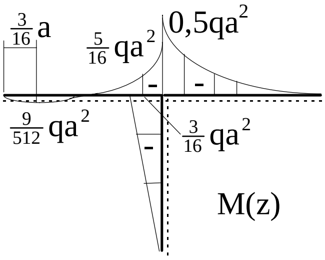
aus 6. folgt mit den GGW $F_{AV} = \frac{3}{16} qa \quad \Rightarrow \quad F_{AH} = F_{BH} = \frac{3}{16} qa$ und $F_{BV} = \frac{29}{16} qa$

Momentenverlauf:

$$M(z_1) = \frac{1}{2} q z_1 \left(\frac{3}{8} a - z_1 \right) \quad F_Q(z_1^*) = \frac{3}{16} qa - q z_1^* = 0 \quad z_1^* = \frac{3}{16} a \quad M\left(z_1^*\right) = \frac{9}{512} qa^2$$

$$M(z_2) = -\frac{1}{2} q \left(a - z_2 \right)^2$$

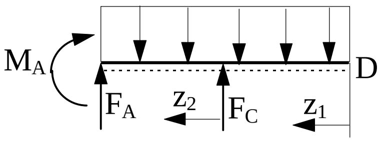
$$M(z_3) = -\frac{3}{16} qa z_3$$



Verschiebung des Punktes C:

$$v_C = v(z_2 = a) = \frac{1}{EI}\left(C_3 a + C_4 \right) = \frac{3qa^4}{16EI}$$

Lösung 4.50



$F_C = v(z_2 = 0) \cdot c$

$$M(z_1) = -\frac{1}{2} q z_1^2 \qquad M(z_2) = -\frac{1}{2} q \left(a + z_2 \right)^2 + F_C z_2$$

$$EIv''(z_1) = \frac{1}{2} q z_1^2 \qquad EIv''(z_2) = \frac{1}{2} q \left(a + z_2 \right)^2 - F_C z_2$$

$$EIv'(z_1) = \frac{1}{6} q z_1^3 + C_1 \qquad EIv'(z_2) = \frac{1}{6} q \left(a + z_2 \right)^3 - \frac{1}{2} F_C z_2^2 + C_3$$

$$EIv(z_1) = \frac{1}{24} q z_1^4 + C_1 z_1 + C_2 \qquad EIv(z_2) = \frac{1}{24} q \left(a + z_2 \right)^4 - \frac{1}{6} F_C z_2^3 + C_3 z_2 + C_4$$

RB/ÜB:

1. $v'(z_2 = a) = 0 \quad \Rightarrow \quad C_3 = \frac{1}{2} F_C a^2 - \frac{4}{3} qa^3$

2. $v(z_2 = a) = 0 \quad \Rightarrow \quad C_4 = \frac{2}{3} qa^4 - \frac{1}{3} F_C a^3$

3. $v'(z_1 = a) = v'(z_2 = 0) \quad \Rightarrow \quad C_1 = C_3 = \frac{1}{2} F_C a^2 - \frac{4}{3} qa^3$

4. $v(z_1 = a) = v(z_2 = 0) \quad \Rightarrow \quad C_2 = C_4 - C_1 a = 2qa^4 - \frac{5}{6} F_C a^3$

$$F_C = v(z_2 = 0)\cdot c = \left(\frac{17qa^4}{24EI} - \frac{F_C a^3}{3EI}\right)\cdot c \quad \Rightarrow \quad F_C = \frac{17qac}{8\left(\frac{3EI}{a^3} + c\right)}$$

$$F_C(c \to \infty) = \frac{17}{8}qa$$

$$v_D = v(z_1 = 0) = \frac{C_2}{EI} = \frac{2qa^4}{EI} - \frac{5F_C a^3}{6EI}$$

$$v_D(c \to 0) = \frac{2qa^4}{EI} \qquad v_D(c \to \infty) = \frac{2qa^4}{EI} - \frac{85qa^4}{48EI} = \frac{11qa^4}{48EI}$$

Lösung 5.6

1. $m_v = A_v \cdot \rho \cdot l = \frac{\pi D^2}{4}\cdot \rho \cdot l \quad m_h = A_h \cdot \rho \cdot l = \frac{\pi}{4}\left(D^2 - d^2\right)\cdot \rho \cdot l$

$$\frac{m_v - m_h}{m_v}\cdot 100\% = \left(\frac{d}{D}\right)^2 \cdot 100\% = 25\%$$

2. $\tau_v = \frac{M_t}{W_{tv}} \quad \tau_h = \frac{M_t}{W_{th}} \qquad W_{tv} = \frac{\pi D^3}{16} \quad W_{th} = W_{tv}\left(1 - \left(\frac{d}{D}\right)^4\right)$

$$\left(\frac{\tau_h - \tau_v}{\tau_v}\right)\cdot 100\% = \left(\frac{W_{tv}}{W_{th}} - 1\right)\cdot 100\% = \left(\frac{1}{1 - \left(\frac{d}{D}\right)^4} - 1\right)\cdot 100\% = \left(\frac{16}{15} - 1\right)\cdot 100\% = 6{,}7\%$$

3. $\left(\frac{\vartheta_h - \vartheta_v}{\vartheta_v}\right)\cdot 100\% = \left(\frac{I_{tv}}{I_{th}} - 1\right)\cdot 100\% = \left(\frac{1}{1 - \left(\frac{d}{D}\right)^4} - 1\right)\cdot 100\% = \left(\frac{16}{15} - 1\right)\cdot 100\% = 6{,}7\%$

Lösung 7.11

Momente an der Einspannstelle: Biege- und Torsionsmoment erreichen an der Einspannstelle ihr Maximum.

$$\left|M_{bmax}\right| = \left|\left(2F_1 - F_2\right)\cdot a\right| = 80\cdot 10^4\,\text{Nmm}$$

$$\left|M_{tmax}\right| = \left|\left(F_1 - F_2\right)\cdot b\right| = 60\cdot 10^4\,\text{Nmm}$$

Spannungen: $\sigma_{max} = \frac{\left|M_{bmax}\right|\cdot 32\cdot D_a}{\pi\left(D_a^4 - D_i^4\right)} \qquad \tau_{max} = \frac{\left|M_{tmax}\right|\cdot 16\cdot D_a}{\pi\left(D_a^4 - D_i^4\right)}$

Der erforderliche Innendurchmesser folgt aus der Beziehung $\sigma_{V4} = \sigma_{max}\sqrt{1 + 3\left(\frac{\tau_{max}}{\sigma_{max}}\right)^2} \le \sigma_{zul}$

$$\sigma_{zul} \geq \frac{32 D_a |M_{bmax}|}{\pi\left(D_a^4 - D_i^4\right)} \sqrt{1 + 3\left(\frac{|M_{tmax}|}{2|M_{bmax}|}\right)^2}$$

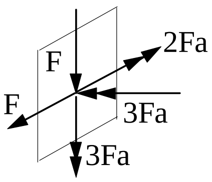
$$D_i \leq \sqrt[4]{D_a^4 - \frac{32 D_a |M_{bmax}|}{\pi\sigma_{zul}} \sqrt{1 + 3\left(\frac{|M_{tmax}|}{2|M_{bmax}|}\right)^2}} = 42{,}3\,\text{mm}$$

Gewählt: $D_i = 42$ mm

Spannungsnachweis: $\sigma_{Vvorh} = 154{,}8 \frac{N}{mm^2} < \sigma_{zul} = 160 \frac{N}{mm^2}$

Lösung 7.13

Biege- und Torsionsmomente an der Einspannstelle:



$F_N = 0$

$|M_x| = 2Fa \quad |M_y| = 3Fa \quad |M_t| = 3Fa$

$$M_{res} = \sqrt{M_x^2 + M_y^2} = Fa\sqrt{4+9} = \sqrt{13}Fa = 1{,}8028 \cdot 10^6\,\text{Nmm}$$

Spannungen:

$$\sigma_{max} = \frac{32 M_{res} \cdot d_a}{\pi\left(d_a^4 - d_i^4\right)} \qquad \tau_{max} = \frac{16 M_t \cdot d_a}{\pi\left(d_a^4 - d_i^4\right)}$$

$$\sigma_{Vmax} = \sqrt{\sigma_{max}{}^2 + 3\tau_{max}{}^2} = \frac{32}{\pi} \frac{d_a}{\left(d_a^4 - d_i^4\right)} \sqrt{M_{res}^2 + 3\left(\frac{M_t}{2}\right)^2}$$

Erforderlicher Innendurchmesser:

Aus $\sigma_{Vmax} \leq \sigma_{zul}$ folgt

$$d_i \leq \sqrt[4]{d_a^4 - \frac{32}{\pi} \frac{d_a}{\sigma_{zul}} \sqrt{M_{res}^2 + 3\left(\frac{M_t}{2}\right)^2}} = 94{,}9\,\text{mm} \qquad d_{igew} = 94\,\text{mm}$$

$$\sigma_{vorh} = 103{,}24 \frac{N}{mm^2} < \sigma_{zul} = 120 \frac{N}{mm^2}$$

Lösungen für Aufgaben zur Technischen Mechanik – Dynamik -

Lösung 1.6

Bereich I: $\ddot{x} = a_A \quad \dot{x} = a_A t + C_1 \quad x = \frac{1}{2} a_A t^2 + C_1 t + C_2$

AB: $t = 0 \quad \dot{x} = 0, \quad x = 0 \quad \Rightarrow \quad C_1 = 0, \quad C_2 = 0$

$t = t_1: \quad \dot{x}(t_1) = v, \quad x(t_1) = s_1 \quad \Rightarrow \quad t_1 = \frac{v}{a_A} = 20\text{s} \quad s_1 = \frac{1}{2} a_A t_1^2 = 160\text{m}$

Bereich II: $\ddot{x} = 0 \quad \dot{x} = C_3 \quad x = C_3 t + C_4$

ÜB $t = t_1 \quad \dot{x} = v, \quad x = s_1 \quad \Rightarrow \quad C_3 = v = 16\frac{\text{m}}{\text{s}}, \quad C_4 = s_1 - v t_1 = -160\text{m}$

$t = t_2: \quad \dot{x}(t_2) = v, \quad x(t_2) = s_2 = v t_2 + s_1 - v t_1 = v(t_2 - t_1) + s_1$

Bereich III: $\ddot{x} = a_B \quad \dot{x} = a_B t + C_5 \quad x = \frac{1}{2} a_B t^2 + C_5 t + C_6$

ÜB $t = t_2 \quad \dot{x} = v, \quad x = s_2 \quad \Rightarrow \quad C_5 = v - a_B t_2 \quad C_6 = s_2 - v t_2 + \frac{1}{2} a_B t_2^2$

Endbedingung: $t = t_3 \quad \dot{x} = 0, \quad x = s_3 = S \quad \Rightarrow \quad a_B(t_3 - t_2) + v = 0 \quad$ **(1)** und

$S = \frac{1}{2} a_B t_3^2 + (v - a_B t_2) t_3 + s_2 - v t_2 + \frac{1}{2} a_B t_2^2 = \frac{1}{2} a_B (t_3 - t_2)^2 + v(t_3 - t_2) + s_2 \quad$ **(2)**

$t_2 = t_3 + \frac{v}{a_B}$ in **(2)** $\Rightarrow \quad t_3 = t_1 + \frac{S - s_1}{v} - \frac{1}{2}\frac{v}{a_B} = (20 + 23 + 8)\text{s} = 51\text{s} = t_{ges}$

$t_2 = (51 - 16)\text{s} = 35\text{s} \quad \Rightarrow \quad s_2 = S - \frac{1}{2} a_B (t_3 - t_2)^2 - v(t_3 - t_2) = 400\text{m}$

Lösung 1.7

1. Bereich: $0 \le t \le t_1$

$a(t) = K_1 t^2 \quad v(t) = \frac{1}{3} K_1 t^3 + C_1 \quad s(t) = \frac{1}{12} K_1 t^4 + C_1 t + C_2$

$t = 0: \quad v(0) = 0 \quad s(0) = 0 \quad \Rightarrow \quad C_1 = 0 \quad C_2 = 0$

$t = t_1: \quad a(t_1) = a_1 \quad \Rightarrow \quad a_1 = K_1 t_1^2 \quad K_1 = \frac{a_1}{t_1^2} = 1\frac{\text{m}}{\text{s}^4}$

$$v(t_1) = \frac{1}{3} K_1 t_1^3 = \frac{1}{3}\frac{\text{m}}{\text{s}} \quad s(t_1) = \frac{1}{12} K_1 t_1^4 = \frac{1}{12}\text{m}$$

2. Bereich $\quad t_1 \le t \le t_2$

$$a(t) = a_1 \quad v(t) = a_1 t + C_3 \quad s(t) = \frac{1}{2} a_1 t^2 + C_3 t + C_4$$

$$\text{ÜB:} \quad t = t_1 \quad v_2(t_1) = v_1(t_1) = \frac{1}{3}\frac{m}{s} \quad \Rightarrow \quad \frac{1}{3}\frac{m}{s} = a_1 t_1 + C_3 = 1\frac{m}{s} + C_3 \quad C_3 = -\frac{2}{3}\frac{m}{s}$$

$$s_2(t_1) = s_1(t_1) = \frac{1}{12} m \quad \Rightarrow \quad \frac{1}{12} m = \frac{1}{2} m - \frac{2}{3} m + C_4 \qquad C_4 = \frac{1}{4} m$$

$$t = t_2 \quad v_2(t_2) = a_1 t_2 + C_3 = 2\frac{m}{s} - \frac{2}{3}\frac{m}{s} = \frac{4}{3}\frac{m}{s}$$

$$s_2(t_2) = \frac{1}{2} a_1 t_2^2 + C_3 t_2 + C_4 = 2m - \frac{4}{3} m + \frac{1}{4} m = \frac{11}{12} m$$

3. *Bereich* $\quad t_2 \le t \le t_3$

$$a(t) = K_2 + K_3 t \quad v(t) = K_2 t + \frac{1}{2} K_3 t^2 + C_5 \quad s(t) = \frac{1}{2} K_2 t^2 + \frac{1}{6} K_3 t^3 + C_5 t + C_6$$

$$t = t_2: \quad a(t_2) = a_1 \quad a_1 = K_2 + K_3 t_2 \Big| -$$

$$t = t_3: \quad a(t_3) = 0 \quad 0 = K_2 + K_3 t_3 \Big| +$$

$$-a_1 = K_3 (t_3 - t_2) \quad \Rightarrow \quad K_3 = \frac{-a_1}{t_3 - t_2} = -1\frac{m}{s^3} K_2 = -K_3 t_3 = 3\frac{m}{s^2}$$

$$\ddot{U}B: \quad t = t_2 \quad v_2(t_2) = v_3(t_2) = \frac{4}{3}\frac{m}{s} \quad \Rightarrow \quad \frac{4}{3}\frac{m}{s} = K_2 t_2 + \frac{1}{2} K_3 t_2^2 + C_5 = 6\frac{m}{s} - 2\frac{m}{s} + C_5$$

$$s_2(t_2) = s_3(t_2) = \frac{11}{12} m \quad \Rightarrow \quad \frac{11}{12} m = -\frac{4}{3} m + 6m + C_5 t_2 + C_6$$

$$C_5 = -\frac{8}{3}\frac{m}{s} \quad C_6 = \frac{19}{12} m$$

Lösung 2.1

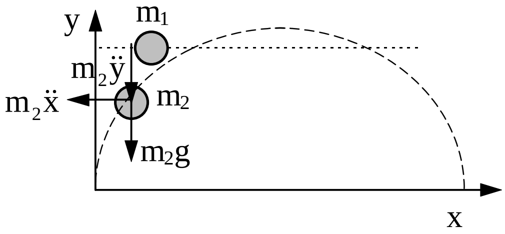
$$\omega_W \cdot \frac{D}{2} = v \quad \Rightarrow \quad \omega_W = \frac{2v}{D}$$

$$\omega_W \cdot r_G = \omega_{ZW} \cdot r_K \quad \Rightarrow \quad \omega_{ZW} = \frac{r_G}{r_K} \cdot \omega_W = \frac{r_G}{r_K} \cdot \frac{2v}{D}$$

$$\omega_{ZW} \cdot r_G = \omega_M \cdot r_K \quad \Rightarrow \quad \frac{r_G}{r_K} \cdot \frac{2v}{D} = \frac{r_K}{r_G} \cdot 2\pi n_M \quad \left(\frac{r_K}{r_G}\right)^2 = \frac{v}{\pi n_M D} = i^2$$

$$i = \frac{r_K}{r_G} = \sqrt{\frac{v}{\pi n_M D}} = 0{,}18 = \frac{z_K}{z_G}$$

Lösung 3.5



$$y_1 = H_0 \quad \dot{x}_1 = v_1 \quad x_1(t) = v_1 t + C_1 \quad C_1 = 0$$

$$\ddot{y}_2 = -g \quad \dot{y}_2 = -gt + C_2 \quad y_2(t) = -\frac{1}{2} g t^2 + C_2 t + C_3$$

$$\ddot{x}_2 = 0 \quad \dot{x}_2 = C_4 \quad x_2(t) = C_4 t + C_5$$

$$AB: \quad t = 0 \quad \dot{x}_2 = v_2 \cos\alpha \quad \Rightarrow \quad C_4 = v_2 \cos\alpha$$

$$\dot{y}_2 = v_2 sin\alpha \quad \Rightarrow \quad C_2 = v_2 sin\alpha$$

$$x_2 = 0 \quad \Rightarrow \quad C_5 = 0$$

$$y_2 = 0 \quad \Rightarrow \quad C_3 = 0$$

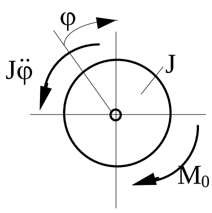
$$EB: \quad t = t_E \quad x_1(t_E) = x_2(t_E) \quad \Rightarrow \quad v_2 = \frac{v_1}{\cos\alpha}$$

$$y_1(t_E) = y_2(t_E) \quad \Rightarrow \quad H_0 = -\frac{1}{2} g t_E^2 + v_1 \tan\alpha \cdot t_E$$

$$t_E^2 - 2\frac{v_1 \tan\alpha}{\mathrm{g}} t_E + 2\frac{H_0}{g} = 0 \quad \Rightarrow \quad t_{E1,2} = \frac{v_1 \tan\alpha}{\mathrm{g}} \pm \sqrt{\left(\frac{v_1 \tan\alpha}{\mathrm{g}}\right)^2 - 2\frac{H_0}{g}}$$

Zahlenwerte: $\mathrm{v}_2 = 1200\,\frac{\mathrm{km}}{\mathrm{h}}$, $\quad \mathrm{t}_{E1} = 3{,}67\mathrm{s} \quad$ **($\mathrm{t}_{E2} = 55{,}1\mathrm{s}$)**

Lösung 3.13



$$\mathrm{J}\ddot{\varphi} = \mathrm{M}_0 \quad \mathrm{J}(\omega - \omega_0) = \mathrm{M}_0(\mathrm{t} - \mathrm{t}_0)$$

$$\mathrm{t}_0 = 0 \quad \omega_0 = 0$$

$$\mathrm{t} = \mathrm{t}_1\mathbf{:} \quad \mathrm{J}\omega_1 = \mathrm{M}_0 \mathrm{t}_1 \quad \Rightarrow \quad \mathrm{M}_0 = \frac{\mathrm{J}\omega_1}{\mathrm{t}_1}$$

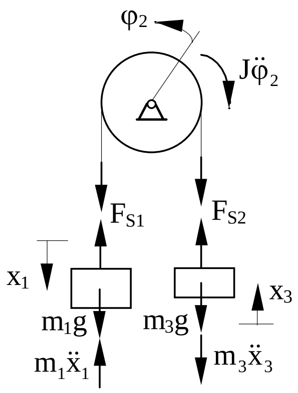
$$\mathrm{J} = \frac{1}{2}\mathrm{m}_1\left(\frac{\mathrm{D}}{2}\right)^2 - 6\left\{\frac{1}{2}\mathrm{m}_2\left(\frac{\mathrm{d}}{2}\right)^2 + \mathrm{m}_2 \mathrm{e}^2\right\}$$

$$\mathrm{m}_1 = \rho \mathrm{a}\frac{\pi \mathrm{D}^2}{4} \quad \mathrm{m}_2 = \rho \mathrm{b}\frac{\pi \mathrm{d}^2}{4} \quad \Rightarrow \quad \mathrm{J} = \frac{\rho\pi}{32}\left[\mathrm{a}\mathrm{D}^4 - 6\mathrm{b}\mathrm{d}^2\left(\mathrm{d}^2 + 8\mathrm{e}^2\right)\right]$$

$$\mathrm{M}_0 = \frac{\omega_1 \rho\pi}{32\mathrm{t}_1}\left[\mathrm{a}\mathrm{D}^4 - 6\mathrm{b}\mathrm{d}^2\left(\mathrm{d}^2 + 8\mathrm{e}^2\right)\right] \qquad \mathrm{n}_1 = \frac{\omega_1}{2\pi}$$

Zahlenwerte**:** $\mathrm{n}_1 = 4{,}77\,\mathrm{s}^{-1} \equiv 286{,}5\,\mathrm{min}^{-1} \quad \mathrm{M}_0 = 0{,}3933\,\mathrm{Nm}$

Lösung 3.20



ZB: $x_3 = x_1 = x \qquad \varphi_2 = \varphi = \frac{x}{r}$

$$J = \frac{1}{2} m_2 r^2$$

$$F_{S1} = m_1 g - m_1 \ddot{x}_1 = m_1 g - m_1 \ddot{x}$$

$$F_{S2} = m_3 g + m_3 \ddot{x}_3 = m_3 g + m_3 \ddot{x}$$

$$J\ddot{\varphi}_2 + F_{S2} r - F_{S1} r = 0 = J\frac{\ddot{x}}{r} + m_3 g r + m_3 \ddot{x} r - m_1 g r + m_1 \ddot{x} r$$

$$\ddot{x} = g \frac{m_1 - m_3}{m_1 + \frac{1}{2} m_2 + m_3}$$

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{2} J\dot{\varphi}_2^2 = \frac{1}{2}\dot{x}^2\left(m_1 + \frac{1}{2} m_2 + m_3\right)$$

$$U = -m_1 g x_1 + m_3 g x_3 = -gx(m_1 - m_3)$$

$$L = T - U = \frac{1}{2}\dot{x}^2\left(m_1 + \frac{1}{2} m_2 + m_3\right) + gx(m_1 - m_3)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

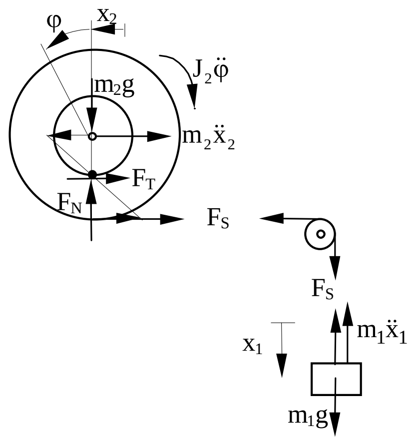
$$\frac{\partial L}{\partial x} = g(m_1 - m_3) \qquad \frac{\partial L}{\partial \dot{x}} = \dot{x}\left(m_1 + \frac{1}{2} m_2 + m_3\right) \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \ddot{x}\left(m_1 + \frac{1}{2} m_2 + m_3\right)$$

$$T + U = \text{konst.} \qquad \Rightarrow \qquad \frac{d}{dt}(T + U) = 0$$

$$T + U = \frac{1}{2}\dot{x}^2\left(m_1 + \frac{1}{2} m_2 + m_3\right) - gx(m_1 - m_3)$$

$$\ddot{x}\dot{x}\left(m_1 + \frac{1}{2} m_2 + m_3\right) - g\dot{x}(m_1 - m_3) = 0$$

Lösung 3.26



$$\varphi = \frac{x_2}{r} \qquad x_1 = (R - r)\varphi = \left(\frac{R}{r} - 1\right) x_2$$

$$F_S = m_1 g - m_1\left(\frac{R}{r} - 1\right)\ddot{x}_2$$

$$F_S(R - r) - J_2 \frac{\ddot{x}_2}{r} - m_2 \ddot{x}_2 r = 0 \,\Big|\cdot \frac{1}{r}$$

$$\ddot{x}_2\left\{m_2 + \frac{J_2}{r^2} + m_1\left(\frac{R}{r} - 1\right)^2\right\} = m_1 g\left(\frac{R}{r} - 1\right)$$

$$J_2 = 2J_A + J_B \qquad J_A = \frac{1}{2} m_A r^2 \qquad J_B = \frac{1}{2} m_B R^2$$

$$m_2 = 2m_A + m_B \quad m_A = \pi r^2 \frac{d}{2}\cdot\rho \quad m_B = \pi R^2 b\cdot\rho$$

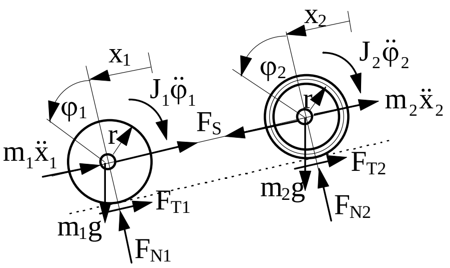
$$J_2 = \frac{1}{2}\pi\rho\left\{dr^4 + bR^4\right\} \qquad m_2 = \pi\rho\left\{dr^2 + bR^2\right\} \quad\Rightarrow\quad \rho = \frac{m_2}{\pi\left\{dr^2 + bR^2\right\}}$$

$$J_2 = \frac{1}{2}m_2\frac{\left\{dr^4 + bR^4\right\}}{\left\{dr^2 + bR^2\right\}}$$

$$\ddot{x}_1 = \left(\frac{R}{r} - 1\right)\ddot{x}_2 = \frac{m_1 g}{m_1 + \frac{m_2}{\left(\frac{R}{r}-1\right)^2} + \frac{J_2}{r^2\left(\frac{R}{r}-1\right)^2}}$$

$$\mathrm{T} = \frac{1}{2}\left(\mathrm{m}_2\dot{\mathrm{x}}_2^2 + \mathrm{m}_1\dot{\mathrm{x}}_1^2 + \mathrm{J}_2\dot{\varphi}^2\right) \qquad \mathrm{U} = -\mathrm{m}_1\mathrm{g}\mathrm{x}_1 \quad \text{z.B.} \quad \frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{T}+\mathrm{U}) = 0$$

Lösung 3.28



$$x_1 = x_2 = x \qquad \varphi_1 = \varphi_2 = \varphi = \frac{x}{r}$$

$$m_1: \quad J_1\ddot{\varphi} + F_S r + m_1\ddot{x}r - m_1 g\sin\alpha\cdot r = 0$$

$$m_2: \quad J_2\ddot{\varphi} - F_S r + m_2\ddot{x}r - m_2 g\sin\alpha\cdot r = 0$$

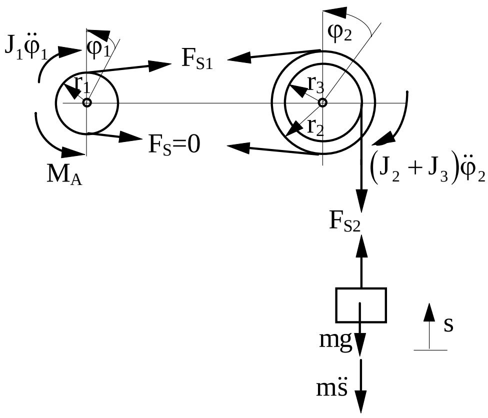
$$J_2\frac{\ddot{x}}{r^2} + m_2\ddot{x} - m_2 g\sin\alpha + J_1\frac{\ddot{x}}{r^2} + m_1\ddot{x} - m_1 g\sin\alpha = 0$$

$$m_1 = m_2 = m \qquad J_1 = \frac{1}{2}mr^2 \qquad J_2 = mr^2$$

$$\ddot{x}\left(m + m + \frac{1}{2}m + m\right) = 2mg\sin\alpha \qquad \ddot{x} = \frac{4}{7}g\sin\alpha$$

$$F_S = m\left(\ddot{x} + \ddot{x} - g\sin\alpha\right) = \frac{1}{7}mg\sin\alpha$$

Lösung 3.31



$$\text{ZB:} \quad \varphi_2 = \frac{\mathrm{s}}{\mathrm{r}_3} \quad \mathrm{r}_2\varphi_2 = \mathrm{r}_1\varphi_1 \quad\Rightarrow\quad \varphi_1 = \frac{\mathrm{r}_2}{\mathrm{r}_1}\frac{\mathrm{s}}{\mathrm{r}_3}$$

$$\text{Masse m:} \quad \mathrm{m}\ddot{\mathrm{s}} + \mathrm{mg} - \mathrm{F}_{S2} = 0$$

$$\text{Scheibe 1:} \quad \mathrm{J}_1\ddot{\varphi}_1 - \mathrm{M}_A + \mathrm{F}_{S1}\mathrm{r}_1 = 0$$

$$\text{Scheibe 2/3:} \quad \left(\mathrm{J}_2 + \mathrm{J}_3\right)\ddot{\varphi}_2 - \mathrm{F}_{S1}\mathrm{r}_2 + \mathrm{F}_{S2}\mathrm{r}_3 = 0$$

$$\mathrm{F}_{S2} = \mathrm{m}\ddot{\mathrm{s}} + \mathrm{mg} \qquad \mathrm{F}_{S1} = \frac{\mathrm{M}_A}{\mathrm{r}_1} - \mathrm{J}_1\frac{\mathrm{r}_2}{\mathrm{r}_1^2}\frac{\ddot{\mathrm{s}}}{\mathrm{r}_3}$$

$$\left(\mathrm{J}_2 + \mathrm{J}_3\right)\frac{\ddot{\mathrm{s}}}{\mathrm{r}_3} - \frac{\mathrm{r}_2}{\mathrm{r}_1}\mathrm{M}_A + \mathrm{J}_1\frac{\mathrm{r}_2^2}{\mathrm{r}_1^2}\frac{\ddot{\mathrm{s}}}{\mathrm{r}_3} + \mathrm{m}\ddot{\mathrm{s}}\mathrm{r}_3 + \mathrm{mgr}_3 = 0$$

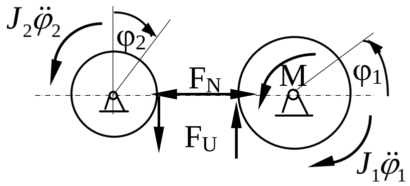
($F_S = 0$, da Riemenabtriebsseite ohne Last)

$$\ddot{s} = \frac{\frac{r_2}{r_1 r_3} M_A - mg}{\left(\frac{r_2}{r_3}\right)^2 \frac{J_1}{r_1^2} + \frac{(J_2 + J_3)}{r_3^2} + m} \qquad 1.\ M_{Amin} \quad \text{für} \quad \ddot{s} = 0 \quad \Rightarrow \quad M_A = M_{Amin} = \frac{r_1 r_3}{r_2} mg$$

$$2.\ M_A = 2M_{Amin} \quad \Rightarrow \quad \ddot{s} = \frac{mg}{\left(\frac{r_2}{r_3}\right)^2 \frac{J_1}{r_1^2} + \frac{(J_2 + J_3)}{r_3^2} + m} = K \qquad \dot{s} = Kt \qquad s = \frac{1}{2} K t^2$$

$$3.\ t = \frac{\dot{s}}{K} \quad \Rightarrow \quad s = \frac{1}{2}\frac{\dot{s}^2}{K} \quad \text{oder} \quad \ddot{s} = K = \frac{d\dot{s}}{ds}\dot{s} \qquad \dot{s}d\dot{s} = Kds \qquad \dot{s} = \sqrt{2Ks}$$

Lösung 3.39



$$r_1\dot{\varphi}_1 = r_2\dot{\varphi}_2 \quad \Rightarrow \quad \ddot{\varphi}_1 = \frac{r_2}{r_1}\ddot{\varphi}_2$$

$$J_1\ddot{\varphi}_1 + F_U r_1 - M = 0$$

$$J_2\ddot{\varphi}_2 - F_U r_2 = 0 \quad \Rightarrow \quad F_U = \frac{J_2}{r_2}\ddot{\varphi}_2$$

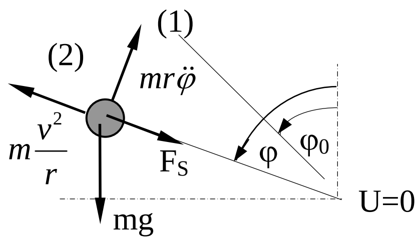
$$\ddot{\varphi}_2\left(J_1\frac{r_2}{r_1} + J_2\frac{r_1}{r_2}\right) - M = 0 \quad \ddot{\varphi}_2 = \frac{M}{\left(J_1\frac{r_2}{r_1} + J_2\frac{r_1}{r_2}\right)} = K$$

$$\ddot{\varphi}_2 = K \qquad \dot{\varphi}_2 = Kt + C \quad \dot{\varphi}_2(0) = 0 \quad \Rightarrow \quad C = 0 \qquad \dot{\varphi}_2 = Kt$$

$$t = t_1: \qquad \dot{\varphi}_2 = \omega_2 \qquad \omega_2 = Kt_1 \quad \Rightarrow \quad M = \frac{\omega_2}{t_1}\left(J_1\frac{r_2}{r_1} + J_2\frac{r_1}{r_2}\right) \qquad \omega_2 = 2\pi n_2$$

$$J_1 = \frac{m_1}{2}r_1^2 = \frac{r_1^4}{2}\pi\rho b_1 \qquad J_2 = \frac{m_2}{2}r_2^2 = \frac{r_2^4}{2}\pi\rho b_2 \qquad M = \frac{\pi^2 n_2 \rho r_1 r_2}{t_1}\left(r_1^2 b_1 + r_2^2 b_2\right) = 4{,}6\,Nm$$

Lösung 4.2



$$U_1 + T_1 = U_2 + T_2$$

$$U_1 = mgr\cos\varphi_0 \qquad T_1 = 0$$

$$U_2 = mgr\cos\varphi \qquad T_2 = \frac{1}{2}mv^2(\varphi)$$

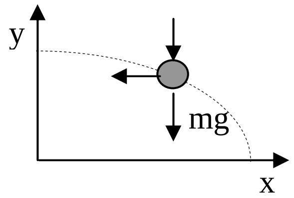
$$mgr\cos\varphi_0 = \frac{1}{2}mv^2(\varphi) + mgr\cos\varphi$$

$$v(\varphi) = \sqrt{2gr(\cos\varphi_0 - \cos\varphi)}$$

$$v(\pi) = \sqrt{2gr(\cos\varphi_0 + 1)} = 6{,}05\frac{m}{s}$$

$$F_S(\varphi) = \frac{m}{r}v^2(\varphi) - mg\cos\varphi = mg(2\cos\varphi_0 - 3\cos\varphi) \qquad F_S(\pi) = mg\left(\sqrt{3} + 3\right) = 928{,}4\,N$$

Lösung 4.10



Energiebilanz im Rohr: $\frac{1}{2}cl^2 = \frac{1}{2}mv_0^2 \quad \Rightarrow \quad v_0 = l\sqrt{\frac{c}{m}}$

$$\ddot{x} = 0 \qquad \dot{x} = C_1 \qquad x = C_1 t + C_2$$

$$\ddot{y} = -g \qquad \dot{y} = -gt + C_3 \qquad y = -\frac{1}{2}gt^2 + C_3 t + C_4$$

$$AB: t = 0 \quad x = 0 \;\Rightarrow\; C_2 = 0 \quad \dot{x} = v_0 \;\Rightarrow\; C_1 = v_0$$

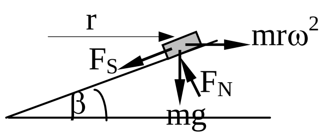
$$y = h \;\Rightarrow\; C_4 = h \quad \dot{y} = 0 \;\Rightarrow\; C_3 = 0$$

EB: $t = t_E \quad x = w \quad y = 0 \quad \Rightarrow \quad w = v_0 t_E \quad t_E = \frac{w}{v_0} \quad \frac{1}{2}g\left(\frac{w}{v_0}\right)^2 - h = 0 \quad l = \sqrt{\frac{w^2 mg}{2ch}}$

Zahlenwerte: l = 9,9 cm; v_0 = 8,09 m/s

Lösung 4.13

Bewegung auf der schiefen Ebene: $mgl\sin\alpha = \frac{1}{2}mv_1^2 \quad \Rightarrow \quad v_1 = \sqrt{2gl\sin\alpha}$



$$\dot{\varphi} = \omega = \frac{v_1}{r}$$

$$\rightarrow: \quad mr\omega^2 - F_S\cos\beta - F_N\sin\beta = 0$$

$$\downarrow: \quad mg + F_S\sin\beta - F_N\cos\beta = 0 \qquad F_N = \frac{mg + F_S\sin\beta}{\cos\beta}$$

$$mr\omega^2 - F_S\cos\beta - \frac{mg + F_S\sin\beta}{\cos\beta}\cdot\sin\beta = 0$$

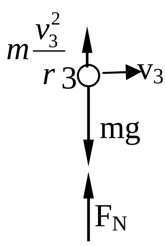
$$F_S = 0 \quad \textit{für} \quad \beta = \beta^* \qquad \tan\beta^* = \frac{r\omega^2}{g} \quad \textit{Überhöhung}: h = w\sin\beta^* = w\frac{\tan\beta^*}{\sqrt{1+\tan^2\beta^*}}$$

$$h = \frac{w}{\sqrt{1+\left(\frac{r}{2l\sin\alpha}\right)^2}} = 20{,}6\,cm$$

Lösung 4.14

$$T_1 + U_1 = T_2 + U_2 \quad \Rightarrow \quad mgh_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 \qquad v_2 = \sqrt{2gh_1 + v_1^2}$$

$$T_1 + U_1 = T_3 + U_3 \quad \Rightarrow \quad mgh_1 + \frac{1}{2}mv_1^2 = mgh_3 + \frac{1}{2}mv_3^2 \qquad v_3 = \sqrt{2g(h_1 - h_3) + v_1^2}$$



$$\uparrow: \quad F_N - mg + m\frac{v_3^2}{r} = 0 \qquad F_N = mg - m\frac{v_3^2}{r}$$

$$F_N = 0 \quad \textit{für} \quad r = r^* \quad \Rightarrow \quad r^* = \frac{v_3^2}{g}$$

$$r > r^* = 2(h_1 - h_3) + \frac{v_1^2}{g} \qquad v_2 = 6{,}95\frac{m}{s} \quad r > 2{,}917m$$